Anomalous $U(1)$ Mediation in Large Volume Compactification

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Abstract

We study the general effects of anomalous $U(1)_A$ gauge symmetry on soft supersymmetry (SUSY) breaking terms in large volume scenario, where the MSSM sector is localized on a small cycle whose volume is stabilized by the D-term potential of the $U(1)_A$. Since it obtains SUSY breaking mass regardless of the detailed form of Kähler potential, the $U(1)_A$ vector superfield acts as a messenger mediating the SUSY breaking in the moduli sector to the MSSM sector. Then, through the loops of $U(1)_A$ vector superfield, there arise soft masses of the order of $m_{3/2}^2/8\pi^2$ for scalar mass squares, $m_{3/2}/(8\pi^2)^2$ for gaugino masses, and $m_{3/2}/8\pi^2$ for A-paramteres. In addition, the massive $U(1)_A$ vector superfield can have non-zero F and D-components through the moduli mixing in the Kähler potential, and this can result in larger soft masses depending upon the details of the moduli mixing. For instance, in the presence of one-loop induced moduli mixing between the visible sector modulus and the large volume modulus, the $U(1)_A$ D-term provides soft scalar mass squares of the order of $m_{3/2}^2$. However, if the visible sector modulus is mixed only with small cycle moduli, its effect on soft terms depends on how to stabilize the small cycle moduli.

I. INTRODUCTION

Moduli stabilization is one of the key steps to understand low energy phenomenology of string theory. So far, several scenarios are suggested and their phenomenological and cosmological consequences are studied extensively. In particular, within the Type IIB theory, fluxes and non-perturbative corrections to the superpotential are considered as crucial ingredients for fixing moduli [1, 2]. Based on this idea, two types of scenario are particularly well-studied. The first is the KKLT type scenario [3] in which the Kähler moduli are stabilized at a supersymmetric AdS minimum by non-perturbative correction to the superpotential and the vacuum is uplifted to de-Sitter spacetime by additional SUSY breaking effect such as an anti-brane. On the other hand, if the overall volume modulus is taken to have a large vacuum value, the non-perturbative superpotential of the large volume modulus will be negligible. Still the large volume modulus can be stabilized at SUSY breaking minimum if there exists a small cycle modulus which admits non-perturbative superpotential and has a correct sign of α' correction to its Kähler potential. This is the so-called Large Volume Scenario (LVS) [4] which we will focus on in this paper.

For both the KKLT and LVS scenarios, in realistic situation, the number of independent and sizable non-perturbative terms in the superpotential might not be enough to stabilize all Kähler moduli. As pointed out in [5], when the non-perturbative superpotential of the visible sector Kähler modulus T_v is generated by E3 instantons, it must be equipped with the standard model (SM) charged matter superfields. Because the vacuum values of the SM charged matter fields should be zero or at most weak scale, the effect of such non-perturbative superpotential on fixing the visible sector modulus must be negligible. One natural solution for fixing T_v in such case is D-term stabilization. If there exists an anomalous $U(1)$ symmetry under which T_v transforms nonlinearly, the corresponding D-term contains the moduli dependent FI-term which is proportional to $\partial_{T_v} K$. If the moduli space of the underlying string compactification admits a solution with vanishing FI-term, which is indeed the case for many of the Type IIB string compactifications, the D-term scalar potential fixes T_v near the point with vanishing FI-term.

Stabilizing moduli in the absence of proper non-perturbative superpotential is not just an issue of moduli stabilization, but directly related to the pattern of soft SUSY breaking parameters in the visible sector. For the KKLT type scenario, the soft terms in case with anomalous $U(1)$ have been studied in [6, 7]. Combining with the SUSY breaking effects of the original

KKLT type models [8–11], it has been noticed that various patterns of soft terms can be realized. On the other hand, for the LVS, the soft terms generated by D-term stabilization have been discussed recently in [12–14].

In [14], the structure of soft terms has been examined for a class of LVS in the presence of one-loop induced moduli mixing between the visible sector modulus and the large volume modulus [13]. It was shown that such moduli mixing induces a $U(1)_A$ D-term of the order of $m_{3/2}^2$, which would provide soft scalar masses of the order of the gravitino mass $m_{3/2}$, while the resulting gaugino masses and A-parameters are of the order of $m_{3/2}/8\pi^2$. Therefore, in such set up, the gravitino mass cannot be much larger than the (multi) TeV scale to realize weak scale SUSY. However, it is also noticed that the specific form of moduli mixing plays the crucial role to determine the size of soft masses. Such mixing-dependent soft terms can be classified as model-dependent contribution of the $U(1)_A$ mediation. Then, it is natural to ask if there exists any model-independent contribution of the $U(1)_A$ mediation, not depending on the detailed form of the moduli Kähler potential. If such contribution exists, it would provide the lower bound of the soft masses in generic LVS with anomalous $U(1)_A$.

The aim of this paper is to extend the previous analysis [14] to more general class of LVS. We first divide the soft terms into the model-independent and the model-dependent parts on the basis of how much they depend on the detailed form of moduli mixing in the Kähler potential. It is shown that the $U(1)_A$ vector supermultiplet gains SUSY-breaking mass splitting regardless of the moduli mixing, so the model-independent soft masses are generated as a result of the loop threshold of the massive $U(1)_A$ vector supermultiplet. The resulting soft scalar masses are of the order of $m_{3/2}/4\pi$, while gaugino masses are of the order of $m_{3/2}/(8\pi^2)^2$ and A-terms are of the order of $m_{3/2}/8\pi^2$. For the model-dependent contributions, as in [14], D-term contribution can appear due to the moduli mixing in the Kähler potential. In addition to the case studied in [14], we study the case that the visible sector Kähler modulus is mixed with other small cycle Kähler moduli at tree-level, and find that its contribution can dominate the soft terms or not, depending on how to stabilize the small cycle Kähler moduli. In any case, we find that the $U(1)_A$ mediated soft terms play an essential role to determine the spectrum of the MSSM soft terms for models with D-term stabilization in LVS.

This paper is organized as follows. In section (II), we review the work of [14], especially focus on the $U(1)_A$ contribution to soft scalar mass. In section(III), we show that there are other types of soft terms induced by $U(1)_A$, not only those discussed in [14]. Section (IV) is devoted to construct the effective action of the light degrees of freedom by integrating out the heavy $U(1)_A$ vector superfield, and calculate the MSSM soft terms discussed in (III) more concretely. Section (V) is the conclusion. Throughout the paper, we will limit ourselves to 4D effective SUGRA.

II. REVIEW OF D-TERM STABILIZATION WITH MODULI MIXING

Before moving to the central part of our argument, it is worth reviewing the previous work [14], in which we studied sparticle spectrum of large volume compactification with loop-induced moduli mixing.

In the large volume scenario (LVS), there are at least two types of Käher moduli superfields. One type is a large volume modulus T_b which determines the overall size of a compactification volume. Another type, T_s, describes the volume of a small 4-cycle which admits non-perturbative effects to the superpotential. Then, $\mathrm{Re}T_b$ can be stabilized at a large vacuum value due to the competition between α' corrections suppressed by the inverse compactification volume and the non-perturbative corrections which are exponentially suppressed. In the large volume limit, $\langle \mathrm{Re}T_b\rangle \gg 1$, the model is given by*

$$\begin{aligned} K &= -3\ln t_b + \frac{(t_s^{3/2}-\xi_{\alpha'})}{t_b^{3/2}} + \mathcal{O}(t_b^{-3}), \\ W &= W_0 + Ae^{-aT_s} \end{aligned} \tag{1}$$

for $t_I = T_I + T_I^*$ $(I = b, s)$. $\xi_{\alpha'}$ represents the leading order α' correction, W_0 is the flux induced constant superpotential, A and a are constants involved in the non-perturbative correction to the superpotential. In this model, the vacuum values of t_b and t_s are fixed as

$$at_s = 2\ln\frac{M_{\mathrm{Pl}}}{|m_{3/2}|} + \mathcal{O}(1), \quad t_s^{3/2} = \xi_{\alpha'}\left(1+\mathcal{O}\Big(\frac{1}{at_s}\Big)\right), \tag{2}$$

where the gravitino mass, $m_{3/2} = e^{K/2}W = W_0 t_b^{-3/2}(1+\mathcal{O}(t_b^{-3/2}))$. It is straightforward to find

$$\frac{F^{T_b}}{t_b} = m_{3/2}^*\Big(1+\mathcal{O}(t_b^{-3/2})\Big),$$

* In most of discussion, detailed dynamics of the string dilaton and complex structure moduli is not important, so we assume that they are stabilized at a supersymmetric solution by fluxes and regarded as fixed values [4, 12]. The effect of backreaction due to the Kähler moduli stabilization is also negligible. In the following, unless specified, we set the 4D Planck scale (in the Einstein frame) $M_{\mathrm{Pl}} = 1$.

$$\frac{F^{T_s}}{t_s} = \frac{m_{3/2}^*}{\ln|M_{\rm Pl}/m_{3/2}|}\left(\frac{3}{4}+\mathcal{O}\Big(\frac{1}{\ln|M_{\rm Pl}/m_{3/2}|}\Big)\right), \tag{3}$$

where the F-component of a generic chiral superfield Φ^I is defined by $F^I = -e^{K/2}K^{I\bar{J}}(D_J W)^*$. An interesting feature of the LVS is a large hierarchy among the mass scales such as

$$\frac{M_{\rm st}}{M_{\rm Pl}} \sim t_b^{-3/4}, \quad \frac{M_{\rm KK}}{M_{\rm Pl}} \sim t_b^{-1}, \quad \frac{M_{\rm wind}}{M_{\rm Pl}} \sim t_b^{-1/2}, \quad \frac{m_{3/2}}{M_{\rm Pl}} \sim |W_0| t_b^{-3/2}, \tag{4}$$

where $M_{\rm st}$ is the string scale, $M_{\rm KK}$ is the bulk KK scale, and $M_{\rm wind}$ is the winding scale.

In order to construct a phenomenologically viable model, we need to specify the visible sector. It is noticed in [5], however, that the MSSM cycle Kähler modulus T_v can not have a non-perturbative superpotential like T_s due to the chiral nature of MSSM matter. Besides, T_v cannot be identified as T_b, since the MSSM gauge couplings at high energy scale are inversely proportional to the vacuum value of the modulus. Therefore in the LVS, the visible sector modulus is not fixed by non-perturbative and α' corrections. In such a situation, D-term stabilization can be used to fix T_v. Elaborating further on the issue in the framework of 4D effective SUGRA, we introduce the anomalous $U(1)_A$ gauge symmetry and suitable gauge transformations as followings.

$$U(1)_A \ : \quad V_A \to V_A + \Lambda_A + \Lambda_A^*, \quad T_v \to T_v + 2\delta_{\rm GS}\Lambda_A, \quad \Phi_i \to e^{-2q_i\Lambda_A}\Phi_i, \tag{5}$$

where V_A is the vector superfield which contains the $U(1)_A$ gauge boson, Λ_A is a chiral superfield parameterizing the $U(1)_A$ transformation on $N=1$ superspace, T_v is the visible sector Kähler modulus chiral superfield which transforms nonlinearly under the $U(1)_A$, $\delta_{\rm GS}$ denotes the constant associated with Green-Schwarz (GS) anomaly cancelation [15], and finally Φ_i stands for generic chiral matter superfields localized on the visible sector 4-cycle with $U(1)_A$ charge q_i. Since $\delta_{\rm GS}$ is determined by GS anomaly cancellation condition which is evaluated at one-loop level, generically

$$\delta_{\rm GS} = \mathcal{O}\Big(\frac{1}{8\pi^2}\Big). \tag{6}$$

Once taking into account the above symmetry, we can write down the gauge invariant Kähler potential and superpotential, including the visible sector fields, proposed by

$$\begin{aligned} K &= -3\ln t_b + \frac{(t_s - \alpha_s \ln t_b)^{3/2} - \xi_{\alpha'}}{t_b^{3/2}} + \mathcal{O}(t_b^{-3}) + \frac{(t_A - \alpha_A \ln t_b)^2}{2t_b^p} + Z_i \Phi_i^* e^{2q_i V_A}\Phi_i + \mathcal{O}(\Phi_i^4), \\ W &= W_0 + Ae^{-aT_s} + \mathcal{O}(\Phi_i^3), \end{aligned} \tag{7}$$

where $t_I = T_I + T_I^*$ $(I = b, s, v)$, and $t_A = t_v - 2\delta_{\rm GS} V_A$ is the gauge invariant combination of the visible sector modulus, p is the modular weight which determines the $U(1)_A$ gauge boson mass scale (8), and α_A (α_s) is the moduli mixing parameter between t_v (t_s) and t_b. Several assumptions were made regarding the $U(1)_A$ sector. First, the Kähler potential allows the limit of vanishing FI-term, $\partial_{T_v} K = 0$. Second, there are radiative corrections to the Kähler potential at one-loop level, which induce the moduli mixing between the visible sector modulus and the large volume modulus, i.e. $t_A \to t_A - \alpha_A \ln t_b$. Then the mixing parameter, $\alpha_A = \mathcal{O}(1/8\pi^2)$.

The first assumption plays a key role in achieving the D-term stabilization of T_v. The idea behind the D-term stabilization is that T_v becomes a part of the massive $U(1)_A$ vector superfield [14]. The imaginary part of the scalar component of T_v is eaten by the $U(1)_A$ gauge boson, gaining a mass through the Stückelberg mechanism. The real part obtains the same mass from the D-term potential. The $U(1)_A$ gaugino and the fermionic component of T_v constitute a Dirac spinor with the same mass as the bosons. In the supersymmetric limit, the mass squared of the $U(1)_A$ vector supermultiplet is given by

$$M_A^2 = \left\langle \frac{g_A^2}{2} \frac{\partial^2 K}{\partial V_A^2} \right\rangle \simeq \left\langle 2 g_A^2 \delta_{\rm GS}^2 \frac{\partial^2 K}{\partial T_v \partial T_v^*} \right\rangle = \left\langle \frac{2 g_A^2 \delta_{\rm GS}^2}{t_b^p} \right\rangle, \tag{8}$$

where g_A is the $U(1)_A$ gauge coupling. As [12], if p is 3/2,

$$M_A \sim M_{\rm st}/8\pi^2 \gg m_{3/2}. \tag{9}$$

Therefore we expect that the $U(1)_A$ vector supermultiplet is much heavier than the remaining Kähler moduli and matters, which indicates that the massive $U(1)_A$ vector superfield is fixed mostly by following superfield equations of motion,

$$\frac{\partial K}{\partial V_A} \simeq -2\delta_{\rm GS} \partial_{T_v} K \simeq 0. \tag{10}$$

We can find the solution of (10) provided by the first assumption. Then, T_v is stabilized near the point with vanishing FI-term.

The second assumption turns out to be important in determining the pattern of soft terms. In order to account for the effect on the soft terms, consider the soft scalar mass squared of Φ_i given by

$$m_i^2 \simeq \frac{2}{3} \langle V_F + \sigma V_{\rm uplift} \rangle - F^{T_I} F^{T_J^*} \partial_{T_I} \partial_{T_J^*} \ln e^{-K/3} Z_i - q_i g_A^2 D_A \quad (I = b, s, v) \tag{11}$$

at tree-level of 4D effective SUGRA. $V_F = e^K \left(K^{I\bar{J}} D_I W (D_J W)^* - 3|W|^2 \right)$ is the F-term scalar potential determined by (7), and $V_{\rm uplift}$ is an additional uplifting potential needed to achieve a phenomenologically viable de-Sitter vacuum $V_0 \approx \langle V_F + V_{\rm uplift}\rangle \simeq 0$. $g_A^2 D_A$ is the auxiliary D-component of the $U(1)_A$ gauge superfield V_A. The constant σ depends on the origin of the uplifting potential. If $V_{\rm uplift}$ originates from an anti-brane (or any SUSY breaking branes) stabilized at the tip of warped throat, then $\sigma = 1$. As a result, the vacuum energy contributions are almost cancelled and the remaining contributions are much suppressed compared to $\langle V_F\rangle$. On the other hand, if $V_{\rm uplfit}$ is made by F-term uplifting, σ is $3/2$. In such cases, the first term in the RHS of (11) is of the order

$$-\frac{1}{3}\langle V_F\rangle = \mathcal{O}\left(\frac{m_{3/2}^2 t_b^{-3/2}}{\ln(M_{\rm Pl}/m_{3/2})}\right). \tag{12}$$

To evaluate the second term (modulus-mediated contribution) and third term (D-term contribution) in the RHS of (11), additional terms should be specified. The matter Kähler metric Z_i is given by

$$Z_i = \frac{\mathcal{Y}_i((t_A - \beta_A \ln t_b))}{t_b}\left(1 + \mathcal{O}\left(t_b^{-3/2}\right)\right). \tag{13}$$

Here $\mathcal{Y}_i((t_A - \beta_A \ln t_b))$ is assumed to be expanded about $(t_A - \beta_A \ln t_b) = 0$ in positive powers of $(t_A - \beta_A \ln t_b)$ to allow the vanishing limit of $(t_A - \beta_A \ln t_b)$ as [12]

$$\mathcal{Y}_i((t_A - \beta_A \ln t_b)) = \mathcal{Y}_i(0) + \mathcal{Y}_i^{(n)}(0)(t_A - \beta_A \ln t_b)^n + \mathcal{Y}_i^{(n+1)}(0)(t_A - \beta_A \ln t_b)^{n+1} + \cdots, \tag{14}$$

where n is the positive integer, and $\mathcal{Y}_i(0),\ \mathcal{Y}_i^{(n)}(0),\ \mathcal{Y}^{(n+1)}(0), \cdots$ are constants of order one. Since the matter fields in visible sector are localized on the small 4-cycle, t_b-dependence of Z_i can be understood by the argument that the physical Yukawa couplings should not have power-dependence on the bulk compactification volume. Logarithmic dependence, however, is allowed at the one-loop level. Hence the moduli mixing parameter β_A is also of the order of $1/8\pi^2$. The auxiliary components, F^{T_v} and $g_A^2 D_A$, are determined dominantly by the superfield equations of motion (10),

$$\partial_{T_v} K \simeq \frac{(t_v - 2\delta_{\rm GS} V_A) - \alpha_A \ln t_b}{t_b^p} \simeq 0. \tag{15}$$

In the Wess-Zumino gauge, the component fields are given by

$$\begin{aligned}
t_v &\simeq \alpha_A \ln t_b && (0 \text{ component}),\\
F^{T_v} &\simeq \alpha_A \frac{F^{T_b}}{t_b} = \alpha_A m_{3/2}^* && (F \text{ component}),\\
g_A^2 D_A &\simeq \frac{\alpha_A}{\delta_{\rm GS}}\left|\frac{F^{T_b}}{t_b}\right|^2 = \frac{\alpha_A}{\delta_{\rm GS}}|m_{3/2}|^2 && (D \text{ component}).
\end{aligned} \tag{16}$$

As stated above, both $\delta_{\rm GS}$ and α_A are generated at the one-loop level, and hence

$$\delta_{\rm GS} \sim \alpha_A = \mathcal{O}\Big(\frac{1}{8\pi^2}\Big) \;\to\; F^{T_v} = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big), \quad g_A^2 D_A = \mathcal{O}(m_{3/2}^2). \tag{17}$$

It is straightforward to estimate the order of the scalar masses using the auxiliary components provided by (3), (16) and (17). We identify that the modulus-mediated contribution is much smaller than $m_{3/2}^2$:

$$\Big| - F^{T_I}F^{T_J^*}\partial_{T_I}\partial_{T_J^*}\ln e^{-K/3}Z_i\Big| \lesssim \mathcal{O}\Big((\beta_A-\alpha_A)\left|\frac{F^{T_b}}{t_b}\right|^2\Big) = \mathcal{O}\Big(\frac{m_{3/2}^2}{8\pi^2}\Big). \tag{18}$$

As a result, the soft scalar mass is dominated by the D-term contribution as

$$m_i^2 \simeq -\frac{q_i\alpha_A}{\delta_{\rm GS}}m_{3/2}^2 = \mathcal{O}(m_{3/2}^2), \tag{19}$$

and this is one of the main result of our previous work [14].

We can interpret the result (19) from the view of effective theory constructed by integrating out T_v and V_A. In the effective theory, the $U(1)_A$ gauge symmetry does not exist anymore, hence no D-term contribution as well. The effect of D-term, however, is transformed to that of modulus mediation, which originates from the effective Kähler metric of light matter fields $\tilde{\Phi}_i = e^{q_iT_v/\delta_{\rm GS}}\Phi_i$,

$$Z_i^{\rm eff} = \frac{\mathcal{Y}_i((\alpha_A-\beta_A)\ln t_b)}{t_b^{1+q_i\alpha_A/\delta_{\rm GS}}}\Big(1+O(t_b^{-3/2})\Big)\,. \tag{20}$$

The soft scalar mass squared is obtained as

$$m_i^2 \simeq -F^{T_b}F^{T_b^*}\partial_{T_b}\partial_{T_b^*}\ln e^{-K/3}Z_i^{\rm eff} \simeq -\frac{q_i\alpha_A}{\delta_{\rm GS}}m_{3/2}^2. \tag{21}$$

III. GENERIC FEATURES OF $U(1)_A$ MEDIATION

A. Model-independent contribtuion

To make brief summary on the previous section, when the visible sector Kähler modulus is stabilized near the point with vanishing FI-term, the SUSY breaking of the large volume modulus can be transmitted to the MSSM sector by the one-loop induced moduli mixing. Its effect appears as the D-term contribution, dominating soft scalar masses. The vanishing limit of FI-term is generic in string compactification [12, 16–18], but the form of moduli mixing among the visible sector modulus and other Kähler moduli is rather model dependent. That is to say that we need to figure out a model independent, i.e. moduli mixing independent, soft term contribution of the $U(1)_A$. In order to do so, let us suppose that there is no moduli mixing, $\alpha_A = \beta_A = 0$. At first sight, the scalar mass squared seems to be much suppressed compared to the gravitino mass squared, since from (20)

$$\begin{aligned} m_i^2 &\sim -F^{T_I} F^{T_J^*} \partial_{T_I} \partial_{T_J^*} \ln e^{-K/3} Z_i^{\rm eff} \\ &= -F^{T_I} F^{T_J^*} \partial_{T_I} \partial_{T_J^*} \ln \mathcal{Y}_i(0) \Big(1 + \mathcal{O}(t_b^{-3/2})\Big) \leq \mathcal{O}\Big(m_{3/2}^2 t_b^{-3/2}\Big), \end{aligned} \tag{22}$$

where $I = (b, s)^\dagger$. We claim that, however, there is model-independent one-loop corrections to the effective Kähler metric of light matter fields to yield

$$Z_i^{\rm eff} = Z_{i({\rm tree})}^{\rm eff} \Big(1 - \epsilon_{Ai} \ln t_b\Big), \tag{23}$$

where $Z_{i({\rm tree})}^{\rm eff}$ is the tree-level effective Kähler metric, given by (20), and ϵ_{Ai} is the constant of $\mathcal{O}(1/8\pi^2)$. Accordingly, the soft scalar mass is not dominated by (22) but modified as follows.

$$\begin{aligned} m_i^2 &\simeq -F^{T_I} F^{T_J^*} \partial_{T_I} \partial_{T_J^*} \ln e^{-K/3} Z_i^{\rm eff} \\ &\simeq -F^{T_b} F^{T_b^*} \partial_{T_b} \partial_{T_b^*} \ln \Big(1 - \epsilon_{Ai} \ln t_b\Big) \simeq -\epsilon_{Ai} m_{3/2}^2 = \mathcal{O}\Big(\frac{m_{3/2}^2}{8\pi^2}\Big). \end{aligned} \tag{24}$$

Note that the values of α_A, β_A are given by string-loop corrections, so they are not calculable in 4D effective SUGRA. On the other hand, ϵ_{Ai} can be computed from the $U(1)_A$ vector supermultiplet threshold at the level of effective field theory. As mentioned in section (II) the massive $U(1)_A$ vector superfield, referred to V_H, obtains the mass of (8) in the SUSY limit. In reality, V_H touchs on the SUSY breaking superfield T_b by means of mass interaction in the Kähler potential, so there is small mass splitting among component fields of V_H. Then, V_H plays the role of a messenger superfield in the visible sector. The MSSM sparticles are communicated to the large volume modulus T_b through the loops of V_H, and have soft masses as (24).

To be more specific, let us fix the modular weight p. In fact the D-term mediated soft scalar masses are not affected by p, and that's why we did not care much about that in the previous work [14]. However, the mass spectrum of the $U(1)_A$ vector supermultiplet is highly dependent on the value of p, hence ϵ_{Ai} and induced soft terms are also influenced by p. If the visible sector 4-cycle is stabilized at a geometric regime, it is natural to fix p at $3/2$ such like the Kähler

† In this case, we must add soft term contributions from the string dilaton and uplifting potential, but it turned out that their corrections are less than or similar to the value given by (22) [12].

potential of T_s in (1). In a singular cycle regime, we might lose the analogy to T_s, but it is quite plausible that the analogy is still valid even in that case. So, we set

$$p = 3/2. \tag{25}$$

As we will see in next section, the corresponding soft scalar mass squared (24) is given by

$$\Delta_{\rm M.I.} m_i^2 = -\frac{g_A^2 q_i^2}{16\pi^2}|m_{3/2}|^2. \tag{26}$$

There are also such contributions for gaugino masses and A-parameters. We call these soft term contributions "model-independent contributions" of the $U(1)_A$, in the sense that they are independent of the specific form of the moduli Kähler potential.

Implication of the model-independent contributions is that (26) provides the lower bound of soft scalar masses, $|m_i| \gtrsim m_{3/2}/4\pi$, (unless the model-independent contribution is canceled by the additional model-dependent string-loop correction), so that the gravitino mass should not exceed more that the scale of (multi) TeV if the weak scale SUSY is realized in nature. Further, since the mass squared of (26) is negative for any nonzero $U(1)_A$ charge assignment, (26) should not dominantly contribute to the MSSM squark and slepton masses.

B. Model-dependent contribution

As being noted above, the model-independent soft terms of the $U(1)_A$ are potentially problematic. However, in [14], we have already argue that the D-term contribution induced by moduli mixing can dominate over the model-independent contributions. Such a non-trivial D-term is originated from the non-trivial Kähler potential of T_v. Based on the viewpoint of moduli mixing between the visible sector modulus and the other SUSY breaking moduli, the visible sector modulus can be mixed not only with the large volume modulus at the one-loop level, but also with small Kähler moduli at the tree-level. Such model-dependence can be accommodated by generalizing the model (7) as follows.

$$\frac{(t_s - \alpha_s \ln t_b)^{3/2} - \xi_{\alpha'}}{t_b^{3/2}} + \frac{(t_A - \alpha_A \ln t_b)^2}{2t_b^{3/2}} \;\to\; \frac{\Delta K(t_{s1}, \cdots t_{sn_s}, t_A, \alpha_A \ln t_b) - \xi_{\alpha'}}{t_b^{3/2}},$$

$$W_0 + Ae^{-aT_s} \;\to\; W_0 + \sum_{j=1}^{n_w} A_j e^{-a_j T_{sj}}, \tag{27}$$

where $t_{sj} = T_{sj} + T_{sj}^*$ $(j = 1, \cdots, n_s)$, n_s is the number of small Kähler moduli, and n_w moduli of them have non-perturbative terms in the superpotential. Even though a number of Kähler moduli are allowed in (27), all the $U(1)_A$ neutral Kähler moduli would be stabilized by the SUSY breaking effect so their masses will be around $m_{3/2}$ which is much smaller than the mass of V_H. Therefore we still make use of the superfield equation (10) to evaluate F-term of T_v and D-term of V_A as functions of the light moduli F-terms. Then,

$$\begin{aligned} F^{T_v} &= -e^{K/2}K^{I\bar{J}}(D_JW)^* \simeq \sum_{I=b,s1,\cdots sn_w} -\left(\frac{\partial_{T_I}\partial_{T_v^*}\Delta K}{\partial_{T_v}\partial_{T_v^*}\Delta K}\right)F^{T_I}, \\ g_A^2 D_A &= -g_A^2\eta^I K_I \simeq \sum_{I,J=b,s1,\cdots sn_w,v} \frac{1}{\delta_{\rm GS}}\left(\frac{\partial_{T_I}\partial_{T_J^*}\partial_{T_v}\Delta K}{\partial_{T_v}\partial_{T_v^*}\Delta K}\right)F^{T_I}F^{T_J^*}, \end{aligned} \tag{28}$$

where $\eta^I = \{\delta_{\rm GS}, -q_i\Phi_i\}$ for $\{T_v, \Phi_i\}$, $\eta^I = 0$ for $T_b,\ T_{s1}, \cdots, T_{sn_w}$. The modulus and D-term mediated soft scalar masses are determined by (11) and (28) after stabilizing the light moduli. Depending on what types of moduli are mixed, the order of each contribution will be different and can be compared with the model-independent contribution.[‡] Notice that for the D-term contribution, there is the enhancement factor $1/\delta_{\rm GS}$ of $\mathcal{O}(8\pi^2)$, so there might be interesting contributions to the soft terms. We have attempted to estimate (28), and its effect on the soft scalar masses for generic moduli mixing by assuming that $\Delta K \sim t_{sj} \sim t_v = \mathcal{O}(1)$, $\partial_{T_I}\Delta K \sim \Delta K/t_I$ for $I = \{s1, \cdots, sn_w, v\}$, and $\partial_{T_b}\Delta K \sim \alpha_A \Delta K/t_b$. Then,

$$\begin{aligned} \frac{F^{T_v}}{t_v} &\sim \frac{F^{T_{sj}}}{t_{sj}}, \\ g_A^2 D_A &\sim \frac{\alpha_A}{\delta_{\rm GS}}\left|\frac{F^{T_b}}{t_b}\right|^2 + \left(\kappa_{bj}\frac{\alpha_A}{\delta_{\rm GS}}\frac{F^{T_b}}{t_b}\frac{F^{T_{sj}^*}}{t_{sj}} + {\rm h.c.}\right) + \frac{\kappa_j}{\delta_{\rm GS}}\left|\frac{F^{T_{sj}}}{t_{sj}}\right|^2, \end{aligned} \tag{29}$$

where κ_{bi}, κ_j are order one. If we take that $\mathcal{Y}_i = e^{-K/3}Z_i$ is also the generic function of t_{sj}, t_v, and $\alpha_A \ln t_b$,

$$-F^IF^{J^*}\partial_I\partial_{\bar{J}}\ln e^{-K/3}Z_i \sim \lambda_b\alpha_A\left|\frac{F^{T_b}}{t_b}\right|^2 + \lambda_{bj}\alpha_A\left(\frac{F^{T_b}}{t_b}\frac{F^{T_{sj}^*}}{t_{sj}} + {\rm h.c.}\right) + \lambda_j\left|\frac{F^{T_{sj}}}{t_{sj}}\right|^2, \tag{30}$$

where λ_b, λ_{bj}, and λ_j are order one. In this naive estimation, soft scalar masses seem to be dominated by the D-term contribution whether $\alpha_A = 0$ or not. However, this is not always true, since ΔK is not a generic function of small moduli. Let us consider a simple example suggested in [5]

$$\begin{aligned} \Delta K &= \Big(t_1 - \delta_{\rm GS}V_A\Big)^{3/2} + \sqrt{5}\Big(t_2 + \delta_{\rm GS}V_A\Big)^{3/2}, \\ W &= W_0 + Ae^{-a(T_1+T_2)}. \end{aligned} \tag{31}$$

[‡] It is noticed that the absolute SUSY breaking scale determined by $\parallel F^I \parallel^2 \approx |K_{I\bar{I}}F^IF^{I^*}|$ should be distinguished from the soft SUSY breaking mass scale determined by $\parallel \Delta_I m_i^2 \parallel \approx |F^IF^{I^*}\partial_I\partial_{\bar{I}}\ln(e^{-K/3}Z_i)|$.

where $t_I = T_I + T_I^*$ ($I = 1, 2$) are small cycle moduli, charged under the $U(1)_A$. We change the basis of small moduli into the $U(1)_A$ neutral modulus $T_s = T_1 + T_2$, and the so-called visible sector modulus $T_v = T_1 - T_2$. In this basis, (31) is rewritten as

$$\begin{aligned} \Delta K &= \frac{1}{2\sqrt{2}}\Big(t_s + t_A\Big)^{3/2} + \frac{\sqrt{5}}{2\sqrt{2}}\Big(t_s - t_A\Big)^{3/2}, \\ W &= W_0 + Ae^{-aT_s}. \end{aligned} \tag{32}$$

It is noticed that in the Kähler potential of (32), there is no moduli mixing between T_v and T_b, but nontrivial mixing between T_v and T_s exists at the tree-level. From (10) and (28), we have

$$t_v \simeq 2t_s/3, \quad \frac{F^{T_v}}{t_v} \simeq \frac{F^{T_s}}{t_s} = \mathcal{O}\Big(\frac{m_{3/2}}{\ln|M_{\rm Pl}/m_{3/2}|}\Big) = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big), \quad g_A^2 D_A \simeq 0. \tag{33}$$

The value of F^{T_s}/t_s is given by (3). The $U(1)_A$ D-term, induced by moduli mixing, is rather suppressed. Consequently, the model-dependent soft scalar mass squared (11) is estimated as

$$\Delta_{\rm M.D.} m_i^2 \simeq -|F^{T_v}|^2 \partial_{T_v}\partial_{T_v^*} \ln \mathcal{Y}_i(t_A) = \mathcal{O}\left(\left|F^{T_v}/t_v\right|^2\right) = \mathcal{O}\Big(\frac{m_{3/2}^2}{(8\pi^2)^2}\Big). \tag{34}$$

Proceeding from what has been said above, it should be concluded that the model-independent contributions (26) still dominate soft scalar masses, even though the non-trivial moduli mixing exists. In (IV B), it is shown that the patter of (34) is generic in case that all small moduli are stabilized by non-perturbative superpotential ($n_s = n_w$), and there is no one-loop induced moduli mixing between T_v and T_b ($\alpha_A = \beta_A = 0$). Thus it points out that there should be additional soft term contribution from the matter sector (e.g. gauge mediation which is not covered in this paper), dominating soft scalar masses.

IV. SOFT SUSY BREAKING TERMS IN D-TERM STABILIZATION

Up to now, we have discussed possible types of soft SUSY breaking terms through the $U(1)_A$ mediation. In what follows, we will provide more concrete formulae of the soft terms discussed in section (III). Because the stabilization procedure of light fields, and induced soft term contributions are rather clearly described by effective theory, we will construct the effective action by integrating out the massive $U(1)_A$ vector supermultiplet. After that, the soft terms will be analyzed in details.

Begining from the generalized action discussed in (III B), there are $n_s + 2$ Kähler moduli. Among them, one modulus T_b has the large vacuum value, and the rest of $n_s + 1$ moduli remain small. The small moduli are classified into one visible sector modulus T_v charged under the $U(1)_A$, n_w moduli $T_{s1}, \cdots, T_{sn_w}$ which have non-perturbative superpotential, and $n_s - n_w$ moduli $T_{sn_w+1}, \cdots, T_{sn_s}$ which do not have non-perturbative terms in the superpotential. The visible sector matter fields Φ_i are localized on a small 4-cycle whose volume is described by T_v. The holomorphic gauge kinetic functions of the $U(1)_A$ and the MSSM gauge groups are referred to f_A and f_a respectively. Then, the Kähler potential, superpotential and gauge kinetic functions are given by

$$\begin{aligned} K &= -3\ln t_b + \frac{\Delta K(\vec{t}_s, t_A, \alpha_A \ln t_b) - \xi_{\alpha'}}{t_b^{3/2}} + \mathcal{O}(t_b^{-3}) + Z_i \Phi_i^* e^{2q_i V_A} \Phi_i + \mathcal{O}(\Phi_i^4), \\ W &= W_0 + \sum_{j=1}^{n_w} A_j e^{-a_j T_{sj}} + \frac{1}{3!}\lambda_{ijk}(\vec{T}_s)\Phi_i\Phi_j\Phi_k + \mathcal{O}(\Phi_i^4), \\ f_A &= k_A T_v + \gamma_A(\vec{T}_s), \quad f_a = k_a T_v + \gamma_a(\vec{T}_s) \end{aligned} \tag{35}$$

where

$$\begin{aligned} t_I &= T_I + T_I^* \qquad \text{for} \quad I = b, s1, \cdots, sn_s, v, \\ \vec{g} &= g_1, \cdots, g_{n_s} \qquad \text{for} \quad g = t_s, T_s, \\ Z_i &= Z_i(\vec{t}_s, t_A, t_b) = \frac{\mathcal{Y}_i(\vec{t}_s, t_A, \beta_A \ln t_b)}{t_b}\left(1 + \mathcal{O}(t_b^{-3/2})\right), \end{aligned} \tag{36}$$

and k_A, k_a are fixed by GS anomaly cancellation conditions,

$$\frac{1}{4\pi^2}\sum_i q_i^3 = k_A \delta_{\rm GS}, \quad \frac{1}{4\pi^2}\sum_i q_i {\rm Tr}(T_a^2(\Phi_i)) = k_a \delta_{\rm GS}, \tag{37}$$

where $\delta_{\rm GS}$ is encoded in the gauge invariant combination $t_A = t_v - 2\delta_{\rm GS} V_A$. We follow the normalization convention of [14], so that the orders of each constant are given by

$$a_1, \cdots, a_{n_w} = \mathcal{O}(8\pi^2), \ \xi_{\alpha'} = \mathcal{O}(1), \ k_{A,a} = \mathcal{O}(1), \ \delta_{\rm GS} \sim \alpha_A \sim \beta_A = \mathcal{O}\Big(\frac{1}{8\pi^2}\Big). \tag{38}$$

Because we are taking bottom-up approach, we can not determine the moduli dependent functions $\gamma_a(\vec{T}_s)$, $\lambda_{ijk}(\vec{T}_s)$, and $\mathcal{Y}_i(\vec{t}_s, t_A, \beta_A \ln t_b)$ whose explicit forms are given by underlying string theory. Although their specific expressions are required to calculate the soft SUSY breaking terms, the order of their contributions can be estimated under the assumption that the functions depend on $\vec{T}_s$ in two ways. One way is that the visible sector cycle is sequestered from the cycles whose volumes are described by $\vec{T}_s$, that γ_a, λ_{ijk}, and $\mathcal{Y}_i$ are independent of $\vec{T}_s$. Another way is that those cycles are not sequestered from each other, so γ_a, λ_{ijk} and $\mathcal{Y}_i$ are of the same order of T_{sj} multiplied by the derivative of them with respect to T_{sj}.

A. Effective theory

Firstly, let us decompose the $U(1)_A$ vector superfield as $V_A = V_0 + V_H$, where V_H is the heavy vector superfield, and V_0 is the background superfield defined by a solution of the superfield equation of motion,

$$\left.\frac{\partial K}{\partial V_A}\right|_{V_A=V_0} = \mathcal{O}(D^2\bar{D}^2V_0). \tag{39}$$

Ignoring the part of the supercovariant derivatives, the Kähler potential is written as

$$K = K|_{V_A=V_0} + \frac{1}{2}\left.\frac{\partial^2 K}{\partial V_A^2}\right|_{V_A=V_0} V_H^2 + \mathcal{O}(V_H^3). \tag{40}$$

By integrating out V_H in a supersymmetric way, we get the tree-level Kähler potential, where V_0 is substituted, as well as the Coleman-Weinberg type Kähler potential at the one-loop level [19, 20]. Thus the effective Kähler potential is given by

$$K_{\text{eff}} = K|_{V_A=V_0} + \frac{\mathcal{M}_A^2}{16\pi^2}\text{Tr}\ln\frac{\mathcal{M}_A^2}{e\mathcal{M}_{\text{UV}}^2} + (\text{two-loops}), \tag{41}$$

where $e = 2.718...$ is Euler's number, $\mathcal{M}_A^2$ is the mass squared superfield for the $U(1)_A$ vector superfield

$$\mathcal{M}_A^2 = \frac{g_A^2}{2}\left.\frac{\partial^2 K}{\partial V_A^2}\right|_{V_A=V_0} \tag{42}$$

and $\mathcal{M}_{\text{UV}}^2$ is the cut-off superfield which will be specified later. To proceed further, let us define several superfields as functions of V_A,

$$\begin{aligned}
\xi_{\text{FI}}(V_A) &= \frac{\delta_{\text{GS}}\Delta K'(\vec{t}_s, t_A, \alpha_A \ln t_b)}{t_b^{3/2}}, \quad \mathcal{M}_{\text{mat}_D}^2(V_A) = q_i\hat{Z}_i\Phi_i^* e^{2q_iV_A}\Phi_i, \\
\mathcal{M}_{\text{GS}}^2(V_A) &= \frac{\delta_{\text{GS}}^2\Delta K''(\vec{t}_s, t_A, \alpha_A \ln t_b)}{t_b^{3/2}}, \quad \mathcal{M}_{\text{mat}}^2(V_A) = q_i^2\tilde{Z}_i\Phi_i^* e^{2q_iV_A}\Phi_i,
\end{aligned} \tag{43}$$

where $q_i\hat{Z}_i = q_iZ_i - \delta_{\text{GS}}Z_i'$, and $q_i^2\tilde{Z}_i = q_i^2Z_i - 2\delta_{\text{GS}}q_iZ_i' + \delta_{\text{GS}}^2Z_i''$. Here, the primed notation denotes the partial derivative with respect to t_A, i.e. $f' = \partial f/\partial t_A$, $f'' = \partial^2 f/\partial t_A^2$. The subscript 'mat$_D$' of $\mathcal{M}_{\text{mat}_D}^2$ stands for matter fields contribution to the D-term. For the model of (35), up to leading order in the volume expansion, (39) is equivalent to

$$\xi_{\text{FI}}(V_0) - \mathcal{M}_{\text{mat}_D}^2(V_0) = \mathcal{O}(D^2\bar{D}^2V_0). \tag{44}$$

And the mass squared superfield (42) is given by

$$\mathcal{M}_A^2 = 2g_A^2\Big(\mathcal{M}_{\text{GS}}^2(V_0) + \mathcal{M}_{\text{mat}}^2(V_0)\Big). \tag{45}$$

Comparing with (8), the expression (45) includes the contribution from the charged matter fields. In order to realize the D-term stabilization of T_v, such matter contribution should be small and treated perturbatively. Therefore we focus on the region :

$$M_{\rm GS}^2 \gg M_{\rm mat}^2, \tag{46}$$

where $M_{\rm GS}^2 = \langle \mathcal{M}_{\rm GS}^2 \rangle$ and $M_{\rm mat}^2 = \langle \mathcal{M}_{\rm mat}^2 \rangle$. (46) implies that the Stückelberg mechanism dominantly determines the $U(1)_A$ gauge boson mass, i.e. $M_A^2 = \langle \mathcal{M}_A^2 \rangle \simeq 2g_A^2 M_{\rm GS}^2$[§]. If there happens to be no cancellation among various terms inside $\mathcal{M}_{{\rm mat}_D}^2$, the orders of $\langle \mathcal{M}_{{\rm mat}_D}^2 \rangle$, $\langle \mathcal{M}_{\rm mat}^2 \rangle$ and $M_{\rm mat}^2$ will be the same. In this case, we can solve (44) perturbatively by decomposing V_0 into $v_0 + \epsilon$, where v_0 is the zeroth order vector superfield which is determined by the moduli sector :

$$\xi_{\rm FI}(v_0) = 0, \tag{47}$$

and ϵ is the small expansion parameter determined by v_0 as follows

$$\begin{aligned} \epsilon = V_0 - v_0 \; &= \; \frac{1}{2} \left(\frac{\xi_{\rm FI}(v_0) - \mathcal{M}_{{\rm mat}_D}^2(v_0)}{\mathcal{M}_{\rm GS}^2(v_0) + \mathcal{M}_{\rm mat}^2(v_0)} \right) \Big(1 + \mathcal{O}(\lesssim \epsilon) \Big) \\ &= \; -\frac{1}{2} \left(\frac{\mathcal{M}_{{\rm mat}_D}^2}{\mathcal{M}_{\rm GS}^2} \right) \left(1 + \mathcal{O}\left(\frac{\mathcal{M}_{\rm mat}^2}{\mathcal{M}_{\rm GS}^2} \right) \right). \end{aligned} \tag{48}$$

where we have omitted v_0 dependence in the last line for the simplicity. Then, up to the order of $\mathcal{M}_{\rm mat}^2/\mathcal{M}_{\rm GS}^2$, the background superfield V_0 can be expanded by

$$V_0 = \frac{1}{2} \left(\frac{t_v - t_A^0}{\delta_{\rm GS}} - \frac{\mathcal{M}_{{\rm mat}_D}^2}{\mathcal{M}_{\rm GS}^2} \right), \tag{49}$$

where $t_A^0 = t_A^0(\vec{t}_s, \alpha_A \ln t_b)$ is the solution of

$$\Delta K'(\vec{t}_s, t_A^0, \alpha_A \ln t_b) = 0. \tag{50}$$

We assume that the solution actually exists inside or on the boundary of Kähler cone. After integrating out V_H, the light degrees of freedom can be described in the $U(1)_A$ gauge invariant field basis. With the matter field redefinition $\Phi_i \to e^{-q_i T_v/\delta_{\rm GS}} \Phi_i$, the one-loop effective Kähler potential (41) is given by

$$\begin{aligned} K_{\rm eff} \; &= \; -3 \ln t_b + \frac{\Delta K_{\rm eff} - \xi_{\alpha'}}{t_b^{3/2}} + \mathcal{O}(1/t_b^3) \\ &+ Z_i^{\rm eff} \Phi_i^* \Phi_i - \frac{(q_i \hat{Z}_i^{\rm eff} \Phi_i^* \Phi_i)^2}{2\mathcal{M}_{\rm GS}^2} \left(1 + \mathcal{O}\left(\frac{g_A^2}{8\pi^2}, \frac{\mathcal{M}_{\rm mat}^2}{\mathcal{M}_{\rm GS}^2} \right) \right) + \mathcal{O}(\Phi_i^4). \end{aligned} \tag{51}$$

[§] Of course, we have to show that the matter fields are really stabilized far below $M_{\rm GS}$. However, this is rather model-dependent question involving details of the matter sector. Since we concentrate on the soft term contributions from the moduli sector, matter field stabilization will not be covered in this paper.

The one-loop correction gives

$$\Delta K_{\rm eff} \;=\; \Delta K(\vec{t}_s, t_A^0, \alpha_A \ln t_b) + \frac{g_A^2 \delta_{\rm GS}^2}{8\pi^2} \Delta K''(\vec{t}_s, t_A^0, \alpha_A \ln t_b) \ln \frac{2 g_A^2 \mathcal{M}_{\rm GS}^2}{e \mathcal{M}_{\rm UV}^2},$$

$$Z_i^{\rm eff} \;=\; e^{-q_i t_A^0/\delta_{\rm GS}} \left(Z_i(\vec{t}_s, t_A^0, t_b) + \frac{g_A^2 q_i^2 \tilde{Z}_i(\vec{t}_s, t_A^0, t_b)}{8\pi^2} \ln \frac{2 g_A^2 \mathcal{M}_{\rm GS}^2}{\mathcal{M}_{\rm UV}^2} \right), \qquad (52)$$

where

$$q_i \hat{Z}_i^{\rm eff} = e^{-q_i t_A^0/\delta_{\rm GS}} \Big(q_i Z_i(\vec{t}_s, t_A^0, t_b) - \delta_{\rm GS} Z_i{}'(\vec{t}_s, t_A^0, t_b) \Big),$$

$$q_i^2 \tilde{Z}_i(\vec{t}_s, t_A^0, t_b) = q_i^2 Z_i(\vec{t}_s, t_A^0, t_b) - 2\delta_{\rm GS} q_i Z_i{}'(\vec{t}_s, t_A^0, t_b) + \delta_{\rm GS}^2 Z_i{}''(\vec{t}_s, t_A^0, t_b). \qquad (53)$$

The effective superpotential and gauge kinetic functions are

$$W_{\rm eff} \;=\; W_0 + \sum_{j=1}^{n_w} A_j e^{-a_j T_{sj}} + \frac{1}{3!} \lambda_{ijk}(\vec{T}_s) \Phi_i \Phi_j \Phi_k + \mathcal{O}(\Phi_i^4),$$

$$f_a^{\rm eff} \;=\; \gamma_a(\vec{T}_s), \qquad (54)$$

where $f_a^{\rm eff}$ is obtained by adding the anomalous pieces generated from the matter fields redefinition.

Let us illustrate the form of $Z_i^{\rm eff}$ in more detail in order to clarify the model-independent contribution. We consider the model of (7). ΔK and Z_i are given by

$$\Delta K(\vec{t}_s, t_A, \alpha_A \ln t_b) \;=\; (t_s - \alpha_s \ln t_b)^{3/2} + (t_A - \alpha_A \ln t_b)^2/2,$$

$$Z_i(t_s, t_A, t_b) \;=\; \frac{1}{t_b} \Big(\mathcal{Y}_i(0) + \mathcal{Y}_i^{(n)}(0)(t_A - \alpha_A \ln t_b)^n + \mathcal{O}\Big((t_A - \alpha_A \ln t_b)^{n+1} \Big) \Big), \qquad (55)$$

where n is the positive integer. In this example, we set $\alpha_A = \beta_A$ which implies that $\ln t_b$-dependence of the matter Kähler metric comes only from moduli redefinition. (43) and (50) yield $\mathcal{M}_{\rm GS}^2 = \delta_{\rm GS}^2 t_b^{-3/2}$ and $t_A^0 = \alpha_A \ln t_b$, respectively. As a result,

$$Z_i^{\rm eff} \;=\; \frac{\mathcal{Y}_i(0)}{t_b^{1+q_i \alpha_A/\delta_{\rm GS}}} \left(1 + \frac{g_A^2 q_i^2}{8\pi^2} \ln \frac{2 g_A^2 \delta_{\rm GS}^2}{\mathcal{M}_{\rm UV}^2 t_b^{3/2}} \right). \qquad (56)$$

The effect of moduli redefinition is encoded in the prefactor of the RHS of (56).

Now, we should specify the cut-off superfield $\mathcal{M}_{\rm UV}^2$ in order to determine the model-independent contribution. We might choose $\mathcal{M}_{\rm UV}^2$ so that $\langle \mathcal{M}_{\rm UV} \rangle \sim M_{\rm string}$. However, the cut-off scale as a "superfield" is rather subtle from the 4D effective field theory point of view. There is no reason to take $\mathcal{M}_{\rm UV}^2 \sim t_b^{-3/2}$. By performing component calculation in appendix (A), we find that $\mathcal{M}_{\rm UV}^2 \sim t_b^{-1}$ is correct choice regardless of moduli redefinition. In (A), it is identified that the tree-level mass splitting of the $U(1)_A$ vector supermultiplet is given by (A4), (A5). Through the vector supermultiplet loops, the matter sector soft terms are generated. In the superfield approach, this is equivalently related to the mismatch between $\mathcal{M}^2_{\rm GS}$ and $\mathcal{M}^2_{\rm UV}$, so that at the one-loop level the matter Kähler metric depends on the large volume modulus T_b as follows

$$Z_i^{\rm eff} = Z^{\rm eff}_{i({\rm tree})}\left(1+\frac{g_A^2 q_i^2}{8\pi^2}\ln\frac{\mathcal{M}^2_{\rm GS}}{\mathcal{M}^2_{\rm UV}}\right) \simeq Z^{\rm eff}_{i({\rm tree})}\left(1-\frac{g_A^2 q_i^2}{16\pi^2}\ln t_b\right). \tag{57}$$

where $Z^{\rm eff}_{i({\rm tree})} = \mathcal{Y}_i(0)t_b^{-(1+q_i\alpha_A/\delta_{\rm GS})}$ is given by (23) and (56). Induced soft terms are the same as those evaluated in the component Lagrangian.

We might infer the UV scale $\langle\mathcal{M}_{\rm UV}\rangle = M_{\rm UV}$ from a running gauge coupling constant. The Kaplunovsky-Louis formula for the physical gauge coupling [22] is given by

$$\frac{1}{g_a^2(\mu)} = {\rm Re}(f_a) + \frac{b_a}{16\pi^2}\ln\frac{e^{K/3}M_{\rm Pl}^2}{\mu^2} - \frac{{\rm Tr}(T_a^2(\Phi_i))}{8\pi^2}\ln e^{-K/3}Z_i(\mu) + \frac{{\rm Tr}(T_a^2(G))}{8\pi^2}\ln g_a^{-2}(\mu), \tag{58}$$

where $b_a = \sum_r {\rm Tr}(T_a^2(\Phi_i)) - {\rm Tr}(T_a(G))$. The combination of $e^{-K/3}Z_i(\mu)\simeq\mathcal{Y}_i$ is nearly independent of t_b at leading order. Thus in the large volume limit, the effective UV scale at which the gauge couplings start to run is neither the string scale $M_{\rm st}\sim M_{\rm Pl}t_b^{-3/4}$ nor the Planck scale, but rather the winding scale $e^{K/6}M_{\rm Pl}\sim M_{\rm Pl}t_b^{-1/2}$ [23]. If the holomorphic gauge kinetic functions f_a are universal as a consequence of GUT, the phyiscal gauge couplings seem to be unified at this scale. On the one hand, when the Planck scale is introduced in the superpotential as the natural suppression scale of higher dimensional operators, the physical suppression scale is not the Planck scale, but scales to $e^{K/6}M_{\rm Pl}$ due to the canonical normalization of the matter fields. With these considerations, we naturally expect the "effective" cut-off of the visible sector is $M_{\rm UV} = e^{K/6}M_{\rm Pl}\simeq t_b^{-1/2}$, and the corresponding cut-off superfield,

$$\mathcal{M}^2_{\rm UV} = e^{K/3}M^2_{\rm Pl} = t_b^{-1}\left(1+\mathcal{O}(t_b^{-3/2})\right). \tag{59}$$

Again, we should address that the moduli-superfields dependence of $\mathcal{M}^2_{\rm UV}$ is not explicitly determined by the real cut-off scale of the effective SUGRA given by underlying string theory. In the language of component calculations, the scalar mass contribution from loops of the $U(1)_A$ vector supermultiplet is the threshold correction generated at the scale of the $U(1)_A$ vector boson mass. Therefore, the real cut-off of the theory does not play the crucial role to determine the value of model-independent scalar masses as long as the cut-off scale is sufficiently bigger than the scale of the $U(1)_A$ vector boson mass¶. Such UV-insensitivity of the correction is the same with that of gauge mediation, where the soft masses are generated at the scale of messenger mass and the UV cut-off scale of the theory is not important. However, we also notice that depending on the cut-off scale of the effective theory, there might be additional string-loop correction which cancels the model-independent contribution obtained by (57). If cancellation is exact at leading order, the soft scalar masses can be further suppressed compared to the gravitino mass. Therefore evaluating such string contributions is very important. Since the calculation is beyond the scope of this paper, we just mention its importance.

It is straightforward to stabilize light scalar fields by minimizing the effective SUGRA potential constructed from (51) and (54),

$$V_F^{\rm eff} = e^{K_{\rm eff}} \left(K_{\rm eff}^{I\bar{J}} D_I W_{\rm eff} D_{\bar{J}} W_{\rm eff}^* - 3|W_{\rm eff}|^2 \right). \tag{60}$$

The light Kähler moduli will be stabilized in the same manner as usual LVS models. The one-loop correction to $\Delta K_{\rm mod}^{\rm eff}$ is actually three-loop suppressed, since $\delta_{\rm GS} = \mathcal{O}(1/8\pi^2)$, so negligible for moduli stabilization. The moduli F-components are mostly determined by vacuum values of the scalar moduli, where the F-term is defined as

$$F^I = -e^{K_{\rm eff}/2} K_{\rm eff}^{I\bar{J}} D_{\bar{J}} W_{\rm eff}^*. \tag{61}$$

Those F-terms play the role of the SUSY breaking sources for the MSSM sector.

B. Soft SUSY breaking terms

We are now ready to calculate the MSSM soft terms induced by moduli stabilization with the $U(1)_A$. For the action described by (51) and (54), the MSSM soft terms take the form

$$\mathcal{L}_{\rm soft} = -\frac{1}{2} M_a \lambda^a \lambda^a - \frac{1}{2} m_i^2 |\phi_i|^2 - \frac{1}{3!} A_{ijk} y_{ijk} \phi_i \phi_j \phi_k + h.c., \tag{62}$$

where λ_a and ϕ_i are canonically normalized gauginos and scalar components of Φ_i respectively, y_{ijk} denote the canonically normalized Yukawa couplings,

$$y_{ijk} = \frac{\lambda_{ijk}(\vec{T}_s)}{\sqrt{e^{-K_0} Z_i^{\rm eff} Z_j^{\rm eff} Z_k^{\rm eff}}}, \tag{63}$$

¶ As we can see in (A), the additional quartic term which depends on the cut-off scale can emerge at one-loop level, but this term does not contribute to the scalar mass.

and the soft SUSY breaking parameters at a scale just below M_A are given by [24–28]

$$\begin{aligned}
\frac{M_a}{g_a^2} &= \frac{1}{2}F^I\partial_I f_a^{\rm eff} - \frac{1}{8\pi^2}\sum_i {\rm Tr}(T_a^2(\Phi_i))F^I\partial_I \ln\left(e^{-K_0/3}Z_i^{\rm eff}\right) + \frac{\Delta_{\rm anom.}M_a}{g_a^2}, \\
m_i^2 &= \frac{2}{3}V_F^{\rm eff} - F^I F^{J^*}\partial_I\partial_{\bar J}\ln\left(e^{-K_0/3}Z_i^{\rm eff}\right) + \Delta_{\rm anom.}m_i^2, \\
A_{ijk} &= -F^I\partial_I \ln\left(\frac{\lambda_{ijk}(\vec{T}_s)}{e^{-K_0}Z_i^{\rm eff}Z_j^{\rm eff}Z_k^{\rm eff}}\right) + \Delta_{\rm anom.}A_{ijk},
\end{aligned} \tag{64}$$

where $K_0 = K_{\rm eff}|_{\Phi_i=0}$ and $I = T_b,\ \vec{T}_s$. The additional contribution to the soft parameters, denoted as $\Delta_{\rm anom.}M_a$, $\Delta_{\rm anom.}m_i^2$ and $\Delta_{\rm anom.}A_{ijk}$, represents the anomaly mediation [28] in which the induced parameters are proportional to

$$\left|m_{3/2}^* + \frac{1}{3}K_I F^I\right| \le \mathcal{O}\Big(m_{3/2}t_b^{-3/2}\Big) \tag{65}$$

multiplied by additional loop suppression factors. Those contributions are strongly suppressed with respect to the prior contributions in the large volume limit due to the no-scale property of the leading order scalar potential, so we neglect its effect from now on. By substituting (52) to (64), and expanding in powers of $1/t_b$, $g_A^2/8\pi^2$, and $\delta_{\rm GS}$, the leading order contributions are obtained as follows.

$$\begin{aligned}
\frac{M_a}{g_a^2} &\simeq -\frac{g_A^2 q_a^2}{(8\pi^2)^2}m_{3/2}^* + \frac{1}{2}\sum_{I=T_b,\vec{T}_s}F^I\partial_I\Big(\gamma_a + k_a t_A^0\Big) \\
&\quad - \frac{1}{8\pi^2}\sum_i {\rm Tr}(T_a^2(\Phi_i))\sum_{I=T_b,\vec{T}_s}F^I\partial_I\left(\ln\mathcal{Y}_i + \frac{g_A^2 q_i^2}{8\pi^2}\ln g_A^2\Delta K''\right), \\
m_i^2 &\simeq -\frac{g_A^2 q_i^2}{16\pi^2}|m_{3/2}|^2 + \sum_{I,J=T_b,\vec{T}_s}F^I F^{\bar J}\partial_I\partial_{\bar J}\left(\frac{q_i}{\delta_{\rm GS}}t_A^0 - \ln\mathcal{Y}_i - \frac{g_A^2 q_i^2}{8\pi^2}\ln g_A^2\Delta K''\right), \\
A_{ijk} &\simeq \frac{g_A^2(q_i^2+q_j^2+q_k^2)}{16\pi^2}m_{3/2}^* - \sum_{I=T_b,\vec{T}_s}F^I\partial_I\left(\ln\frac{\lambda_{ijk}}{\mathcal{Y}_i\mathcal{Y}_j\mathcal{Y}_k} - \frac{g_A^2(q_i^2+q_j^2+q_k^2)}{8\pi^2}\ln g_A^2\Delta K''\right).
\end{aligned} \tag{66}$$

where

$$\begin{aligned}
&q_a^2 = \sum_i q_i^2{\rm Tr}(T_a^2(\Phi_i)), \quad \gamma_a = \gamma_a(\vec{T}_s), \quad \lambda_{ijk} = \lambda_{ijk}(\vec{T}_s), \quad t_A^0 = t_A^0(\vec{t}_s, \alpha_A\ln t_b), \\
&\mathcal{Y}_i = e^{-K_0/3}Z_i(\vec{t}_s, t_A^0, t_b) = \mathcal{Y}_i(\vec{t}_s, t_A^0, \beta_A\ln t_b), \quad \Delta K'' = \Delta K''(t_a, t_A^0, \alpha_A\ln t_b).
\end{aligned} \tag{67}$$

For each soft parameters in (66), first terms of the RHS are the model-independent contributions induced by the $U(1)_A$ threshold correction. These contributions are estimated as

$$\Delta_{\rm M.I.}M_a = \mathcal{O}\Big(\frac{m_{3/2}}{(8\pi^2)^2}\Big), \quad \Delta_{\rm M.I.}m_i^2 = \mathcal{O}\Big(\frac{m_{3/2}^2}{8\pi^2}\Big), \quad \Delta_{\rm M.I.}A_{ijk} = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big). \tag{68}$$

On the other hand, the remaining contributions are determined after specifying the forms of γ_a, λ_{ijk}, $\mathcal{Y}_i$, $\Delta K''$ and t_A^0. The soft terms which depend on t_A^0 are easily understood from (49) :

$$\begin{aligned} F^{T_v} &= \sum_{I=T_b,\vec{T}_s} F^I \partial_I \left(t_A^0 + \delta_{\rm GS} \frac{\mathcal{M}^2_{{\rm mat}_D}}{\mathcal{M}^2_{\rm GS}} \right), \\ -q_i g_A^2 D_A &= \sum_{I,J=T_b,\vec{T}_s} F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} \left(\frac{q_i}{\delta_{\rm GS}} t_A^0 + q_i \frac{\mathcal{M}^2_{{\rm mat}_D}}{\mathcal{M}^2_{\rm GS}} \right). \end{aligned} \tag{69}$$

In the perspective of UV theory, they are identified as the modulus and D-term mediated soft masses induced by moduli mixing. In (66), the contributions from the SUSY breaking of matter fields are not included, because we focus on the soft terms generated from the moduli sector at a energy scale just below M_A. In the effective theory, matter contributions come from the higher dimensional operator of (51), and can be included consistently. Their contributions should be critical in the case that the effect of moduli mixing is suppressed.

In order to estimate model-dependent contributions, let us look at the following cases. First, consider the case when there is no moduli mixing and the visible sector is sequestered from the other moduli sector. Then, γ_a, λ_{ijk}, $\mathcal{Y}_i$, $\Delta K''$ and t_A^0 are independent of T_b, $\vec{T}_s$. As a result,

$$\Delta^{(1)}_{\rm M.D.} M_a = \mathcal{O}(m_{3/2} t_b^{-3/2}), \quad \Delta^{(1)}_{\rm M.D.} m_i^2 \leq \mathcal{O}(m_{3/2}^2 t_b^{-3/2}), \quad \Delta^{(1)}_{\rm M.D.} A_{ijk} = \mathcal{O}(m_{3/2} t_b^{-3/2}). \tag{70}$$

The model-independent contribution dominates overall soft terms. The second case is that the visible sector is still sequestered from the small moduli sector, but the one-loop induced moduli mixing between the visible sector modulus and the large volume modulus [13, 14] gives rise to $t_A^0 \simeq \alpha_A \ln t_b$, $\mathcal{Y}_i \simeq \mathcal{Y}_i((\alpha_A - \beta_A)\ln t_b)$. Then, gaugino and sfermion masses are dominated by the model-dependent contribution :

$$\begin{aligned} \Delta^{(2)}_{\rm M.D.} M_a &\simeq \frac{g_A^2}{2} F^{T_b} \partial_{T_b} \left(k_a t_A^0 \right) \simeq \frac{g_A^2 k_a \alpha_A}{2} \frac{F^{T_b}}{t_b} = \frac{g_A^2 k_a \alpha_A}{2} m_{3/2}^*, \\ \Delta^{(2)}_{\rm M.D.} m_i^2 &\simeq \frac{q_i}{\delta_{\rm GS}} F^{T_b} F^{T_b^*} \partial_{T_b} \partial_{T_b^*} t_A^0 \simeq -\frac{q_i \alpha_A}{\delta_{\rm GS}} |m_{3/2}|^2, \end{aligned} \tag{71}$$

whereas the model-dependent contribution to A-terms might be comparable with the model-independent contribution :

$$\begin{aligned} \Delta^{(2)}_{\rm M.D.} A_{ijk} &\simeq -F^{T_b} \partial_{T_b} \ln \frac{\lambda_{ijk}}{\mathcal{Y}_i \mathcal{Y}_j \mathcal{Y}_k} \\ &\simeq (\alpha_A - \beta_A) m_{3/2}^* \Big(\partial_t \ln \mathcal{Y}_i(t) \mathcal{Y}_j(t) \mathcal{Y}_k(t) \Big)_{t=(\alpha_A-\beta_A)\ln t_b}. \end{aligned} \tag{72}$$

The resulting model-dependent contributions are estimated as

$$\Delta^{(2)}_{\rm M.D.} M_a = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big), \quad \Delta^{(2)}_{\rm M.D.} m_i^2 = \mathcal{O}(m_{3/2}^2), \quad \Delta^{(2)}_{\rm M.D.} A_{ijk} = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big). \tag{73}$$

The third case is that there is no one-loop induced moduli mixing with the large volume modulus ($\alpha_A = \beta_A = 0$), but the visible sector is not sequestered from the small moduli sector. Hence $\{\gamma_a,\, \lambda_{ijk}\}$ and $\{\mathcal{Y}_i,\, \Delta K'',\, t_A^0\}$ are generic functions of $\vec{T}_s$ and $\vec{t}_s$ respectively. Most controllable situation is that $n_s = n_w$, i.e. the number of small moduli is equal to the number of non-perturbatively generated terms in the superpotential. In appendix (B), we show that due to the no-scale property of the Kähler potential, the leading order $F^{T_{sj}}/t_{sj}$ are universal as

$$\frac{F^{T_{sj}}}{t_{sj}} = \frac{m_{3/2}^*}{\ln|M_{\rm Pl}/m_{3/2}|}\left(\frac{3}{4} + \mathcal{O}\Big(\frac{1}{\ln|M_{\rm Pl}/m_{3/2}|}\Big)\right) = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big) \quad \text{for } j = 1, \cdots, n_s = n_w. \tag{74}$$

At first sight, the model-dependent (D-term) contribution to the soft scalar mass seems to be comparable with the model-independent contribution by following estimation.

$$\Delta_{\rm M.D.} m_i^2 \simeq \frac{q_i}{\delta_{\rm GS}} \sum_{I,J=\vec{T}_s} F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} t_A^0(\vec{t}_s) = \mathcal{O}\Big(\frac{1}{\delta_{\rm GS}}\left|\frac{F^{T_s}}{t_s}\right|^2\Big) = \mathcal{O}\Big(\frac{m_{3/2}^2}{8\pi^2}\Big), \tag{75}$$

where $1/\delta_{\rm GS} = \mathcal{O}(8\pi^2)$, $t_{sj} t_{sk} \partial_{t_{sj}} \partial_{t_{sk}} t_A^0 \sim t_A^0 = \mathcal{O}(1)$ according to our normalization convention. However, this is not easily achieved. The no-scale property of the tree-level Kähler potential imply that $\Delta K(\lambda\vec{t}_s, \lambda t_A) \approx \lambda^{3/2} \Delta K(\vec{t}_s, t_A)$. So, the solution of (50) ($\Delta K'(\vec{t}_s, t_A^0) = 0$) also scales as $t_A^0(\lambda\vec{t}_s) \approx \lambda t_A^0(\vec{t}_s)$. Then, the leading order contributions of (75) cancel out :

$$\sum_{I,J=\vec{T}_s} F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} t_A^0(\vec{t}_s) \simeq \left|\frac{F^{T_s}}{t_s}\right|^2 \sum_{t_{sj}, t_{sk}} \Big(t_{sj} t_{sk} \partial_{t_{sj}} \partial_{t_{sk}} t_A^0\Big) = 0 \tag{76}$$

thanks to the universality of F^{T_s}/t_s and the scaling behavior of t_A^0 at leading order. Consequently, the D-term contribution is at most of the same order as the model-dependent modulus mediated soft term. Thus,

$$\Delta^{(3)}_{\rm M.D.} M_a = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big), \quad \Delta^{(3)}_{\rm M.D.} m_i^2 = \mathcal{O}\Big(\frac{m_{3/2}^2}{(8\pi^2)^2}\Big), \quad \Delta^{(3)}_{\rm M.D.} A_{ijk} = \mathcal{O}\Big(\frac{m_{3/2}}{8\pi^2}\Big). \tag{77}$$

For gaugino masses and A-terms, the model-dependent contributions are of the same order as those of the second case,

$$\begin{aligned}
\Delta^{(3)}_{\rm M.D.} M_a &\simeq -\frac{g_A^2}{2} \sum_j F^{T_{sj}} \partial_{T_{sj}} \Big(\gamma_a(\vec{T}_s) + k_a t_A^0(\vec{t}_s)\Big) \\
&\simeq -\sum_j \frac{F^{T_{sj}} \partial_{T_{sj}} \Big(\gamma_a(\vec{T}_s) + k_a t_A^0(\vec{t}_s)\Big)}{2{\rm Re}(\gamma_a(\vec{T}_s)) + k_a t_A^0(\vec{t}_s)} \sim \frac{F^{T_{sj}}}{t_{sj}} = \mathcal{O}\left(\frac{m_{3/2}}{\ln(M_{\rm Pl}/m_{3/2})}\right), \\
\Delta^{(3)}_{\rm M.D.} A_{ijk} &\simeq -\sum_j F^{T_{sj}} \partial_{T_{sj}} \ln \frac{\lambda_{ijk}}{\mathcal{Y}_i \mathcal{Y}_j \mathcal{Y}_k} \sim \frac{F^{T_{sj}}}{t_{sj}} = \mathcal{O}\left(\frac{m_{3/2}}{\ln(M_{\rm Pl}/m_{3/2})}\right),
\end{aligned} \tag{78}$$

where we assume that $\gamma_a(\vec{T}_s)$, $\mathcal{Y}_i(\vec{t}_s, t^0_A(\vec{t}_s)), \lambda_{ijk}(\vec{T}_s)$ are generic functions of $\vec{T}_s$, in the sense that $t_{sj}\partial_{T_{sj}}(\gamma_a(\vec{T}_s) + k_a t^0_A(\vec{t}_s)) \sim (\gamma_a(\vec{T}_s) + k_a t^0_A(\vec{t}_s))$, $t_{sj}\partial_{T_{sj}}\mathcal{Y}_i(\vec{t}_s, t^0_A(\vec{t}_s)) \sim \mathcal{Y}_i(\vec{t}_s, t^0_A(\vec{t}_s))$, $t_{sj}\partial_{T_{sj}}\lambda_{ijk}(\vec{T}_s) \sim \lambda_{ijk}(\vec{T}_s)$. Look at the final case when $n_s > n_w$ so that some of small moduli do not admit non-perturbative superpotential. In such a case, the moduli might be stabilized via several corrections to the Kähler potential which break no-scale structure [29, 30]. Even though the situation is less controllable, we generally expect that if the moduli are stabilized by the Kähler potential, the corresponding F-terms will be of order of $m_{3/2}$. Unlike the third case, there is no scaling property or symmetry to suppress the D-term contribution, and $1/\delta_{\rm GS}$ enhancement effect with respect to the ordinary modulus mediation will be realized. Therefore, we expect

$$\Delta^{(4)}_{\rm M.D.} M_a = \mathcal{O}(m_{3/2}), \quad \Delta^{(4)}_{\rm M.D.} m_i^2 = \mathcal{O}(8\pi^2 m^2_{3/2}), \quad \Delta^{(4)}_{\rm M.D.} A_{ijk} = \mathcal{O}(m_{3/2}), \tag{79}$$

and these contributions dominate overall soft terms.

V. CONCLUSION

To conclude, the central to this paper has been the study of soft term structure of the LVS models in which the visible sector Kähler modulus is dominantly stabilized by the D-term potential of the anomalous $U(1)_A$ gauge symmetry. This analysis has led to the following observations : Regardless of the detailed form of the Kähler potential, there are unavoidable soft term contributions coming from the $U(1)_A$ vector supermultiplet threshold correction. These model-independent contributions are of the order of $m_{3/2}/4\pi$ for soft scalar masses, $m_{3/2}/(8\pi^2)^2$ for gaugino masses, and $m_{3/2}/8\pi^2$ for A-parameters. However, the corresponding soft scalar mass squares are negative for any non-zero $U(1)_A$ charge assignment. In order to prevent charge and color breaking of the MSSM sector, the additional model-dependent contributions must be needed. We get such contributions from the moduli sector. As studied in [14], the moduli mixing between the visible sector modulus and the large volume modulus in the Kähler potential provides sfermion masses of the order of $m_{3/2}$. But, if the visible sector modulus is mixed only with small moduli stabilized by non-perturbative corrections to the superpotential, the corresponding model dependent contribution is of the order of $m_{3/2}/8\pi^2$. In this case, we still need an additional contribution from the matter sector to compensate for the model-independent sfermion mass squared.

An inevitable consequence of our paper is that due to the model-independent contribution, in order to obtain TeV-scale gaugino mass, the gravitino mass $m_{3/2} \simeq |W_0| t_b^{-3/2}$ is bounded from the above by the scale of the order of 10^6GeV. This means that if the effective UV scale of the visible sector $M_{\rm UV} \sim M_{\rm Pl} t_b^{-1/2}$ is identified as the GUT scale $M_{\rm GUT} \sim 2 \times 10^{16}$GeV, the flux induced superpotential W_0 should be much smaller than the value of $\mathcal{O}(1)$. In other words, if W_0 is given by $\mathcal{O}(1)$ so that $m_{3/2} \sim 10^{11}$GeV for $M_{\rm UV} \sim M_{\rm GUT}$, we need to fine-tune the visible sector model parameters to get correct orders of soft terms. So there is still a tension between the natural large volume scenario and the idea of grand unification. Of course, this conclusion can be wrong, if there are additional (model-dependent) string-loop corrections which cancel the above model-independent contributions. Since the detailed calculation should be performed in string theory, we leave it as a further work.

In this paper, we did not discuss stabilization of D-flat directions. As a remnant of the $U(1)_A$, there is anomalous global $U(1)_{\rm PQ}$ symmetry for the D-flat directional light matter fields. Since the $U(1)_{\rm PQ}$ should be spontaneously broken above 10^9GeV by astrophysical considerations [31], in [14], we introduced the PQ sector which consists of the $U(1)_A$ charged but the SM singlet matter fields. They dominantly break the $U(1)_{\rm PQ}$ and the QCD axion [32–34] is generated. Similar approach can be made here. After that, we can estimate the soft SUSY breaking terms coming from the PQ sector. They might be keystones when the moduli mixing effect is suppressed and the model-independent contribution dominates soft scalar masses.

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Appendix A: Model-independent soft scalar masses

Starting from the Kähler potential and superpotential given by (35), let us try deriving the scalar mass squared (26) at a component level. In order to see the effect of the $U(1)_A$ threshold correction clearly, we only consider a single small modulus and matter superfield, and take ΔK, Z_i and W as simple as possible. However, we allow the loop-induced moduli redefinition of the visible sector modulus as a probe for model-dependence. Then,

$$\begin{aligned} K &= -3\ln t_b + \frac{(t_s^{3/2}-\xi_{\alpha'}) + (t_A - \alpha_A \ln t_b)^2/2}{t_b^{3/2}} + \mathcal{O}(1/t_b^{-3}) + \frac{\Phi_i^* e^{2q_i V_A}\Phi_i}{t_b}, \\ W &= W_0 + Ae^{-aT_s}. \end{aligned} \tag{A1}$$

The large volume modulus T_b and the small cycle modulus T_s are stabilized in the usual manner. In this background, we can extract the effective tree-level Lagrangian for component fields of T_v, Φ_i, V_A. Since the background spacetime is nearly flat due to the no-scale structure of large volume stabilization, the leading order Lagrangian can be derived in flat spacetime limit. In other words, we neglect any soft terms of the order of $\Delta m_i^2 \sim m_{3/2}^2 t_b^{-3/2}$ and possible gravitational effect.

The tree-level Lagrangian for the canonically normalized component fields is written as follows.

$$\begin{aligned} \mathcal{L}_{\rm tree} \simeq\ & \frac{1}{2}\Big(t_v\Box t_v + \varphi_v\Box\varphi_v\Big) + \phi_i^*\Box\phi_i + i\Big(\partial_\mu\bar{\psi}_v\bar{\sigma}^\mu\partial_\mu\psi_v + \partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + \partial_\mu\bar{\lambda}_A\bar{\sigma}^\mu\lambda_A\Big) \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\Big(2g_A^2(M_{\rm GS}^2 + q_i^2|\phi_i|^2)\Big)A^\mu A_\mu + J_A^\mu A_\mu \\ & - \Big((\sqrt{2}g_A M_{\rm GS})\psi_v\lambda_A + (\sqrt{2}q_i\phi_i^*)\psi_i\lambda_A + {\rm h.c.}\Big) - \frac{g_A^2}{2}\Big(\sqrt{2}M_{\rm GS}t_v - q_i|\phi_i|^2\Big)^2 \\ & - \Big(\frac{1}{4}m_{3/2}\psi_v^2 + {\rm h.c.}\Big) + \frac{1}{4}m_{3/2}^2 t_v^2 + \Big(\frac{\alpha_A}{\delta_{\rm GS}}m_{3/2}^2\Big)\sqrt{2}M_{\rm GS}t_v, \end{aligned} \tag{A2}$$

where $\{t_v+i\varphi_v, \psi_v\}$ is the visible sector modulus supermultiplet, t_v and φ_v are the real part and the imaginary part of the scalar modulus respectively, ψ_v is the fermion component, $\{\phi_i, \psi_i\}$ is the chiral matter field supermultiplet, ϕ_i is the complex scalar, ψ_i is the fermion component, $\{A_\mu, \lambda_A\}$ is the $U(1)_A$ gauge supermultiplet in the WZ gauge, A_μ is the gauge field and λ_A is the gaugino, the $U(1)_A$ current $J_A^\mu = \Big(q_i\bar{\psi}_i\bar{\sigma}^\mu\psi_i + iq_i(\phi_i^*\partial^\mu\phi_i - \phi_i\partial^\mu\phi_i^*) + \sqrt{2}M_{\rm GS}\partial^\mu\varphi_v\Big)$, and finally $M_{\rm GS}$ is the square root of the vacuum value of $\mathcal{M}_{\rm GS}^2$ in (43) : $M_{\rm GS} = \delta_{\rm GS}t_b^{-3/4}$.

The first three lines of (A2) represent a supersymmetric part of the Lagrangian, while the last line is induced by the SUSY breaking of the large volume modulus T_b. Note that the modulus t_v has a soft mass squared of the order $m_{3/2}^2$, whereas the matter field ϕ_i does not have such term. This difference can be easily understood as follows. Since the matter field is localized on the MSSM 4-cycle, the Kähler metric of Φ_i is suppressed by t_b^{-1} as in (A2). Thus, the SUSY breaking of T_b is not transmitted to Φ_i and no soft terms are generated at the tree-level. On the other hand, the Kähler metric of T_v is suppressed by a inverse power of the Calabi-Yau volume $t_b^{-3/2}$. In this case, due to the additional suppression factor $t_b^{-1/2}$, sequestering does not work. The resulting SUSY breaking mass squared is of the order of $|F^{T_b}/t_b|^2 \sim m_{3/2}^2$. The SUSY breaking Majorana mass term of ψ_v also comes from the Kähler potential for the same reason. The linear term of t_v which is proportional to α_A originates from the moduli mixing term, $(t_A - \alpha_A \ln t_b)$ in the Kähler potential.

The mass squared of the $U(1)_A$ gauge boson A_μ is given by the supersymmetric contribution, $2g_A^2(M_{\rm GS}^2 + q_i^2|\phi_i|^2)$. We want to consider the case that the gauge boson gets its mass mostly from the Stückelberg mechanism, i.e. $M_{\rm GS}^2 \gg q_i^2|\phi_i|^2$. In this limit, one-loop correction to the scalar potential is generated as follows.

$$\Delta V_{1-{\rm loop}}(\phi_i) = -\frac{\Lambda^4}{128\pi^2}{\rm Str}1 + \frac{\Lambda^2}{64\pi^2}{\rm Str}M^2 + \frac{1}{64\pi^2}{\rm Str}M^4\left(\ln\frac{M^2}{\Lambda^2} - \frac{3}{2}\right), \tag{A3}$$

where Λ is the cut-off scale which is independent of ϕ_i, and $M^2 = M^2(|\phi_i|^2)$ is the ϕ_i dependent tree-level mass squared matrix for A_μ, t_v, ϕ_i, ψ_v, λ_A, and ψ_i. Since the visible sector is localized on the vanishing cycle, the natural cut-off scale of the 4D effective field theory is the string scale, $\Lambda \sim M_{\rm string} \sim t_b^{-3/4}$. The mass of the $U(1)_A$ gauge boson is of the order of $M_{\rm GS} \sim \delta_{\rm GS} t_b^{-3/4} \sim M_{\rm string}/8\pi^2$ which is quite below the cut-off scale, so we can safely calculate the one-loop correction of (A3) including all fields discussed above. In [35], it was argued that in the case of D3 branes at orbifold singularities, the cut-off scale is given by the winding scale $\Lambda \sim M_{\rm wind} \sim t_b^{-1/2}$ which is much bigger than the mass of the $U(1)_A$ gauge boson. For all cases, the $U(1)_A$ vector superfield can be included in the effective field theory. In order to see the cut-off dependence of the soft terms explicitly, we do not fix Λ as a specific value during calculation. After calculation, we will discuss its effect on the soft terms.

If we ignore the SUSY breaking terms specified in the last line of (A2), the vacuum will be described by D-flat condition, $\sqrt{2}M_{\rm GS}t_v = q_i|\phi_i|^2$. Then, a complex scalar field which spans the D-flat direction, and a linear combination of ψ_v and ψ_i which does not appear in the third line of (A2) remain massless. Masses of the other fields are all the same as $2g_A^2(M_{\rm GS}^2 + q_i^2|\phi_i|^2)$, and hence (A3) is vanishing. Now let us correctly count the SUSY breaking terms. By diagonalizing $M^2(|\phi_i|^2)$ and expanding the mass eigenvalues in powers of $q_i^2|\phi_i|^2$, we get the following mass squared spectrum at the leading order. For bosons,

$$A_\mu \quad : \quad M_A^2 = 2g_A^2\Big(M_{\rm GS}^2 + q_i^2|\phi_i|^2\Big),$$

$$t_v \;:\; M_{t_v}^2 = M_A^2 - m_{3/2}^2\left(\frac{1}{2} - \frac{q_i^2|\phi_i|^2}{2M_{\rm GS}^2}\right),$$
$$\phi_i \;:\; M_{\phi_i}^2 = -m_{3/2}^2\left(\frac{q_i\alpha_A}{\delta_{\rm GS}} + \frac{q_i^2|\phi_i|^2}{2M_{\rm GS}^2}\right), \tag{A4}$$

and for fermions,

$$\lambda_A^1 \;:\; M_{\lambda_A^1}^2 = M_A^2 + \frac{1}{8}m_{3/2}^2 + m_{3/2}M_A\left(\frac{1}{2} - \frac{q_i^2|\phi_i|^2}{2M_{\rm GS}^2}\right),$$
$$\lambda_A^2 \;:\; M_{\lambda_A^2}^2 = M_A^2 + \frac{1}{8}m_{3/2}^2 - m_{3/2}M_A\left(\frac{1}{2} - \frac{q_i^2|\phi_i|^2}{2M_{\rm GS}^2}\right),$$
$$\psi_i \;:\; M_{\psi_i}^2 = \mathcal{O}\left(\frac{m_{3/2}^2|\phi_i|^4}{M_{\rm GS}^4}\right), \tag{A5}$$

where λ_A^1, λ_A^2 are the mass eigenstates of the heavy fermions. In this mass spectrum, the SUSY breaking effect induced by the last line of (A2) is reflected on the terms proportional to $m_{3/2}$. Although (A4) and (A5) are evaluated directly from (A2), one can calculate $M_{\phi_i}^2$ from the tree-level effective scalar potential of ϕ_i in which t_v is integrated out along the D-flat direction, $\sqrt{2}M_{\rm GS}t_v \approx q_i|\phi_i|^2$. Then, the effective scalar potential $V_{\rm eff}(\phi_i) = -(q_i\alpha_A/\delta_{\rm GS})m_{3/2}^2|\phi_i|^2 - (m_{3/2}^2/8M_{\rm GS}^2)q_i^2|\phi_i|^4$ and $M_{\phi_i}^2 = \partial_{\phi_i}\partial_{\phi_i^*}V_{\rm eff}(\phi_i) = -m_{3/2}^2\left(q_i\alpha_A/\delta_{\rm GS} + q_i^2|\phi_i|^2/2M_{\rm GS}^2\right)$ is obtained.

It is straightforward to calculate $\Delta V_{1-{\rm loop}}$ by substituting (A4), (A5) to (A3). There is no soft mass contribution from $\Lambda^2{\rm Str}M^2/64\pi^2$. However, the last term of the RHS of (A3),

$$\begin{aligned}
&\frac{1}{64\pi^2}{\rm Str}M^4\ln\frac{M^2}{e^{3/2}\Lambda^2} \\
= \;&\frac{1}{64\pi^2}\left(3M_A^4\ln\frac{M_A^2}{e^{3/2}\Lambda^2} + M_{t_v}^4\ln\frac{M_{t_v}^2}{e^{3/2}\Lambda^2} - 2M_{\lambda_A^1}^4\ln\frac{M_{\lambda_A^1}^2}{e^{3/2}\Lambda^2} - 2M_{\lambda_A^2}^4\ln\frac{M_{\lambda_A^2}^2}{e^{3/2}\Lambda^2}\right) \\
&+ \frac{1}{64\pi^2}\left(M_{\phi_i}^4\ln\frac{M_{\phi_i}^2}{\Lambda^2} - 2M_{\psi_i}^4\ln\frac{M_{\psi_i}^2}{\Lambda^2}\right),
\end{aligned} \tag{A6}$$

should be carefully treated. Notice that the masses of heavy fields (A_μ, t_v, and $\lambda_A^{1,2}$) are independent of α_A, i.e. independent of moduli mixing. Thus soft SUSY breaking terms induced by the second line of the RHS of (A6) can be called model-independent contribution. We find that the induced soft mass for ϕ_i is also cut-off independent at the one-loop level. This is quite reasonable, since it corresponds to the $U(1)_A$ threshold correction. Suppose $\alpha_A = 0$, then there is no soft scalar mass contribution from the third line. Even if α_A is nonzero, its contribution is suppressed by $(m_{3/2}^2/M_{\rm GS}^2)$ compared to the model-independent contribution. Accordingly,

$$\Delta V_{1-{\rm loop}}(\phi_i) = {\rm constant} - \frac{g_A^2q_i^2}{16\pi^2}m_{3/2}^2|\phi_i|^2 + \mathcal{O}\left(\frac{m_{3/2}^2\Lambda^2|\phi_i|^4}{8\pi^2M_{\rm GS}^4}, \frac{m_{3/2}^2|\phi_i|^4}{8\pi^2M_{\rm GS}^2}, \frac{\alpha_Am_{3/2}^4|\phi_i|^2}{\delta_{\rm GS}8\pi^2M_{\rm GS}^2}\right), \tag{A7}$$

where "constant" implies that the value is independent of ϕ_i, the scalar mass squared,

$$\Delta m_i^2 = -\frac{g_A^2 q_i^2}{16\pi^2} m_{3/2}^2, \tag{A8}$$

comes from the second line of the RHS of (A6). The scalar mass contribution from the last term of (A7) can be ignored. The term which depends on the cut-off scale is the quartic potential of $|\phi_i|$, so its effect on the scalar mass is negligible regardless of taking Λ as $M_{\rm string}$ or $M_{\rm wind}$. The value (A8) is identical with the model-independent contribution of (66) obtained by setting $\mathcal{M}_{\rm UV}^2 \simeq t_b^{-1}$.

Appendix B: Small moduli F-components in the LVS

In this appendix, we will exhibit the result of (74) explicitly. We begin from the effective Kähler potential (51) and superpotential (54) constructed by integrating out the $U(1)_A$ vector superfield. The model consists of a single large volume modulus T_b, and n_s small moduli T_{sj}. For each T_{sj}, there exists non-perturbative correction to the superpotential ($n_s = n_w$). Also there is no one-loop induced moduli mixing between T_{sj} and T_b ($\alpha_A = 0$). The matter sector does not have an important role for evaluating moduli F-terms, so we can ignore it. Then, the Kähler potential and superpotential for the moduli sector are given by

$$\begin{aligned} K_{\rm eff} &= -3\ln t_b + \frac{\hat{K}(\vec{t}_s) - \xi_{\alpha'}}{t_b^{3/2}} + \mathcal{O}(t_b^{-3}), \\ W &= W_0 + \sum_j A_a e^{-a_j T_{sj}}, \end{aligned} \tag{B1}$$

where

$$\begin{aligned} \hat{K}(\vec{t}_s) &= \Delta K(\vec{t}_s, t_A^0(\vec{t}_s)) + \frac{g_A^2 \delta_{\rm GS}^2}{8\pi^2} \Delta K''(\vec{t}_s, t_A^0(\vec{t}_s)) \ln \frac{2 g_A^2 \delta_{\rm GS}^2 \Delta K''(\vec{t}_s, t_A^0(\vec{t}_s))}{e t_b^{1/2}} \\ &= \Delta K(\vec{t}_s, t_A^0(\vec{t}_s)) \left(1 + \mathcal{O}\Big(\frac{1}{(8\pi^2)^3}\Big)\right). \end{aligned} \tag{B2}$$

Since the $U(1)_A$ threshold correction is three-loop suppressed, we set the argument of $\hat{K}$ just $\vec{t}_s$ as ΔK. Due to the no-scaler structure of the tree-level Kähler potential, $\hat{K}$ has the following scaling behavior

$$\hat{K}(\lambda \vec{t}_s) = \lambda^{3/2} \hat{K}(\vec{t}_s) \tag{B3}$$

up to (string) loop corrections. The corresponding scalar potential is given by

$$
\begin{aligned}
V_F &= e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} W^* - 3|W|^2 \right) \\
&= \frac{1}{t_b^{3/2}} \left(\hat{K}^{ij} (\partial_{T_{si}} W)(\partial_{T^*_{sj}} W^*) - \frac{1}{2} \Big(\hat{K}^{ij} \hat{K}_j (\partial_{T_{si}} W)(W^* t_b^{-3/2}) + h.c. \Big) \right. \\
&\qquad \left. + \frac{1}{4} \Big(\hat{K}^{ij} \hat{K}_i \hat{K}_j - 3\hat{K} + 3\xi_{\alpha'} \Big) \left| W t_b^{-3/2} \right|^2 \right) + \mathcal{O}(t_b^{-6})
\end{aligned} \tag{B4}
$$

where $\hat{K}_i = \partial_{t_{si}} \hat{K}(\vec{t}_s)$, $\hat{K}_{ij} = \partial_{t_{si}} \partial_{t_{sj}} \hat{K}$, $\hat{K}^{ij} = (\hat{K}_{ij})^{-1}$. In leading order of $1/t_b$ expansion, the stationary condition $\partial_{T_b} V_F = \partial_{T_{sj}} V_F = 0$ gives

$$
\begin{aligned}
\partial_{T_{sk}} V_F = 0 \ : \ & \Big(\hat{K}^{ij} (\partial_{T^*_{sj}} W^*) - \frac{1}{2} \hat{K}^{ij} \hat{K}_j (W^* t_b^{-3/2}) \Big) \Big(\partial_{T_{sk}} \partial_{T_{si}} W \Big) \\
& + \frac{1}{2} \Big(\hat{K}^{ij}_k (\partial_{T^*_{sj}} W^*) - (\hat{K}^{ij}_k \hat{K}_j + \delta^i_k)(W^* t_b^{-3/2}) \Big) \Big(\partial_{T_{si}} W \Big) + h.c. \\
& + \frac{1}{4} \Big(\hat{K}^{ij}_k \hat{K}_i \hat{K}_j - \hat{K}_k \Big) \left| W t_b^{-3/2} \right|^2 = 0, \\
\partial_{t_b} V_F = 0 \ : \ & \hat{K}^{ij} (\partial_{T_{si}} W)(\partial_{T^*_{sj}} W^*) - 2 \Big(\hat{K}^{ij} K_j (\partial_{T_{si}} W)(W^* t_b^{-3/2}) + h.c. \Big) \\
& + \frac{3}{4} \Big(\hat{K}^{ij} \hat{K}_i \hat{K}_j - 3\hat{K} + 3\xi_{\alpha'} \Big) \left| W t_b^{-3/2} \right|^2 = 0.
\end{aligned} \tag{B5}
$$

In our field basis, $t_{sk} \partial_{T_{sk}} \partial_{T_{si}} W = -(a_i t_{si}) \delta_{ki} \partial_{T_{si}} W$. We would like to find the solution in the large volume limit. Such limit corresponds to $a_j t_{sj} \gg \mathcal{O}(1)$, and $|t_{si} \partial^2_{T_{si}} W| \gg |\partial_{T_{si}} W|$. Then the solution can be evaluated perturbatively as follows.

$$
\begin{aligned}
\partial_{T_{si}} W &= -a_i A_i e^{-a_i T_{si}} = \frac{1}{2} (1 - \epsilon_i) \hat{K}_i (W t_b^{-3/2}), \\
\sum_j \left(\frac{\hat{K}^{ij} \hat{K}_j}{t_i} \right) \epsilon_j &= \frac{1}{a_i t_{si}} \left(2 - \sum_{j,k} \frac{\hat{K}^{jk}_i \hat{K}_j \hat{K}_k}{2\hat{K}_i} \right) \Big(1 + \mathcal{O}(\epsilon_i) \Big), \\
\xi_{\alpha'} &= \hat{K} \left(1 + \sum_{ij} \left(\frac{2\hat{K}^{ij} \hat{K}_i \hat{K}_j}{9\hat{K}} \right) \epsilon_i + \mathcal{O}(\epsilon_i^2) \right).
\end{aligned} \tag{B6}
$$

Notice that there is no sum for an index i. We assume that A_i and the vacuum value of $\hat{K}_i$ are of order one. However, the gravitino mass $m_{3/2} = e^{K/2} W = (W_0 t_b^{-3/2})(1 + \mathcal{O}(t_b^{-3/2}))$ would be around TeV so that it is hierarchically much smaller than one. From the first equation of (B6),

$$
a_i t_{si} = 2 \ln \frac{M_{\rm Pl}}{|m_{3/2}|} - 2 \ln \frac{\hat{K}_i (1 - \epsilon_i)}{2 |a_i A_i|} = 2 \ln \frac{M_{\rm Pl}}{|m_{3/2}|} + \mathcal{O}(1). \tag{B7}
$$

$a_i t_{si}$ are universal at the leading order. On the one hand, due to the scaling behavior of $\hat{K}$ given by (B3), it is easily identified that $\sum_{ij} \hat{K}^{ij}_k \hat{K}_i \hat{K}_j = \hat{K}_k$. Then, the F-terms of the small moduli can be obtained as

$$\begin{aligned}\frac{F^{T_{si}}}{t_{si}} &= -\frac{1}{t_{si}} e^{K/2} K^{T_{si}\bar{J}} D_{\bar{J}} W^* = \sum_j \left(\frac{\hat{K}^{ij}\hat{K}_j}{t_{si}} \right) \epsilon_j^* (W^* t_b^{-3/2}) \\ &= \frac{m_{3/2}^*}{\ln |M_{Pl}/m_{3/2}|} \left(\frac{3}{4} + \mathcal{O}\left(\frac{1}{\ln|M_{\rm Pl}/m_{3/2}|}, \frac{\hat{K}_{\rm loop}}{\hat{K}} \right) \right),\end{aligned} \tag{B8}$$

where $\hat{K}_{\rm loop}$ stands for the perturbative correction to $\hat{K}$ which breaks the no-scale form of Kähler potential.

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[1] J. P. Derendinger, L. E. Ibanez and H. P. Nilles, "On the low-energy D = 4, N=1 supergravity theory extracted from the D = 10, N=1 superstring," Phys. Lett. B **155**, 65 (1985); Nucl. Phys. B **267**, 365 (1986); M. Dine, R. Rohm, N. Seiberg and E. Witten, "Gluino condensation in superstring models," Phys. Lett. B **156**, 55 (1985).

[2] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097]; K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088].

[3] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, "de Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].

[4] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," JHEP **0503**, 007 (2005) [arXiv:hep-th/0502058]; J. P. Conlon, F. Quevedo and K. Suruliz, "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking," JHEP **0508**, 007 (2005) [arXiv:hep-th/0505076].

[5] R. Blumenhagen, S. Moster and E. Plauschinn, "Moduli stabilisation versus chirality for MSSM like Type IIB orientifolds," JHEP **0801**, 058 (2008) [arXiv:0711.3389 [hep-th]].

[6] K. Choi and K. S. Jeong, "Supersymmetry breaking and moduli stabilization with anomalous U(1) gauge symmetry," JHEP **0608**, 007 (2006) [arXiv:hep-th/0605108].

[7] K. Choi, K. S. Jeong, K. -I. Okumura, M. Yamaguchi, "Mixed Mediation of Supersymmetry Breaking with Anomalous U(1) Gauge Symmetry," JHEP **1106** (2011) 049. [arXiv:1104.3274 [hep-ph]].

[8] K. Choi, K. S. Jeong, "String theoretic QCD axion with stabilized saxion and the pattern of supersymmetry breaking," JHEP **0701** (2007) 103. [hep-th/0611279]

[9] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, “Stability of flux compactifications and the pattern of supersymmetry breaking,” JHEP **0411**, 076 (2004) [arXiv:hep-th/0411066].

[10] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, “Soft supersymmetry breaking in KKLT flux compactification,” Nucl. Phys. B **718**, 113 (2005) [arXiv:hep-th/0503216].

[11] K. Choi, K. S. Jeong and K. i. Okumura, “Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models,” JHEP **0509**, 039 (2005) [arXiv:hep-ph/0504037]; M. Endo, M. Yamaguchi and K. Yoshioka, “A bottom-up approach to moduli dynamics in heavy gravitino scenario: Superpotential, soft terms and sparticle mass spectrum,” Phys. Rev. D **72**, 015004 (2005) [arXiv:hep-ph/0504036]; A. Falkowski, O. Lebedev and Y. Mambrini, “SUSY phenomenology of KKLT flux compactifications,” JHEP **0511**, 034 (2005) [arXiv:hep-ph/0507110]; O. Loaiza-Brito, J. Martin, H. P. Nilles and M. Ratz, “log(M(Pl/m(3/2))),” AIP Conf. Proc. **805**, 198 (2006) [arXiv:hep-th/0509158].

[12] R. Blumenhagen, J. P. Conlon, S. Krippendorf, S. Moster and F. Quevedo, “SUSY breaking in local string/F-theory models,” JHEP **0909**, 007 (2009) [arXiv:0906.3297 [hep-th]].

[13] J. P. Conlon and F. G. Pedro, “Moduli redefinitions and moduli stabilisation,” JHEP **1006**, 082 (2010) [arXiv:1003.0388 [hep-th]].

[14] K. Choi, H. P. Nilles, C. S. Shin, M. Trapletti, “Sparticle Spectrum of Large Volume Compactification,” JHEP **1102** (2011) 047. [arXiv:1011.0999 [hep-th]].

[15] M. B. Green and J. H. Schwarz, “Anomaly cancellation in supersymmetric D=10 gauge theory and superstring Theory,” Phys. Lett. B **149**, 117 (1984).

[16] M. Cvetic, G. Shiu and A. M. Uranga, “Chiral four-dimensional N=1 supersymmetric type 2A orientifolds from intersecting D6 branes,” Nucl. Phys. B **615**, 3 (2001) [arXiv:hep-th/0107166]; D. Cremades, L. E. Ibanez and F. Marchesano, “SUSY quivers, intersecting branes and the modest hierarchy problem,” JHEP **0207**, 009 (2002) [arXiv:hep-th/0201205]; G. Villadoro and F. Zwirner, “D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications,” JHEP **0603**, 087 (2006) [arXiv:hep-th/0602120].

[17] L. B. Anderson, J. Gray, A. Lukas and B. Ovrut, “Stability Walls in Heterotic Theories,” JHEP **0909**, 026 (2009) [arXiv:0905.1748 [hep-th]].

[18] G. Aldazabal, A. Font, L. E. Ibanez and G. Violero, “D = 4, N = 1, type IIB orientifolds,” Nucl. Phys. B **536**, 29 (1998) [arXiv:hep-th/9804026]; Z. Lalak, S. Lavignac and H. P. Nilles,

"String dualities in the presence of anomalous U(1) symmetries," Nucl. Phys. B **559**, 48 (1999) [arXiv:hep-th/9903160]

[19] M. T. Grisaru, M. Rocek, R. von Unge, "Effective Kahler potentials," Phys. Lett. **B383** (1996) 415-421. [hep-th/9605149].

[20] A. Brignole, "One loop Kahler potential in non renormalizable theories," Nucl. Phys. B **579** (2000) 101 [arXiv:hep-th/0001121].

[21] N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, "Supersymmetry breaking loops from analytic continuation into superspace," Phys. Rev. D **58** (1998) 115005 [arXiv:hep-ph/9803290].

[22] V. Kaplunovsky and J. Louis, "Field dependent gauge couplings in locally supersymmetric effective quantum field theories," Nucl. Phys. B **422** (1994) 57 [arXiv:hep-th/9402005].

[23] J. P. Conlon and E. Palti, "On Gauge Threshold Corrections for Local IIB/F-theory GUTs," Phys. Rev. D **80**, 106004 (2009) [arXiv:0907.1362 [hep-th]].

[24] V. S. Kaplunovsky and J. Louis, "Model independent analysis of soft terms in effective supergravity and in Phys. Lett. B **306**, 269 (1993) [arXiv:hep-th/9303040]; A. Brignole, L. E. Ibanez and C. Munoz, "Towards a theory of soft terms for the supersymmetric Standard Model," Nucl. Phys. B **422**, 125 (1994) [Erratum-ibid. B **436**, 747 (1995)] [arXiv:hep-ph/9308271].

[25] Y. Kawamura and T. Kobayashi, "Soft scalar masses in string models with anomalous $U(1)$ symmetry," Phys. Lett. B **375**, 141 (1996) [Erratum-ibid. B **388**, 867 (1996)] [arXiv:hep-ph/9601365]; P. Binetruy and E. Dudas, "Gaugino condensation and the anomalous U(1)," Phys. Lett. B **389**, 503 (1996) [arXiv:hep-th/9607172]; G. R. Dvali and A. Pomarol, "Anomalous U(1) as a mediator of supersymmetry breaking," Phys. Rev. Lett. **77**, 3728 (1996) [arXiv:hep-ph/9607383].

[26] T. Higaki, Y. Kawamura, T. Kobayashi and H. Nakano, "Anomalous U(1) D-term contribution in type I string models," Phys. Rev. D **69**, 086004 (2004) [arXiv:hep-ph/0308110]; B. Kors and P. Nath, "Hierarchically split supersymmetry with Fayet-Iliopoulos D-terms in string theory," Nucl. Phys. B **711**, 112 (2005) [arXiv:hep-th/0411201]; K. S. Babu, T. Enkhbat and B. Mukhopadhyaya, "Split supersymmetry from anomalous U(1)," Nucl. Phys. B **720**, 47 (2005) [arXiv:hep-ph/0501079]; E. Dudas and S. K. Vempati, "Large D-terms, hierarchical soft spectra and moduli stabilisation," Nucl. Phys. B **727**, 139 (2005) [arXiv:hep-th/0506172]; C. A. Scrucca, "Soft masses in superstring models with anomalous U(1) symmetries," JHEP **0712**, 092 (2007) [arXiv:0710.5105 [hep-th]]; E. Dudas, Y. Mambrini, S. Pokorski, A. Romagnoni and M. Trapletti,

"Gauge vs. Gravity mediation in models with anomalous U(1)'s," JHEP **0903**, 011 (2009) [arXiv:0809.5064 [hep-th]].

[27] J. A. Bagger, T. Moroi and E. Poppitz, "Anomaly mediation in supergravity theories," JHEP **0004**, 009 (2000) [arXiv:hep-th/9911029]; K. Choi and H. P. Nilles, "The gaugino code," JHEP **0704**, 006 (2007) [arXiv:hep-ph/0702146]

[28] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155]; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, "Gaugino mass without singlets," JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442]; J. P. Conlon, M. Goodsell and E. Palti, "Anomaly mediation in superstring theory," arXiv:1008.4361 [hep-th].

[29] M. Cicoli, J. P. Conlon, F. Quevedo, "Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications," JHEP **0801** (2008) 052. [arXiv:0708.1873 [hep-th]];

[30] M. Cicoli, J. P. Conlon and F. Quevedo, "General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation," JHEP **0810** (2008) 105 [arXiv:0805.1029 [hep-th]].

[31] For a recent review on axions and the strong CP problem, see J. E. Kim and G. Carosi, "Axions and the strong CP problem," arXiv:0807.3125 [hep-ph].

[32] R. D. Peccei and H. R. Quinn, "Constraints imposed by CP conservation in the presence of instantons," Phys. Rev. D **16**, 1791 (1977); "CP conservation in the presence of instantons," Phys. Rev. Lett. **38**, 1440 (1977).

[33] S. Weinberg, "A new light boson?," Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, "Problem of strong P and T invariance in the presence of instantons," Phys. Rev. Lett. **40**, 279 (1978).

[34] J. E. Kim, "Weak interaction singlet and strong CP invariance," Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, "Can confinement ensure natural CP invariance of strong interactions?," Nucl. Phys. B **166**, 493 (1980); M. Dine, W. Fischler and M. Srednicki, "A simple solution to the strong CP problem with a harmless axion," Phys. Lett. B **104**, 199 (1981); A. R. Zhitnitsky, "On possible suppression of the axion hadron interactions. (in Russian)," Sov. J. Nucl. Phys. **31**, 260 (1980) [Yad. Fiz. **31**, 497 (1980)].

[35] J. P. Conlon, "Gauge Threshold Corrections for Local String Models," JHEP **0904**, 059 (2009) [arXiv:0901.4350 [hep-th]].