

Effective interactions of axion supermultiplet and thermal production of axino dark matter

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ABSTRACT: We discuss the effective interactions of axion supermultiplet, which might be important for analyzing the cosmological aspect of supersymmetric axion model. Related to axino cosmology, it is stressed that three seemingly similar but basically different quantities, the Wilsonian axino-gluino-gluon coupling, the 1PI axino-gluino-gluon amplitude, and the PQ anomaly coefficient, should be carefully distinguished from each other for correct analysis of the thermal production of axinos in the early Universe. It is then noticed that the 1PI axino-gluino-gluon amplitude at energy scale p in the range $M_\Phi < p < v_{\text{PQ}}$ is suppressed by M_Φ^2/p^2 in addition to the well-known suppression by $p/16\pi^2 v_{\text{PQ}}$, where M_Φ is the mass of the heaviest PQ-charged and gauge-charged matter supermultiplet in the model, which can be well below the PQ scale v_{PQ} . As a result, axino production at temperature $T > M_\Phi$ is dominated by the production by matter supermultiplet, not by the production by gauge supermultiplet. Still the axino production rate is greatly reduced if $M_\Phi \ll v_{\text{PQ}}$, which would make the subsequent cosmology significantly altered. This would be most notable in the supersymmetric DFSZ model in which M_Φ corresponds to the Higgsino mass which is around the weak scale, however a similar reduction is possible in the KSVZ model also. We evaluate the relic axino density for both the DFSZ and KSVZ models while including the axino production in the processes involving the heaviest PQ-charged and gauge-charged matter supermultiplet.

KEYWORDS: Supersymmetry Phenomenology

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1 Introduction

Supersymmetric extension of the standard model provides perhaps the most appealing solution to the gauge hierarchy problem [1–3]. Furthermore it can easily accommodate the axion solution to the strong CP problem [4–8] which is another naturalness problem of the standard model. Supersymmetric axion model [9–11] necessarily contains the superpartners of axion, the axino and saxion, which can have a variety of cosmological implications. In particular, axino can be a good candidate for cold dark matter [12] in the Universe, depending upon the mechanism of supersymmetry breaking and the presumed cosmological scenario [13–18].

One of the key issues in axino cosmology is the thermal production of axino by scattering or decay of particles in thermal equilibrium in the early Universe. Most of the previous analysis of thermal axino production [19–22] is based on the following form of local effective interaction between the axion supermultiplet and the gauge supermultiplet:

$$\int d^2\theta \frac{1}{32\pi^2} \frac{A}{v_{\text{PQ}}} W^{a\alpha} W_\alpha^a, \tag{1.1}$$

where v_{PQ} is the scale of spontaneous PQ breakdown, and $A = (s + ia)/\sqrt{2} + \sqrt{2}\theta\tilde{a} + \theta^2 F^A$ is the axion superfield which contains the axion a , the saxion s , and the axino \tilde{a} as its component fields. If axinos were in thermal equilibrium, their relic density can be determined in a straightforward manner, and the result does not depend on the details of axino interactions. For the other case that axinos were not in thermal equilibrium, one usually assumes that the production is mostly due to the effective interaction (1.1).

Then a simple dimensional analysis implies that the axino production rate per unit volume scales as $\Gamma_{\tilde{a}} \propto T^6/(16\pi^2 v_{\text{PQ}})^2$, which results in a relic axino number to entropy ratio $Y_{\tilde{a}} \propto T_R/(16\pi^2 v_{\text{PQ}})^2$, where the reheat temperature T_R is assumed to be lower than both v_{PQ} and the axino decoupling temperature. If axino is a stable dark matter (DM), this result leads to a severe upper limit on $m_{\tilde{a}} T_R$ [13–18]. Even when axino is not stable, thermal axino production in the early Universe can affect the later stage of cosmological evolution in various ways, e.g. there can be a late decay of axino into lighter particles in the minimal supersymmetric standard model (MSSM) or into the lighter gravitino and axion [23–31], which might affect the relic DM density, or the Big-Bang nucleosynthesis, or the structure formation.

In this paper, we discuss the effective interactions of axion supermultiplet in a general context, and examine the implications for cosmological axino production. It is stressed that three seemingly similar but basically different quantities, the Wilsonian axino-gaugino-gauge boson coupling, the 1PI axino-gaugino-gauge boson amplitude, and the PQ anomaly coefficient, should be carefully distinguished from each other for correct analysis of the cosmological axino production. At any given scale, the Wilsonian coupling is local by construction, but it depends severely on the field basis adopted to define the effective theory. On the other hand, the 1PI amplitude is an observable quantity, and therefore is basis-independent, while it generically contains non-local piece and has different value at different energy scale of external particles. The PQ anomaly coefficient is an intrinsic property of the PQ symmetry of the model, which is basis-independent and has a common value at all scales as a consequence of the anomaly matching condition.

At any temperature below v_{PQ} , axino production by gauge supermultiplet is determined by the 1PI axino-gaugino-gauge boson amplitude at the corresponding energy scale. Our key observation is that the 1PI axino-gaugino-gauge boson amplitude at energy scale p in the range $M_{\Phi} < p < v_{\text{PQ}}$ does not scale like the Wilsonian effective interaction (1.1), but is suppressed further by the factor M_{Φ}^2/p^2 , where M_{Φ} is the mass of the heaviest PQ-charged and gauge-charged matter field in the model, which can be well below v_{PQ} in general. This is a simple consequence of that the axion supermultiplet is decoupled from gauge supermultiplets (and from charged matter supermultiplets also) in the limit $M_{\Phi} \rightarrow 0$, which is not manifest if one considers the effective interaction (1.1) alone, but recovered in the full analysis taking into account all interactions of the axion supermultiplet together. As a result, if $M_{\Phi} \ll v_{\text{PQ}}$, which is possible in most cases, axino production by gauge supermultiplets is greatly reduced. This would be most dramatic in the supersymmetric Dine-Fishler-Srednicki-Zhitnitsky (DFSZ) model [32, 33] as the model does not contain any exotic PQ-charged and gauge-charged matter field other than the matter and Higgs fields in the MSSM, and therefore M_{Φ} is around the weak scale. A similar suppression is possible in the supersymmetric Kim-Shifman-Vainshtein-Zakharov (KSVZ) axion model [34, 35] also, in which M_{Φ} corresponds to the mass of an exotic PQ-charged quark multiplet, which in principle can take any value between the PQ scale and the weak scale.

An immediate consequence of the above observation is that axino production in the temperature range $M_{\Phi} < T < v_{\text{PQ}}$ is dominated *not* by the production by gauge supermultiplets, *but* by the production by the heaviest PQ-charged and gauge-charged matter

multiplet Φ . If $M_\Phi \ll v_{\text{PQ}}$, axino production rate at $T > M_\Phi$ is greatly reduced compared to the previous result based on the effective interaction (1.1) alone. This can significantly alter the cosmological aspect of the model. For instance, a high reheat temperature which has been considered to produce too much axino dark matter can be allowed if $M_\Phi \ll v_{\text{PQ}}$. Also, our result can relieve the upper bound on the reheat temperature, which has been obtained in [27] by considering the gravitino and axino productions together, and might have implication for the cosmological bounds on axion supermultiplet in the presence of small R -parity breaking, which have been discussed in [31]. Since the interaction rate between the axino and the thermal bath of gauge-charged particles is reduced in the limit $M_\Phi \ll T$, the axino decoupling temperature can take a higher value than the previous estimate based on (1.1). Similar observations apply to the axion and saxion cosmology also, and our result might affect some of the results of the recent analysis of thermal axion production [36].

The organization of this paper is as follows. In the next section, we discuss in a general context the effective interactions of axion supermultiplet which will be crucial for later discussion of cosmological axino production. In section 3, we provide an analysis of thermal axino production for both the KSVZ and DFSZ models, and determine the relic axino density including the contribution from the processes involving the heaviest PQ-charged and gauge-charged matter multiplet at $T > M_\Phi$. Section 4 is the conclusion.

2 Effective interactions of axion supermultiplet

In this section, we discuss the generic feature of the effective interactions of axion supermultiplet [37], which will be relevant for our later discussion of cosmological axino production. For the purpose of illustration, we first consider a simple specific model, a supersymmetric extension of the KSVZ axion model [34, 35], and later generalize the discussion to generic supersymmetric axion models.

At the fundamental scale M_* , which is presumed to be of the order of the reduced Planck scale $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV, the supersymmetric part of our model is described by the following high scale lagrangian

$$\mathcal{L}(M_*) = \int d^4\theta K + \left[\int d^2\theta \left(\frac{1}{4} f_a W^{a\alpha} W_\alpha^a + W(\Phi_I) \right) + \text{h.c.} \right], \quad (2.1)$$

where W_α^a are the gluon superfields, and the Kähler potential K , the holomorphic gauge kinetic function f_a , and the superpotential W are given by

$$\begin{aligned} K &= \sum_I \Phi_I^\dagger \Phi_I, & f_a &= \frac{1}{\hat{g}_s^2(M_*)}, \\ W &= hZ \left(XY - \frac{v_{\text{PQ}}^2}{2} \right) + \lambda XQQ^c, \end{aligned} \quad (2.2)$$

where $\hat{g}_s^2(M_*)$ is the holomorphic QCD coupling at M_* , and Φ_I stand for the chiral matter superfields in the model, $\{\Phi_I\} = \{Z, X, Y, Q, Q^c\}$. Among these matter fields, Z, X and

Y are gauge singlet, while $Q + Q^c$ are colored quark multiplet. Here we distinguish the holomorphic QCD coupling \hat{g}_s from the physical QCD coupling g_s for later discussion. Also we ignore non-renormalizable operators suppressed by $1/M_*$, as well as the similarly suppressed supergravity effects, under the assumption that the PQ scale v_{PQ} is far below M_* , so that any effect suppressed by v_{PQ}/M_* can be safely ignored.

The model is invariant under the global PQ symmetry¹

$$U(1)_{\text{PQ}} : \quad \Phi_I \rightarrow e^{ix_I\alpha}\Phi_I, \quad (2.3)$$

where the PQ charge x_I are given by

$$(x_Z, x_X, x_Y, x_Q + x_{Q^c}) = (0, -1, 1, 1). \quad (2.4)$$

This PQ symmetry is explicitly broken by the QCD anomaly as

$$\partial_\mu J_{\text{PQ}}^\mu = \frac{g^2}{16\pi^2} C_{\text{PQ}} F^{a\mu\nu} \tilde{F}_{\mu\nu}^a, \quad (2.5)$$

where J_{PQ}^μ is the Noether current for the symmetry transformation (2.3) defined in a gauge invariant regularization scheme, and then the anomaly coefficient is given by

$$C_{\text{PQ}} = 2 \sum_I x_I \text{Tr}(T_c^2(\Phi_I)) = 2(x_Q + x_{Q^c}) \text{Tr}(T_c^2(Q)) = N_Q, \quad (2.6)$$

where $T_c(\Phi_I)$ is the generator of the $SU(3)_c$ transformation of Φ_I , and N_Q is the number of $Q + Q^c$. The QCD instantons then generate an axion potential which has a minimum at the axion vacuum expectation value (VEV) which cancels the QCD vacuum angle, so the strong CP problem is solved [8]. In our convention, the PQ anomaly coefficient C_{PQ} corresponds to the number of the discrete degenerate vacua of the axion potential, which can have a nontrivial cosmological implication [38–40]. In the following, we will assume $N_Q = 1$ for simplicity.

In the limit to ignore SUSY breaking effects, the model has a supersymmetric ground state

$$\langle XY \rangle = \frac{v_{\text{PQ}}^2}{2}, \quad \langle Z \rangle = 0. \quad (2.7)$$

Including SUSY breaking effects, the VEV of the ratio X/Y can be fixed and a small but nonzero VEV of Z can be induced also. In fact, SUSY breaking part of the model is not essential for our discussion of axino effective interactions, although it is crucial for the determination of the axino and saxion masses [41–46]. We thus simply assume that soft SUSY breaking terms in the model fix the VEVs as

$$\langle X \rangle \sim \langle Y \rangle \quad (2.8)$$

¹In addition to the PQ symmetry, we can introduce an approximate $U(1)_R$ symmetry under which $\theta^\alpha \rightarrow e^{i\beta}\theta^\alpha$ and $Z \rightarrow e^{2i\beta}Z$ to justify the form of the superpotential. In the present form, the model is invariant under another $U(1)$ symmetry which makes the massive Dirac quark $Q + Q^c$ stable. However it is straightforward to break this $U(1)$, while keeping the PQ symmetry unbroken, when the model is combined with the MSSM, making $Q + Q^c$ decay fast enough.

with appropriate (model-dependent, but light) axino and saxion masses, for which the axion scale is given by v_{PQ} . Also, to illustrate our main point, we further assume that $h = \mathcal{O}(1)$, but $\lambda \ll 1$, so the model involves two widely separated mass scales, the axion scale v_{PQ} and the quark mass M_Q which can be far below v_{PQ} :

$$M_Q \equiv \frac{\lambda v_{\text{PQ}}}{\sqrt{2}} \ll v_{\text{PQ}}. \quad (2.9)$$

All physical consequences of the model can be determined in principle by the high scale lagrangian (2.1). However, for low energy dynamics at scales below v_{PQ} , it is convenient to construct an effective lagrangian at a cutoff scale Λ *just below* v_{PQ} , which can be obtained by integrating out the massive fields heavier than Λ as well as the high momentum modes of light fields at scales above Λ . In our case, two chiral superfields among X , Y and Z get a mass of $\mathcal{O}(v_{\text{PQ}})$, while the remained combination provides the axion supermultiplet. To integrate out the heavy fields, one can parameterize them as

$$X = \frac{1}{\sqrt{2}}(v_{\text{PQ}} + \rho_1) e^{-A/v_{\text{PQ}}}, \quad Y = \frac{1}{\sqrt{2}}(v_{\text{PQ}} + \rho_1) e^{A/v_{\text{PQ}}}, \quad Z = \rho_2, \quad (2.10)$$

for which the Kähler potential and superpotential at the UV scale M_* are given by

$$\begin{aligned} K &= |v_{\text{PQ}} + \rho_1|^2 \cosh\left(\frac{A + A^\dagger}{v_{\text{PQ}}}\right) + \rho_2^\dagger \rho_2 + Q^\dagger Q + Q^{c\dagger} Q^c, \\ W &= h v_{\text{PQ}} \rho_1 \rho_2 + \frac{1}{2} h \rho_2 \rho_1^2 + \left(M_Q + \sqrt{2} \frac{M_Q}{v_{\text{PQ}}} \rho_1\right) e^{-A/v_{\text{PQ}}} Q Q^c, \end{aligned} \quad (2.11)$$

where ρ_i ($i = 1, 2$) are massive chiral matter fields, and

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\theta\tilde{a} + \theta^2 F^A \quad (2.12)$$

is the axion superfield which contains the axion a , the saxion s and the axino \tilde{a} as its component fields.

After integrating out the heavy ρ_i and the high momentum modes of light fields, the resulting Wilsonian effective lagrangian can be chosen to take the following form

$$\begin{aligned} \mathcal{L}_{\text{eff}}(\Lambda) &= \int d^4\theta \left[K_A(A + A^\dagger) + Z_Q(A + A^\dagger) Q^\dagger Q + Z_{Q^c}(A + A^\dagger) Q^{c\dagger} Q^c \right] \\ &+ \left[\int d^2\theta \left(\frac{1}{4} f_a^{\text{eff}}(A) W^{a\alpha} W_\alpha^a + W_{\text{eff}} \right) + \text{h.c.} \right], \end{aligned} \quad (2.13)$$

where

$$\begin{aligned} K_A &= \frac{1}{2}(A + A^\dagger)^2 + \mathcal{O}\left(\frac{(A + A^\dagger)^3}{v_{\text{PQ}}}\right) \\ f_a^{\text{eff}} &= \frac{1}{\hat{g}_s^2(\Lambda)} - \frac{C_W}{8\pi^2} \frac{A}{v_{\text{PQ}}}, \\ W_{\text{eff}} &= M_Q e^{-(\tilde{x}_Q + \tilde{x}_{Q^c})A/v_{\text{PQ}}} Q Q^c. \end{aligned} \quad (2.14)$$

With the above form of effective lagrangian, the PQ symmetry is realized as

$$U(1)_{PQ} : \quad A \rightarrow A + i\alpha v_{PQ}, \quad \Phi \rightarrow e^{i\tilde{x}_\Phi \alpha} \Phi \quad (\Phi = Q, Q^c), \quad (2.15)$$

and the PQ anomaly in the effective theory is given by

$$C_{PQ} = C_W + \tilde{x}_Q + \tilde{x}_{Q^c}, \quad (2.16)$$

which should be same as the PQ anomaly (2.6) in the underlying UV theory. For generic form of effective lagrangian consistent with the PQ symmetry (2.15), there can be three types of axino effective interactions at the order of $1/v_{PQ}$:

$$\Delta_1 \mathcal{L}(\Lambda) = - \int d^2\theta \frac{C_W}{32\pi^2} \frac{A}{v_{PQ}} W^{a\alpha} W_\alpha^a + \text{h.c.}, \quad (2.17)$$

$$\Delta_2 \mathcal{L}(\Lambda) = \int d^4\theta \frac{(A + A^\dagger)}{v_{PQ}} \left(\tilde{y}_Q Q^\dagger Q + \tilde{y}_{Q^c} Q^{c\dagger} Q^c \right), \quad (2.18)$$

$$\Delta_3 \mathcal{L}(\Lambda) = - \int d^2\theta (\tilde{x}_Q + \tilde{x}_{Q^c}) M_Q \frac{A}{v_{PQ}} Q Q^c + \text{h.c.}, \quad (2.19)$$

where

$$\tilde{y}_\Phi \equiv v_{PQ} \left. \frac{\partial \ln Z_\Phi}{\partial A} \right|_{A=0} \quad (\Phi = Q, Q^c). \quad (2.20)$$

In our particular case, one easily finds

$$C_W = 0, \quad \tilde{x}_\Phi = x_\Phi, \quad \tilde{y}_\Phi(\Lambda) = \mathcal{O} \left(\frac{1}{16\pi^2} \frac{M_Q^2}{v_{PQ}^2} \ln \left(\frac{M_*}{\Lambda} \right) \right), \quad (2.21)$$

where the small \tilde{y}_Φ is induced by the radiative corrections involving the Yukawa coupling $\lambda = \sqrt{2}M_Q/v_{PQ}$ at UV scales above Λ . This result shows that the axion supermultiplet is decoupled from the gauge multiplet and charged-matter multiplet in the limit $M_Q \rightarrow 0$, which is manifest in the UV theory (2.11). Note that the PQ symmetry splits into two global U(1) symmetries in the limit $M_Q \rightarrow 0$, the axial U(1) symmetry of Q, Q^c and the anomaly free U(1) not involving the transformation of Q, Q^c , and the axion becomes a massless Goldstone boson of the anomaly-free U(1) part in the limit $M_Q \rightarrow 0$.

In fact, one can consider a different description of the same effective theory, for instance a scheme using different form of PQ symmetry under which all light fields at Λ are invariant except the axion superfield. In our case, such description can be achieved by the A -dependent field redefinition $\Phi \rightarrow e^{x_\Phi A/v_{PQ}} \Phi$, making all matter fields except A invariant under the PQ symmetry. Such form of PQ symmetry can be convenient in certain respects since $\tilde{x}_\Phi = 0$ ($\Phi = Q, Q^c$), so the PQ anomaly originates entirely from the variation of the local effective interaction (2.17), which results in $C_{PQ} = C_W$. However such field basis is not convenient for a discussion of physics at energy scales above M_Q since the decoupling of the axion supermultiplet from the gauge and charged-matter multiplets in the limit $M_Q \rightarrow 0$ is not manifest, but is achieved by the cancellation between the contributions from $C_W = N_Q$ and $\tilde{y}_Q + \tilde{y}_{Q^c} = 1$.

To get more insights, let us consider a more general description associated with the field redefinition

$$\Phi \rightarrow e^{cx_\Phi A/v_{\text{PQ}}} \Phi \quad (\Phi = Q, Q^c), \quad (2.22)$$

where c is an arbitrary real constant parameterizing the field basis. After this field redefinition, the PQ symmetry is realized as

$$\text{U}(1)_{\text{PQ}} : \quad A \rightarrow A + i\alpha v_{\text{PQ}}, \quad \Phi \rightarrow e^{i(1-c)x_\Phi \alpha} \Phi, \quad (2.23)$$

and there appears an axion-dependent local counter term in the effective lagrangian

$$- \int d^2\theta \frac{c(x_Q + x_{Q^c})}{32\pi^2} \frac{A}{v_{\text{PQ}}} W^{a\alpha} W_\alpha^a + \text{h.c.}, \quad (2.24)$$

which is due to the Konishi anomaly [47] for the field redefinition (2.22). Note that this Konishi anomaly term is required to match the PQ anomaly of (2.23) with the PQ anomaly (2.6). Including the Konishi anomaly, Wilsonian effective couplings of the axion superfield in the redefined field basis are given by

$$\begin{aligned} C_W &= c(x_Q + x_{Q^c}), \quad \tilde{x}_\Phi = (1-c)x_\Phi, \\ \tilde{y}_\Phi(\Lambda) &= cx_\Phi + \mathcal{O}\left(\frac{1}{16\pi^2} \frac{M_Q^2}{v_{\text{PQ}}^2} \ln\left(\frac{M_*}{\Lambda}\right)\right). \end{aligned} \quad (2.25)$$

Note that the PQ anomaly in the redefined field basis is determined by two contributions:

$$C_{\text{PQ}} = c(x_Q + x_{Q^c}) + (1-c)(x_Q + x_{Q^c}), \quad (2.26)$$

where the first piece is from the variation of the local effective interaction (2.24), while the second piece is from the axial anomaly of the PQ transformation of Q, Q^c in (2.23). Obviously each contribution depends on the field basis parameter c , but their sum is independent of c as it should be.

Now, if one ignores the part of \tilde{y}_Φ suppressed by M_Φ^2/v_{PQ}^2 , Wilsonian effective interactions of the axion supermultiplet are given by

$$\Delta_1 \mathcal{L}(\Lambda) = - \int d^2\theta \frac{c(x_Q + x_{Q^c})}{32\pi^2} \frac{A}{v_{\text{PQ}}} W^{a\alpha} W_\alpha^a + \text{h.c.}, \quad (2.27)$$

$$\Delta_2 \mathcal{L}(\Lambda) = \int d^4\theta \frac{(A + A^\dagger)}{v_{\text{PQ}}} \left(cx_Q Q^\dagger Q + cx_{Q^c} Q^{c\dagger} Q^c \right) \quad (2.28)$$

$$\Delta_3 \mathcal{L}(\Lambda) = - \int d^2\theta (1-c)(x_Q + x_{Q^c}) M_Q \frac{A}{v_{\text{PQ}}} Q Q^c + \text{h.c.} \quad (2.29)$$

With these, one can compute the axino production rate at any temperature $T < \Lambda$, and the result should be independent of the field basis parameter c since all physical quantities should be basis-independent. However, most of the previous studies use only the effective interaction of the form (2.27), which can lead to a misleading result as we will see below. In fact, considering only the effective interaction (2.27) leads to a highly overestimated

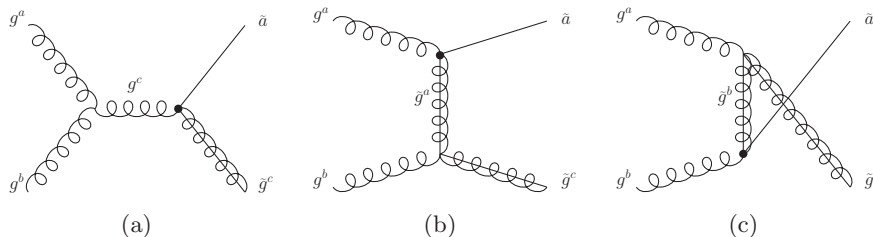


Figure 1. Diagrams for the process $g + g \rightarrow \tilde{g} + \tilde{a}$.

result for the axino production at energy scale $p \gg M_Q$. At such high energy scale, one can take a limit $M_Q/p \rightarrow 0$ in which the axion supermultiplet is decoupled from the gauge multiplets and charged-matter multiplets. Although such decoupling is not manifest in the effective interactions (2.27)–(2.29) with $c \neq 0$, it is manifest in the field basis with $c = 0$, and also in the underlying UV theory (2.11). This means that axino production by gauge supermultiplets at $p > M_Q$ should be suppressed by some powers of M_Q/p if one takes into account the interactions (2.27)–(2.29) altogether to get a correct c -independent result. On the other hand, the interaction (2.27) by itself does not involve any suppression by M_Q/p , and therefore the analysis using (2.27) alone gives a highly overestimated result in the limit $p \gg M_Q$.

More explicitly, the effective interaction (2.27) gives the Wilsonian axino-gluino-gluon amplitude

$$\mathcal{A}_W(k, q, p) = -\frac{g^2 c(x_Q + x_{Q^c})}{16\pi^2 \sqrt{2} v_{\text{PQ}}} \delta^4(k + q + p) \bar{u}(k) \sigma_{\mu\nu} \gamma_5 v(q) \epsilon^\mu p^\nu, \quad (2.30)$$

where p^μ and ϵ^μ are the gluon momentum and polarization, and $u(k)$ and $v(q)$ are the 4-component Majorana spinor wavefunction of the axino and gluino, respectively. If one uses this amplitude alone to compute the axino production rate in the process gluon + gluon \rightarrow gluino + axino (see figure 1), a simple dimensional analysis tells that the rate (per unit volume) is given by

$$\Gamma(gg \rightarrow \tilde{g}\tilde{a}) = c^2 (x_Q + x_{Q^c})^2 \frac{\xi g_s^6 T^6}{(16\pi^2 v_{\text{PQ}})^2}, \quad (2.31)$$

where ξ is a dimensionless coefficient which is independent of c . However this can not be the correct answer as it depends on the field-basis parameter c , and there should be additional contribution which removes the c -dependence of the result. Indeed, including the contributions from the axino-gluon-gluino amplitude due to the loops of Q, Q^c (see figure 2), which involve the c -dependent axino-matter couplings in (2.28) and (2.29), we find that the c -dependence in the production rate disappears as required, and the final result takes the form

$$\Gamma(gg \rightarrow \tilde{g}\tilde{a}) = \gamma^2 \frac{\xi g_s^6 T^6}{(16\pi^2 v_{\text{PQ}})^2}, \quad (2.32)$$

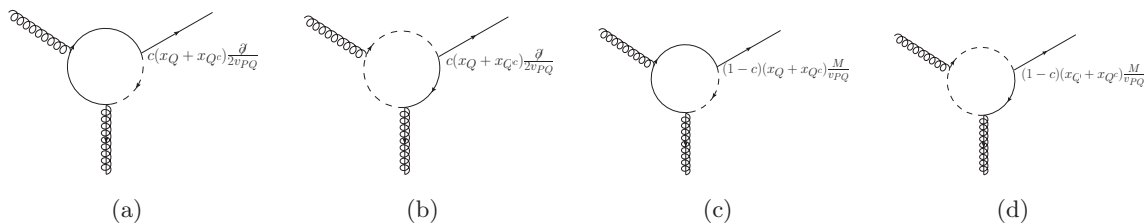


Figure 2. Contributions to the 1PI axino-gluino-gluon amplitudes from the loops of Q, Q^c .

where

$$\begin{aligned} \gamma &= \mathcal{O}\left(\frac{M_Q^2}{T^2} \ln^2\left(\frac{T}{M_Q}\right)\right) \quad \text{for } T > M_Q, \\ \gamma &= (x_Q + x_{Q^c}) + \mathcal{O}\left(\frac{T^2}{M_Q^2}\right) \quad \text{for } T < M_Q, \end{aligned} \quad (2.33)$$

which shows that the production rate is indeed suppressed by M_Q^2/T^2 at $T > M_Q$. This result can be obtained by replacing the axino-gluino-gluon vertex in figure 1 with the one-particle-irreducible (1PI) axino-gluino-gluon amplitude

$$\mathcal{A}_{1\text{PI}}(k, q, p) = -\frac{g^2 \tilde{C}_{1\text{PI}}(k, q, p)}{16\pi^2 \sqrt{2} v_{PQ}} \delta^4(k + q + p) \bar{u}(k) \sigma_{\mu\nu} \gamma_5 v(q) \epsilon^\mu p^\nu \quad (2.34)$$

which has the following limiting behavior (see the appendix A) when the axino and gluino are on mass-shell:

$$\begin{aligned} \tilde{C}_{1\text{PI}}(k^2 = q^2 = 0; |p^2| \gg M_Q^2) &= (x_Q + x_{Q^c}) \frac{M_Q^2}{|p^2|} \left[\ln^2\left(\frac{|p^2|}{M_Q^2}\right) + \mathcal{O}\left(\frac{M_Q^2}{|p^2|} \ln\left(\frac{|p^2|}{M_Q^2}\right)\right) \right], \\ \tilde{C}_{1\text{PI}}(k^2 = q^2 = 0; |p^2| \ll M_Q^2) &= x_Q + x_{Q^c} + \mathcal{O}\left(\frac{p^2}{M_Q^2}\right). \end{aligned} \quad (2.35)$$

Generically the 1PI amplitude due to light particle loops includes non-local and non-analytic piece in the limit $p^2 \gg M_{\text{light}}^2$, while the contribution from heavy particle loop allows a local expansion in the limit $p^2 \ll M_{\text{heavy}}^2$. For the 1PI axino-gaugino-gauge boson amplitude, it takes the following form (see the appendix A) when the axino and gaugino (or gauge boson) are on mass-shell:

$$\tilde{C}_{1\text{PI}} = C_{1\text{PI}} + \mathcal{O}\left(\frac{M_{\text{light}}^2}{p^2} \ln^2\left(\frac{p^2}{M_{\text{light}}^2}\right)\right) + \mathcal{O}\left(\frac{p^2}{M_{\text{heavy}}^2}\right), \quad (2.36)$$

where $C_{1\text{PI}}$ can have a logarithmic p -dependence due to higher loop effects. The constant (or logarithmic) $C_{1\text{PI}}$ can be encoded in the PQ-invariant real 1PI gauge kinetic function \mathcal{F}_a which defines the 1PI gauge kinetic term [48–50]:

$$\int d^4\theta \frac{1}{16} \mathcal{F}_a(\ln p^2, A + A^\dagger) W^{a\alpha} \frac{D^2}{p^2} W_\alpha^a, \quad (2.37)$$

and then $C_{1\text{PI}}^a$ is determined as

$$C_{1\text{PI}} = -8\pi^2 v_{\text{PQ}} \left. \frac{\partial \mathcal{F}_a}{\partial A} \right|_{A=0}. \quad (2.38)$$

This 1PI operator should be distinguished from the Wilsonian gauge kinetic term

$$\int d^2\theta \frac{1}{4} f_a^{\text{eff}}(\ln \Lambda, A) W^{a\alpha} W_\alpha^a + \text{h.c.} \quad (2.39)$$

which determines C_W as

$$C_W = -8\pi^2 v_{\text{PQ}} \left. \frac{\partial f_a^{\text{eff}}}{\partial A} \right|_{A=0}. \quad (2.40)$$

As we have noticed, C_W is a basis-dependent Wilsonian coupling which changes under an A -dependent holomorphic redefinition of charged matter fields, while $C_{1\text{PI}}$ is an observable amplitude invariant under the redefinition of matter fields.

In the above, we find in the context of a simple supersymmetric KSVZ axion model that the 1PI axino-gaugino-gauge boson amplitude at energy scale in the range $M_\Phi < p < v_{\text{PQ}}$ is suppressed by M_Φ^2/p^2 in addition to the well-known suppression by $p/16\pi^2 v_{\text{PQ}}$, where M_Φ is the supersymmetric mass of the heaviest PQ-charged and gauge-charged matter field, which can be well below v_{PQ} . On the other hand, the 1PI amplitude at energy scales below M_Φ has the same scaling behavior as the Wilsonian amplitude (2.30), which can be noticed from (2.35), although the precise coefficient can be different in general. This observation applies also to another type of popular axion model, the DFSZ model [32, 33]. To see this, let us consider a supersymmetric extension of the DFSZ axion model described by the following form of the Kähler potential, gauge kinetic function and superpotential at the UV scale M_* :

$$\begin{aligned} K &= \sum_I \Phi_I^\dagger \Phi_I, & f_a &= \frac{1}{\hat{g}_a^2(M_*)}, \\ W &= hZ \left(XY - \frac{v_{\text{PQ}}^2}{2} \right) + \kappa \frac{X^2}{M_*} H_u H_d + \dots, \end{aligned} \quad (2.41)$$

where $\{\Phi_I\}$ denote the matter fields in the model, including the MSSM Higgs, quark and lepton superfields, and the ellipsis stands for the PQ-invariant Yukawa couplings for the quark and lepton masses. The matter fields transform under the PQ symmetry as

$$\text{U}(1)_{\text{PQ}} : \quad \Phi_I \rightarrow e^{ix_I \alpha} \Phi_I \quad (2.42)$$

with the PQ charges

$$(x_Z, x_X, x_Y, x_{H_u}, x_{H_d}) = (0, -1, 1, 1, 1), \quad (2.43)$$

and the PQ charges of the quark and lepton superfields can be fixed by the MSSM Yukawa couplings. Here the Higgs μ -term is generated by the spontaneous breakdown of the PQ symmetry [10, 11, 51–53], yielding

$$\mu \sim \frac{\kappa v_{\text{PQ}}^2}{2M_*}. \quad (2.44)$$

The most notable feature of this DFSZ model is that the Higgs doublets correspond to the heaviest PQ-charged and gauge-charged matter field, so $M_\Phi = \mu$ which should be around the weak scale. It is straightforward to repeat our discussion for this DFSZ model, and then one finds the 1PI axino-gaugino-gauge boson amplitude at $p > \mu$ is suppressed by μ^2/p^2 . This would lead to a dramatic reduction of the cosmological axino production, when compared to the result of the previous analysis using only the effective interaction of the form (2.17). It should be noted that a similar reduction is possible for the KSVZ model also as $M_\Phi = M_Q$ can be in principle comparable to the weak scale.

Our result can have important implications for the cosmology of supersymmetric axion model. For instance, certain parameter range of the model and/or the cosmological scenario, which has been considered to give a too large relic axino mass density in the previous analysis, can be safe if the model is assumed to have $M_\Phi \ll v_{\text{PQ}}$. It also implies that at temperature $T > M_\Phi$, axinos are produced dominantly by the processes involving the heaviest PQ-charged and gauge-charged matter supermultiplet, e.g. $Q + \text{gluon} \rightarrow \tilde{Q} + \tilde{a}$ for the KSVZ model and $\text{higgs} + W \rightarrow \text{higgsino} + \tilde{a}$ for the DFSZ model, since the amplitude of such process does not involve the loop suppression factor and is suppressed only by a single power of M_Φ/v_{PQ} . In the next section, we compute the axino production rate and the resulting relic axino number density while including the production by the heaviest PQ-charged and gauge-charged matter multiplet for both the KSVZ and DFSZ models.

It is in fact straightforward to generalize our discussion to generic supersymmetric axion models. For this, let us consider a general form of Wilsonian effective lagrangian of the axion superfield at a scale Λ just below v_{PQ} , which takes the form

$$\begin{aligned} \mathcal{L}_{\text{eff}}(\Lambda) = & \int d^4\theta \left(K_A(A + A^\dagger) + Z_n(A + A^\dagger)\Phi_n^\dagger\Phi_n \right) \\ & + \left[\int d^2\theta \left(\frac{1}{4}f_a^{\text{eff}}(A)W^{a\alpha}W_\alpha^a + W_{\text{eff}} \right) + \text{h.c.} \right], \end{aligned} \quad (2.45)$$

where $\{\Phi_n\}$ denote the light gauge-charged matter fields at Λ , and

$$\begin{aligned} K_A &= \frac{1}{2}(A + A^\dagger)^2 + \mathcal{O}\left(\frac{(A + A^\dagger)^3}{v_{\text{PQ}}}\right), \\ \ln Z_n &= \ln Z_n|_{A=0} + \tilde{y}_n \frac{(A + A^\dagger)}{v_{\text{PQ}}} + \mathcal{O}\left(\frac{(A + A^\dagger)^2}{v_{\text{PQ}}^2}\right), \\ f_a^{\text{eff}} &= \frac{1}{\hat{g}_a^2(\Lambda)} - \frac{C_W^a}{8\pi^2} \frac{A}{v_{\text{PQ}}}, \\ W_{\text{eff}} &= \frac{1}{2}e^{-(\tilde{x}_n + \tilde{x}_m)A/v_{\text{PQ}}} M_{mn} \Phi_m \Phi_n + \frac{1}{6}e^{-(\tilde{x}_n + \tilde{x}_m + \tilde{x}_p)A/v_{\text{PQ}}} \lambda_{mnp} \Phi_m \Phi_n \Phi_p. \end{aligned} \quad (2.46)$$

The PQ symmetry in this effective theory is realized as

$$\text{U}(1)_{\text{PQ}} : \quad A \rightarrow A + i\alpha v_{\text{PQ}}, \quad \Phi_n \rightarrow e^{i\tilde{x}_n\alpha} \Phi_n, \quad (2.47)$$

and the Wilsonian effective interactions of the axion superfield are given by

$$\Delta_1 \mathcal{L}(\Lambda) = - \int d^2\theta \frac{C_W^a}{32\pi^2} \frac{A}{v_{\text{PQ}}} W^{\alpha\alpha} W_\alpha^a, \quad (2.48)$$

$$\Delta_2 \mathcal{L}(\Lambda) = \int d^4\theta \tilde{y}_n \frac{(A + A^\dagger)}{v_{\text{PQ}}} \Phi_n^\dagger \Phi_n, \quad (2.49)$$

$$\begin{aligned} \Delta_3 \mathcal{L}(\Lambda) = & - \int d^2\theta \frac{A}{v_{\text{PQ}}} \left[\frac{1}{2} (\tilde{x}_m + \tilde{x}_n) M_{mn} \Phi_n \Phi_m \right. \\ & \left. + \frac{1}{6} (\tilde{x}_m + \tilde{x}_n + \tilde{x}_p) \lambda_{mnp} \Phi_m \Phi_n \Phi_p \right]. \end{aligned} \quad (2.50)$$

According to our discussion, there are three quantities which are all related to the axino coupling to gauge supermultiplets, but basically different from each other:

$$\{ C_W^a, C_{\text{PQ}}^a, C_{\text{1PI}}^a \}, \quad (2.51)$$

where C_W^a are the Wilsonian couplings in (2.48), C_{PQ}^a are the PQ anomaly coefficients defined as

$$\partial_\mu J_{\text{PQ}}^\mu = \frac{g^2}{16\pi^2} C_{\text{PQ}}^a F^{a\mu\nu} \tilde{F}_{\mu\nu}^a, \quad (2.52)$$

and finally C_{1PI}^a determines the leading part of the 1PI axino-gaugino-gauge boson amplitude

$$\mathcal{A}_{\text{1PI}}^a(k, q, p) = - \frac{g^2}{16\pi^2 \sqrt{2} v_{\text{PQ}}} \tilde{C}_{\text{1PI}}^a \delta^4(k + q + p) \bar{u}(k) \sigma_{\mu\nu} \gamma_5 v(q) \epsilon^\mu p^\nu \quad (2.53)$$

which shows the behavior

$$\begin{aligned} \tilde{C}_{\text{1PI}}^a(k^2 = q^2 = 0, M_{\text{light}}^2 < p^2 < M_{\text{heavy}}^2) \\ = C_{\text{1PI}}^a + \mathcal{O} \left(\frac{M_{\text{light}}^2}{p^2} \ln^2 \left(\frac{p^2}{M_{\text{light}}^2} \right) \right) + \mathcal{O} \left(\frac{p^2}{M_{\text{heavy}}^2} \right), \end{aligned} \quad (2.54)$$

where M_{light} and M_{heavy} denote the masses of matter fields in the effective theory (2.45). It is then straightforward to find

$$C_{\text{PQ}}^a = C_W^a + 2 \sum_n \tilde{x}_n \text{Tr}(T_a^2(\Phi_n)), \quad (2.55)$$

$$C_{\text{1PI}}^a(M_\Phi^2 < p^2 < \Lambda^2) = C_W^a - 2 \sum_n \tilde{y}_n \text{Tr}(T_a^2(\Phi_n)), \quad (2.56)$$

where M_Φ is the mass of the heaviest PQ-charged and gauge-charged matter field in the model. Here the expression of C_{PQ}^a is exact and valid at any scale below v_{PQ} , while the expression of C_{1PI}^a is derived in 1-loop approximation. It is also straightforward (see the appendix A) to compute C_{1PI}^a at lower momentum scale, which yields

$$\begin{aligned} C_{\text{1PI}}^a(p^2 < M_\Phi^2) = & C_W^a(\Lambda) - 2 \sum_{M_n^2 < p^2} \tilde{y}_n(\Lambda) \text{Tr}(T_a^2(\Phi_n)) \\ & + 2 \sum_{M_n^2 > p^2} \tilde{x}_n(\Lambda) \text{Tr}(T_a^2(\Phi_n)) \end{aligned} \quad (2.57)$$

in 1-loop approximation.

In fact, one can easily derive an exact (in perturbation theory) expression of C_{1PI}^a , including the piece depending logarithmically on the momentum scale p . We already noticed that C_{1PI}^a can be determined by the real 1PI gauge kinetic function \mathcal{F}_a as

$$C_{\text{1PI}}^a = -8\pi^2 v_{\text{PQ}} \left. \frac{\partial \mathcal{F}_a}{\partial A} \right|_{A=0}. \quad (2.58)$$

Solving the Novikov-Shifman-Vainshtein-Zakharov RG equation in the limit $M_\Phi^2/p^2 \rightarrow 0$, one finds [48–50]

$$\mathcal{F}_a = \text{Re}(f_a^{\text{eff}}) + \frac{b_a}{16\pi^2} \ln\left(\frac{p^2}{\Lambda^2}\right) - \sum_n \frac{\text{Tr}(T_a^2(\Phi_n))}{8\pi^2} \ln \mathcal{Z}_n + \frac{\text{Tr}(T_a^2(G))}{8\pi^2} \ln \mathcal{F}_a, \quad (2.59)$$

where $b_a = -3\text{Tr}(T_a^2(G)) + \sum_n \text{Tr}(T_a^2(\Phi_n))$ is the one-loop beta function coefficient, and $\gamma_n = d \ln \mathcal{Z}_n / d \ln p^2$ is the anomalous dimension of Φ_n for the 1PI wavefunction coefficient \mathcal{Z}_n which can be chosen to satisfy the matching condition

$$\mathcal{Z}_n(p^2 = \Lambda^2) = Z_n(\Lambda). \quad (2.60)$$

We then find

$$C_{\text{1PI}}^a(M_\Phi^2 < p^2 < \Lambda^2) = \frac{C_W^a - 2 \sum_n \tilde{y}_n(p) \text{Tr}(T_a^2(\Phi_n))}{1 - \text{Tr}(T_a^2(G))g_a^2(p)/8\pi^2}, \quad (2.61)$$

where

$$\tilde{y}_n(p) = v_{\text{PQ}} \left. \frac{\partial \ln \mathcal{Z}_n}{\partial A} \right|_{A=0}, \quad (2.62)$$

which gives (2.56) at 1-loop approximation.

Within the effective theory (2.45), one can make a holomorphic redefinition of matter fields

$$\Phi_n \rightarrow e^{z_n A / v_{\text{PQ}}} \Phi_n, \quad (2.63)$$

after which the PQ symmetry takes the form

$$\text{U}(1)_{\text{PQ}} : \quad A \rightarrow A + i\alpha v_{\text{PQ}}, \quad \Phi_n \rightarrow e^{i(\tilde{x}_n - z_n)\alpha} \Phi_n, \quad (2.64)$$

and the Wilsonian couplings of the axion superfield are changed as

$$\begin{aligned} C_W^a &\rightarrow C_W^a + 2 \sum_n z_n \text{Tr}(T_a^2(\Phi_n)), \\ \tilde{y}_n &\rightarrow \tilde{y}_n + z_n, \quad \tilde{x}_n \rightarrow \tilde{x}_n - z_n. \end{aligned} \quad (2.65)$$

This shows that one can always choose a field basis with $\tilde{x}_n = 0$, for which only the axion superfield transforms under the PQ symmetry, and therefore $C_W^a = C_{\text{PQ}}^a$. Another interesting choice would be the field basis with $\tilde{y}_n = 0$, for which $C_W^a = C_{\text{1PI}}^a$. Note that C_{PQ}^a and C_{1PI}^a are directly linked to observables, and therefore invariant under the reparametrization (2.65) of the Wilsonian couplings associated with the field redefinition (2.63). It should

be noticed also that for a given theory, C_{PQ}^a have common values at all scales, while C_{1PI}^a can have different values at different momentum scales. On the other hand, C_W^a are lagrangian parameters which can take different values in different field basis (or in different UV regularization) within the same theory.

A key result of our discussion, which has direct implication for cosmological axino production, is that the 1PI axino-gaugino-gauge boson amplitude in the momentum range $M_\Phi^2 < p^2 < v_{\text{PQ}}^2$ is suppressed by M_Φ^2/p^2 , more specifically²

$$\begin{aligned} \tilde{C}_{\text{1PI}}^a(M_\Phi^2 < p^2 < v_{\text{PQ}}^2) &= C_{\text{1PI}}^a(M_\Phi^2 < p^2 < v_{\text{PQ}}^2) + \mathcal{O}\left(\frac{M_\Phi^2}{p^2} \ln^2\left(\frac{M_\Phi^2}{p^2}\right)\right) \\ &= \mathcal{O}\left(\frac{M_\Phi^2}{p^2} \ln^2\left(\frac{M_\Phi^2}{p^2}\right)\right), \end{aligned} \tag{2.66}$$

where M_Φ is the supersymmetric mass of the heaviest PQ-charged and gauge-charged matter field in the model, which can be well below v_{PQ} in most cases. As we will see, there can be contributions to the above 1PI amplitude from UV dynamics at scales above v_{PQ} , but they are generically of $\mathcal{O}(M_\Phi^2 \ln(M_*/v_{\text{PQ}})/8\pi^2 v_{\text{PQ}}^2)$ or $\mathcal{O}(v_{\text{PQ}}^2/M_*^2)$, where M_* is the fundamental scale of the model, e.g. the Planck scale or the GUT scale, which is presumed to be far above the PQ scale. These UV contributions are either smaller than (2.66) or negligible by itself when M_* is comparable to the Planck or GUT scale. As we will argue below, the result (2.66) applies to generic supersymmetric axion model if the model has a UV realization at M_* , in which (i) the PQ symmetry is linearly realized in the standard manner,

$$\text{U(1)}_{\text{PQ}} : \quad \Phi_I \rightarrow e^{ix_I \alpha} \Phi_I, \tag{2.67}$$

where $\{\Phi_I\}$ stand for generic chiral matter superfields, and (ii) all higher dimensional operators of the model are suppressed by appropriate powers of $1/M_*$.

To proceed, let $\{\Phi_A\}$ denote the gauge-singlet but generically PQ-charged matter fields, whose VEVs break U(1)_{PQ} spontaneously, and $\{\Phi_n\}$ denote the gauge-charged matter fields in the model. Then the Kähler potential and superpotential at the UV scale M_* can be expanded in powers of the gauge-charged matter fields as follows

$$\begin{aligned} K &= K_{\text{PQ}}(\Phi_A^\dagger, \Phi_A) + \left(1 + \frac{\kappa_{\bar{A}Bn}}{M_*^2} \Phi_A^\dagger \Phi_B + \dots\right) \Phi_n^\dagger \Phi_n + \dots, \\ W &= W_{\text{PQ}}(\Phi_A) + \frac{1}{2} \left(\hat{\lambda}_{Amn} \Phi_A + \frac{\hat{\lambda}_{ABmn}}{M_*} \Phi_A \Phi_B + \dots\right) \Phi_m \Phi_n \\ &\quad + \frac{1}{6} \left(\hat{\lambda}_{mnp} + \frac{\hat{\lambda}_{Amnp}}{M_*} \Phi_A + \dots\right) \Phi_m \Phi_n \Phi_p + \dots, \end{aligned} \tag{2.68}$$

where K_{PQ} and W_{PQ} are the Kähler potential and superpotential of the PQ sector fields $\{\Phi_A\}$, and the ellipses stand for higher dimensional terms. Here we assume that the Kähler

² This is for the kinematic regime with a gauge boson (or gaugino) 4-momentum having $|p^2| > M_\Phi^2$, while the axino and gaugino (or gauge boson) 4-momenta are on mass-shell. \tilde{C}_{1PI}^a can have a bit different behavior in other kinematic regimes, but always suppressed by M_Φ^2/p^2 if any of the external particles has a 4-momentum p with $|p^2| > M_\Phi^2$. For instance, if any of the external particles has a vanishing 4-momentum, while the other particles have $|p^2| > M_\Phi^2$, the amplitude is of $\mathcal{O}\left(\frac{M_\Phi^2}{p^2} \ln(p^2/M_\Phi^2)\right)$.

metric of the gauge-charged matter fields Φ_n is flavor-diagonal for simplicity. Also, to be complete, we include the leading higher dimensional operators suppressed by $1/M_*$. Under the assumption that K_{PQ} and W_{PQ} provide a proper dynamics to break the PQ symmetry spontaneously, we can parameterize the PQ sector fields as follows

$$\Phi_A = \left(\frac{1}{\sqrt{2}} v_A + U_{Ai} \rho_i \right) e^{x_A A / v_{\text{PQ}}}, \quad (2.69)$$

where $v_A = \langle \Phi_A \rangle$ with $v_{\text{PQ}}^2 = \sum_A x_A^2 |v_A|^2$, ρ_i denote the massive chiral superfields in the PQ sector, and U_{Ai} are the mixing coefficients which are generically of order unity. For this parametrization of the PQ sector fields, the Kähler potential and superpotential at M_* take the form

$$\begin{aligned} K &= K_{\text{PQ}}(\rho_i^\dagger, \rho_i, A + A^\dagger) + \left(Z_n^{(0)} + \mathcal{O}\left(\frac{v_{\text{PQ}}}{M_*^2}(A + A^\dagger), \frac{v_{\text{PQ}} \rho_i}{M_*^2}, \frac{v_{\text{PQ}} \rho_i^\dagger}{M_*^2}\right) \right) \Phi_n^\dagger \Phi_n + \dots, \\ W &= W_{\text{PQ}}(\rho_i) + \frac{1}{2} \left(M_{mn} + \mathcal{O}\left(\frac{M_{mn} \rho_i}{v_{\text{PQ}}}\right) \right) e^{-(x_m + x_n)A / v_{\text{PQ}}} \Phi_m \Phi_n \\ &\quad + \frac{1}{6} \left(\lambda_{mnp} + \mathcal{O}\left(\frac{\rho_i}{M_*}\right) \right) e^{-(x_m + x_n + x_p)A / v_{\text{PQ}}} \Phi_m \Phi_n \Phi_p + \dots, \end{aligned} \quad (2.70)$$

where $Z_n^{(0)}$ and M_{mn} are field-independent constants, and the Yukawa coupling constants λ_{mnp} obeys the PQ selection rule

$$(x_m + x_n + x_p) \lambda_{mnp} = (x_m + x_n + x_p) \left(\hat{\lambda}_{mnp} + \mathcal{O}\left(\frac{v_{\text{PQ}}}{M_*}\right) \right) = \mathcal{O}\left(\frac{v_{\text{PQ}}}{M_*}\right). \quad (2.71)$$

One can now integrate out the massive ρ_i as well as the high momentum modes of light fields, and also make the field redefinition (2.63) to derive an effective theory in generic field basis. The resulting effective lagrangian at the scale Λ just below v_{PQ} takes the form of (2.46) with

$$\begin{aligned} C_W^a &= -8\pi^2 v_{\text{PQ}} \frac{\partial f_a^{\text{eff}}}{\partial A} = 2 \sum_n z_n \text{Tr}(T_a^2(\Phi_n)), \\ \tilde{x}_n &= x_n - z_n, \\ \tilde{y}_n(\Lambda) &= v_{\text{PQ}} \frac{\partial \ln Z_n}{\partial A} \Big|_{A=0} = z_n + \mathcal{O}\left(\frac{(x_m + x_n) M_{mn}^2}{8\pi^2 v_{\text{PQ}}^2} \ln\left(\frac{M_*}{\Lambda}\right)\right) + \mathcal{O}\left(\frac{v_{\text{PQ}}^2}{M_*^2}\right) \\ &= z_n + \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}\right) + \mathcal{O}\left(\frac{v_{\text{PQ}}^2}{M_*^2}\right), \end{aligned} \quad (2.72)$$

where we have set $\ln(M_*/v_{\text{PQ}}) = \mathcal{O}(\pi^2)$ and used $(x_m + x_n)M_{mn} \lesssim \mathcal{O}(M_\Phi)$ for M_Φ denoting the mass of the heaviest PQ-charged and gauge-charged matter field in the model. Here C_W^a and the first part of \tilde{y}_n are due to the field redefinition (2.63), the second part of \tilde{y}_n is from the loops involving the Yukawa couplings between the PQ sector fields and the gauge-charged matter fields, which are generically of $\mathcal{O}(M_{mn}/v_{\text{PQ}})$ and depend on the axion superfield through the combination $(x_m + x_n)A$, and finally the last part of \tilde{y}_n is from

the higher dimensional operator in the Kähler potential of Φ_n . There can be additional contribution to \tilde{y}_n from the loops involving the Yukawa couplings λ_{mnp} obeying the PQ selection rule (2.71), which is still within the estimate of \tilde{y}_n in (2.72). We then have

$$\begin{aligned} C_{\text{1PI}}^a(p^2 = \Lambda^2) &= C_W^a(\Lambda) - 2 \sum_n \tilde{y}_n(\Lambda) \text{Tr}(T_a^2(\Phi_n)) \\ &= \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}\right) + \mathcal{O}\left(\frac{v_{\text{PQ}}^2}{M_*^2}\right) \end{aligned} \quad (2.73)$$

at the cutoff scale Λ just below v_{PQ} , and the PQ selection rule (2.71) takes the form

$$(\tilde{x}_m + \tilde{x}_n + \tilde{x}_p + \tilde{y}_m + \tilde{y}_n + \tilde{y}_p)\lambda_{mnp} = \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}\right) + \mathcal{O}\left(\frac{v_{\text{PQ}}}{M_*}\right). \quad (2.74)$$

In the appendix B, we examine the 1PI RG evolution of C_{1PI}^a including higher loop effects, and show that the above estimate of C_{1PI}^a is valid at generic momentum scale in the range $M_\Phi < p < v_{\text{PQ}}$. This implies that \tilde{C}_{1PI}^a in the momentum range $M_\Phi < p < v_{\text{PQ}}$ is indeed dominated by the piece of $\mathcal{O}\left(\frac{M_\Phi^2}{p^2} \ln^2(p^2/M_\Phi^2)\right)$, so the estimate (2.66) of \tilde{C}_{1PI}^a is valid even when higher loop effects are taken into account. We thus conclude that in generic supersymmetric axion model with a PQ scale hierarchically lower than the UV scale M_* , which is presumed to be around the Planck scale or the GUT scale, the 1PI axino-gaugino-gauge boson amplitude at momentum scales in the range $M_\Phi < p < v_{\text{PQ}}$ is suppressed by M_Φ^2/p^2 , in addition to the suppression by $p/16\pi^2 v_{\text{PQ}}$, where M_Φ is the mass of the heaviest PQ-charged and gauge-charged matter field in the model. With the boundary condition (2.73) at $p^2 = \Lambda^2$, one can determine C_{1PI}^a at lower momentum scale $p < M_\Phi$ by computing the threshold correction. Using the result obtained in the appendix A, we find the leading constant part of C_{1PI}^a at generic momentum scale below v_{PQ} is given by

$$\begin{aligned} C_{\text{1PI}}^a(p) &= C_W^a(\Lambda) - 2 \sum_{M_n^2 < p^2} \tilde{y}_n(\Lambda) \text{Tr}(T_a^2(\Phi_n)) + 2 \sum_{M_m^2 > p^2} \tilde{x}_m(\Lambda) \text{Tr}(T_a^2(\Phi_m)) \\ &= 2 \sum_{M_m^2 > p^2} \left(\tilde{x}_m(\Lambda) + \tilde{y}_m(\Lambda) \right) \text{Tr}(T_a^2(\Phi_m)), \end{aligned} \quad (2.75)$$

where $C_W^a(\Lambda)$, $\tilde{y}_n(\Lambda)$ and $\tilde{x}_n(\Lambda)$ are the Wilsonian couplings in the effective lagrangian (2.45) at the cutoff scale Λ just below v_{PQ} . Note that this general result correctly reproduces the 1PI amplitude (2.35) at $p < M_Q$ in the KSVZ model.

3 Thermal production of axino

In this section, we examine the thermal production of axino with the effective interactions which generically take the form (2.48)–(2.50). As we have noticed, if the model has a UV completion (at a scale $M_* \gg v_{\text{PQ}}$) in which the PQ symmetry is linearly realized in the standard manner and all non-renormalizable interactions are suppressed by the powers of $1/M_*$, the effective interactions (2.48)–(2.50) are constrained by the matching

condition (2.73) at the scale Λ just below v_{PQ} . Then one can choose a field basis in which the Wilsonian couplings of axion supermultiplet at Λ are given by

$$C_W(\Lambda) = 0, \quad \tilde{x}_n = x_n, \quad \tilde{y}_n(\Lambda) = \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}, \frac{v_{\text{PQ}}^2}{M_*^2}\right). \quad (3.1)$$

Of course, one can choose any other field basis, for instance the one with

$$C_W = 2 \sum_n z_n \text{Tr}(T_a^2(\Phi_n)), \quad \tilde{x}_n = x_n - z_n, \quad \tilde{y}_n = z_n + \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}, \frac{v_{\text{PQ}}^2}{M_*^2}\right), \quad (3.2)$$

which would be obtained by the field redefinition (2.63). Then the above Wilsonian couplings for arbitrary real values of $\{z_n\}$ should give the same physical results as those of (3.1). The field basis (3.1) is convenient for describing the physics at energy scales above M_Φ since the decoupling of the axion supermultiplet in the limit $M_\Phi \rightarrow 0$ is manifest. However, for physics at lower energy scales below M_Φ , it is often more convenient to choose the field basis (3.2) with $\tilde{x}_n = x_n - z_n = 0$ for which $C_W = C_{\text{PQ}}$.

Let Φ, Φ^c denote the heaviest PQ-charged and gauge-charged matter superfield with a supersymmetric mass M_Φ . In the field basis (3.1), the relevant effective interaction of axion supermultiplet takes a simple form

$$- \int d^2\theta (x_\Phi + x_{\Phi^c}) M_\Phi \frac{A}{v_{\text{PQ}}} \Phi \Phi^c + \text{h.c.}, \quad (3.3)$$

where we have ignored the small $\tilde{y}_n = \mathcal{O}(M_\Phi^2/v_{\text{PQ}}^2, v_{\text{PQ}}^2/M_*^2)$. In the KSVZ model (2.2), Φ, Φ^c correspond to an exotic vector-like quark multiplet with $x_\Phi + x_{\Phi^c} = 1$, while in the DFSZ model (2.41), Φ, Φ^c correspond to the Higgs doublet superfields H_u, H_d with $M_\Phi = \mu$ and $x_\Phi + x_{\Phi^c} = 2$. A key element for the axino production by gauge supermultiplet is the 1PI axino-gaugino-gauge boson amplitude which is given by

$$\mathcal{A}_{1\text{PI}}(k, q, p) = - \frac{g^2}{16\pi^2 \sqrt{2} v_{\text{PQ}}} \delta^4(k + q + p) \tilde{C}_{1\text{PI}}(k, q, p) \tilde{u}(k) \sigma_{\mu\nu} \gamma_5 v(q) \epsilon^\mu(p) p^\nu, \quad (3.4)$$

with

$$\tilde{C}_{1\text{PI}}(k^2 = q^2 = 0; p^2 \gg M_\Phi^2) \simeq (x_\Phi + x_{\Phi^c}) \frac{M_\Phi^2}{p^2} \ln^2\left(\frac{p^2}{M_\Phi^2}\right) \quad (3.5)$$

$$\tilde{C}_{1\text{PI}}(k^2 = q^2 = 0; p^2 \ll M_\Phi^2) = x_\Phi + x_{\Phi^c} + \mathcal{O}\left(\frac{p^2}{M_\Phi^2}\right). \quad (3.6)$$

With the above 1PI amplitude and also the axino-matter coupling (3.3), we can calculate the thermal production of axinos in the temperature range of our interest. Following [21], here we consider the axino production processes listed in table 1. (See figure 3–7 for corresponding Feynman diagrams.) Among these processes, the processes A, B and F produce axino through the 1-loop transition $g \rightarrow \tilde{g} + \tilde{a}$ (or $\tilde{g} \rightarrow g + \tilde{a}$) whose amplitude is given by the 1PI amplitude (3.4). On the other hand, other processes produce axino through both the tree-level transition $\Phi \rightarrow \tilde{\Phi} + \tilde{a}$ (or $\tilde{\Phi} \rightarrow \Phi + \tilde{a}$) and the 1-loop transition

$g \rightarrow \tilde{g} + \tilde{a}$ (or $\tilde{g} \rightarrow g + \tilde{a}$). To compute the amplitudes of these axino production processes, we will use the field basis (3.1) for which the decoupling of the axino in the limit $M_\Phi \rightarrow 0$ is manifest.

The 1PI amplitude (3.5) implies that the amplitude of the axino production through the transition $g \rightarrow \tilde{g} + \tilde{a}$ in the temperature range $M_\Phi \ll T < v_{\text{PQ}}$ is suppressed by M_Φ^2/T^2 . As a result, in this temperature range, axinos are produced mostly by the transition $\Phi \rightarrow \tilde{\Phi} + \tilde{a}$ (or $\tilde{\Phi} \rightarrow \Phi + \tilde{a}$) with an amplitude $\mathcal{A}_{\Phi\tilde{\Phi}\tilde{a}} \propto (x_\Phi + x_{\Phi^c})M_\Phi/v_{\text{PQ}}$, and then the production rate is given by

$$\begin{aligned} \Gamma_{\tilde{a}}(M_\Phi \ll T < v_{\text{PQ}}) &= \sum_{IJ} \langle \sigma(I + J \rightarrow \tilde{a} + \dots) v \rangle n_I n_J \\ &= \mathcal{O}(1) \times (x_\Phi + x_{\Phi^c})^2 \frac{g^2 M_\Phi^2 T^4}{\pi^5 v_{\text{PQ}}^2}, \end{aligned} \quad (3.7)$$

where n_I is the number density of the I -th particle species in thermal equilibrium. On the other hand, at lower temperature $T \ll M_\Phi$, the matter multiplet Φ is not available anymore, and axinos are produced either by $g \rightarrow \tilde{g} + \tilde{a}$ or by $q \rightarrow \tilde{q} + \tilde{a}$, where q denotes a generic light matter multiplet with $M_q < T$. For the temperature range $8\pi^2 M_q < T \ll M_\Phi$, looking at the magnitudes of the involved transition amplitudes, one easily finds that axinos are produced mostly by the transition $g \rightarrow \tilde{g} + \tilde{a}$ (or $\tilde{g} \rightarrow g + \tilde{a}$) with an amplitude $\mathcal{A}_{g\tilde{g}\tilde{a}} \propto (x_\Phi + x_{\Phi^c})/16\pi^2 v_{\text{PQ}}$, which results in

$$\Gamma_{\tilde{a}}(8\pi^2 M_q < T \ll M_\Phi) = \mathcal{O}(1) \times (x_\Phi + x_{\Phi^c})^2 \frac{g^6 T^6}{64\pi^7 v_{\text{PQ}}^2}, \quad (3.8)$$

where the $\mathcal{O}(1)$ factor includes the thermal field theoretic effects discussed in [22].

Solving the Boltzmann equation, the relic axino number density over the entropy density can be determined as (see the appendix C)

$$Y_{\tilde{a}}(T_0) \equiv \frac{n_{\tilde{a}}(T_0)}{s(T_0)} = \int_{T_0}^{T_R} \frac{dT}{T} \frac{\Gamma_{\tilde{a}}}{s(T)H(T)} \quad (3.9)$$

where T_R is the reheat temperature, $s(T) = 2\pi^2 g_* T^3/45$ is the entropy density, and $H(T) = \sqrt{\pi^2 g_*/90} T^2/M_{\text{Pl}}$ is the Hubble parameter for the effective degrees of freedom g_* and the reduced Planck mass $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV. We then find

$$Y_{\tilde{a}}(8\pi^2 M_q < T_R \ll M_\Phi) = \mathcal{O}(1) \times (x_\Phi + x_{\Phi^c})^2 \frac{\bar{g} g^6 M_{\text{Pl}}}{64\pi^7 v_{\text{PQ}}^2} T_R, \quad (3.10)$$

$$Y_{\tilde{a}}(M_\Phi \ll T_R \ll v_{\text{PQ}}) = \mathcal{O}(1) \times (x_\Phi + x_{\Phi^c})^2 \frac{\bar{g} g^2 M_{\text{Pl}}}{2\pi^4 v_{\text{PQ}}^2} M_\Phi, \quad (3.11)$$

where $\bar{g} = 135\sqrt{10}/2\pi^3 g_*^{3/2}$. Here we used the result of [22] for the first result, while the second result is derived in the appendix C.

Figure 8 summarizes our discussion of the relic axino density. It shows $Y_{\tilde{a}} \propto T_R$ for $T_R \lesssim 0.1M_\Phi$, which is due to that in the temperature range $8\pi^2 M_q < T \lesssim 0.1M_\Phi$, axinos are produced mostly through the transition $g \rightarrow \tilde{g} + \tilde{a}$ (or $\tilde{g} \rightarrow g + \tilde{a}$) with an amplitude

	Process	Feynman diagrams	$ \mathcal{M} ^2(8\pi^2 M_q \ll T \ll M_\Phi)$	$ \mathcal{M} ^2(M_\Phi \ll T \ll v_{\text{PQ}})$
A	$g+g \rightarrow \tilde{a}+\tilde{g}$	figure 3	$4C_1(s+2t+2t^2/s)$	suppressed
B	$g+\tilde{g} \rightarrow \tilde{a}+g$	crossing of A	$-4C_1(t+2s+2s^2/t)$	suppressed
C	$\tilde{q}+g \rightarrow \tilde{a}+q$	figure 4	$2sC_2$	$-C(1 + \frac{s-M_\Phi^2}{t-M_\Phi^2})$
D	$q+g \rightarrow \tilde{a}+\tilde{q}$	crossing of C	$-2tC_2$	$C(1 + \frac{t-M_\Phi^2}{s-M_\Phi^2})$
E	$\tilde{q}+q \rightarrow \tilde{a}+g$	crossing of C	$-2tC_2$	$-C\frac{s-M_\Phi^2}{t-M_\Phi^2}$
F	$\tilde{g}+\tilde{g} \rightarrow \tilde{a}+\tilde{g}$	figure 5	$-8C_1(s^2+t^2+u^2)^2/stu$	suppressed
G	$q+\tilde{g} \rightarrow \tilde{a}+q$	figure 6	$-4C_2(s+s^2/t)$	$C(4 + \frac{2M_\Phi^2}{s-M_\Phi^2} + \frac{2M_\Phi^2}{t-M_\Phi^2})$
H	$\tilde{q}+\tilde{g} \rightarrow \tilde{a}+\tilde{q}$	figure 7	$-2C_2(t+2s+2s^2/t)$	$C(2 - \frac{t-3M_\Phi^2}{s-M_\Phi^2} - \frac{s-3M_\Phi^2}{t-M_\Phi^2})$
I	$q+\tilde{q} \rightarrow \tilde{a}+\tilde{g}$	crossing of G	$-4C_2(t+t^2/s)$	$C(4 + \frac{2M_\Phi^2}{u-M_\Phi^2} + \frac{2M_\Phi^2}{t-M_\Phi^2})$
J	$\tilde{q}+\tilde{q} \rightarrow \tilde{a}+\tilde{g}$	crossing of H	$2C_2(s+2t+2t^2/s)$	$C(2 - \frac{t-3M_\Phi^2}{u-M_\Phi^2} - \frac{u-3M_\Phi^2}{t-M_\Phi^2})$

Table 1. Processes of axino production. Here $C = 8g^2 M_\Phi^2 |T_{ij}(\Phi)^a|^2 / v_{\text{PQ}}^2$, $C_1 = g^6 |f^{abc}|^2 / 256\pi^4 v_{\text{PQ}}^2$, and $C_2 = g^6 |T_{ij}^a(q)|^2 / 256\pi^4 v_{\text{PQ}}^2$, where Φ denotes the heaviest PQ-charged and gauge-charged matter multiplet with a gauge-charge matrix given by $T_{ij}^a(\Phi)$. For $T \ll M_\Phi$, q stands for generic gauge-charged matter with $M_q < T$, while it means Φ for $M_\Phi \ll T \ll v_{\text{PQ}}$.

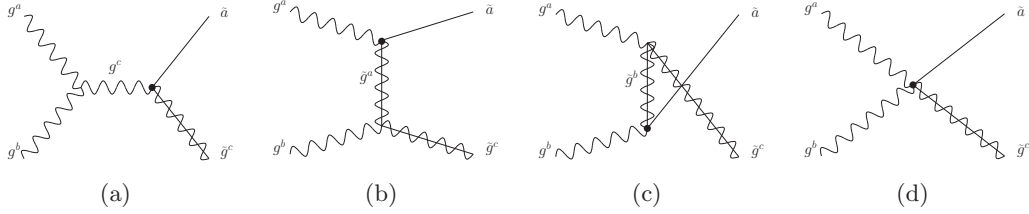


Figure 3. Diagrams for the process A. Here black dots represent the 1PI axino-gluino-gluon amplitude.

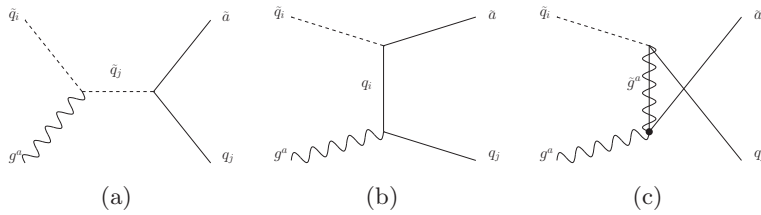


Figure 4. Diagrams for the process C.

$\mathcal{A}_{g\tilde{g}\tilde{a}} = \mathcal{O}(T/16\pi^2 v_{\text{PQ}})$. (Here M_q corresponds to the mass of the next heaviest PQ-charged and gauge-charged matter multiplet.) If one uses only the effective interaction of the form $\frac{1}{32\pi^2 v_{\text{PQ}}} \int d^2\theta AW^a W^a$ to evaluate the axino production by $g \rightarrow \tilde{g} + \tilde{a}$ (or $\tilde{g} \rightarrow g + \tilde{a}$), as one did in most of the previous analysis, one would get $Y_{\tilde{a}} \propto T_R$ even for higher $T_R \gtrsim 0.1M_\Phi$, as represented by the dashed line. Taking it into account that the 1PI axino-gluino-gluon

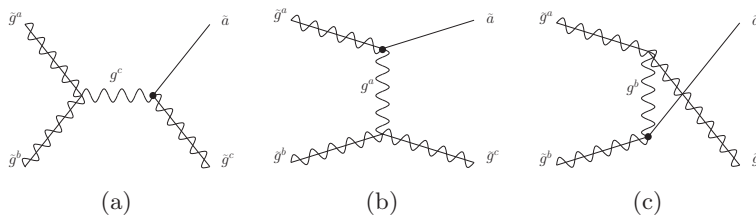


Figure 5. Diagrams for the process F.

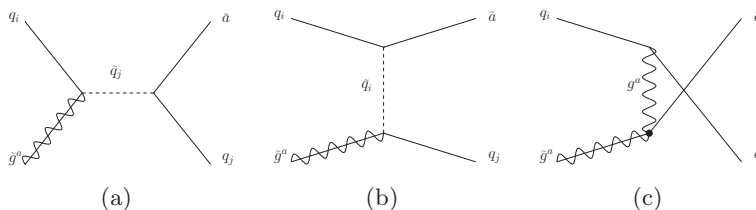


Figure 6. Diagrams for the process G.

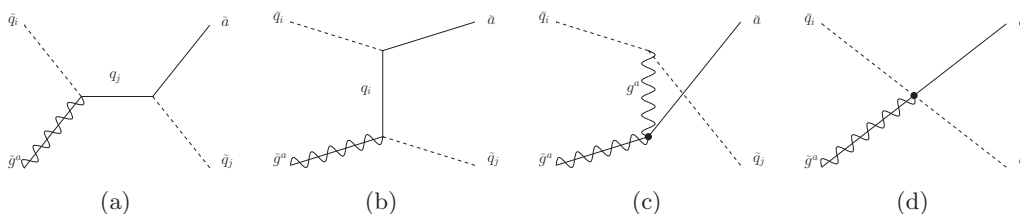


Figure 7. Diagrams for the process H.

amplitude $\mathcal{A}_{g\tilde{g}\tilde{a}} = \mathcal{O}(M_\Phi^2 \ln^2(T/M_\Phi)/16\pi^2 T v_{\text{PQ}})^3$ for $T > M_\Phi$, and therefore the axino production at $T > M_\Phi$ is mostly due to the transition $\Phi \rightarrow \tilde{\Phi} + \tilde{a}$ (or $\tilde{\Phi} \rightarrow \Phi + \tilde{a}$) with an amplitude $\mathcal{A}_{\Phi\tilde{\Phi}\tilde{a}} = \mathcal{O}(M_\Phi/v_{\text{PQ}})$, one can easily understand the behavior of $Y_{\tilde{a}}$ for $T_R > 10M_\Phi$, which is nearly independent of T_R . Note that the dashed line crosses the correct solid line at $T_R \sim 10^3 M_\Phi$, implying that the previous analysis based on the effective interaction $\frac{1}{32\pi^2 v_{\text{PQ}}} \int d^2\theta AW^a W^a$ alone gives rise to an overestimated axion relic density for the reheat temperature $T_R \gtrsim 10^3 M_\Phi$, while it gives an underestimated $Y_{\tilde{a}}$ for $0.1M_\Phi \lesssim T_R \lesssim 10^3 M_\Phi$.

We now proceed to apply our results to the two well-known types of axion models, i.e. KSVZ [34, 35] and DFSZ [32, 33] models. In DFSZ model, the heaviest PQ-charged and gauge-charged matter can be identified as the MSSM Higgs doublets, and then $M_\Phi = \mu$ is around the weak scale. On the other hand, in KSVZ model, Φ, Φ^c correspond to an exotic quark multiplet with M_Φ which can take any value between the weak scale and v_{PQ} . Since our results can have more important cosmological implication when M_Φ/v_{PQ} is smaller, we first discuss the case of DFSZ model.

³This estimate is based on the simple identification $p^2 \sim T^2$, which might not work for instance for the t-channel processes with the final state axino momentum parallel to the initial state gluino (or gluon) momentum [54]. However, including the thermal mass of intermediate state gluon (or gluino), which is of $\mathcal{O}(gT)$, this simple approach does work for the purpose of the order of magnitude estimation.

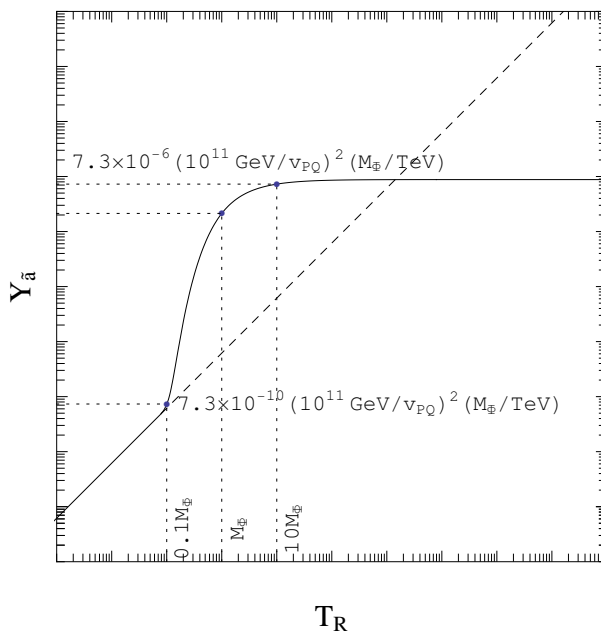


Figure 8. Relic axino number density over the entropy density vs the reheating temperature T_R (solid line). The dashed line represents the result one would get by using only the effective interaction of the form $\frac{1}{32\pi^2 v_{PQ}} \int d^2\theta AW^a W^a$.

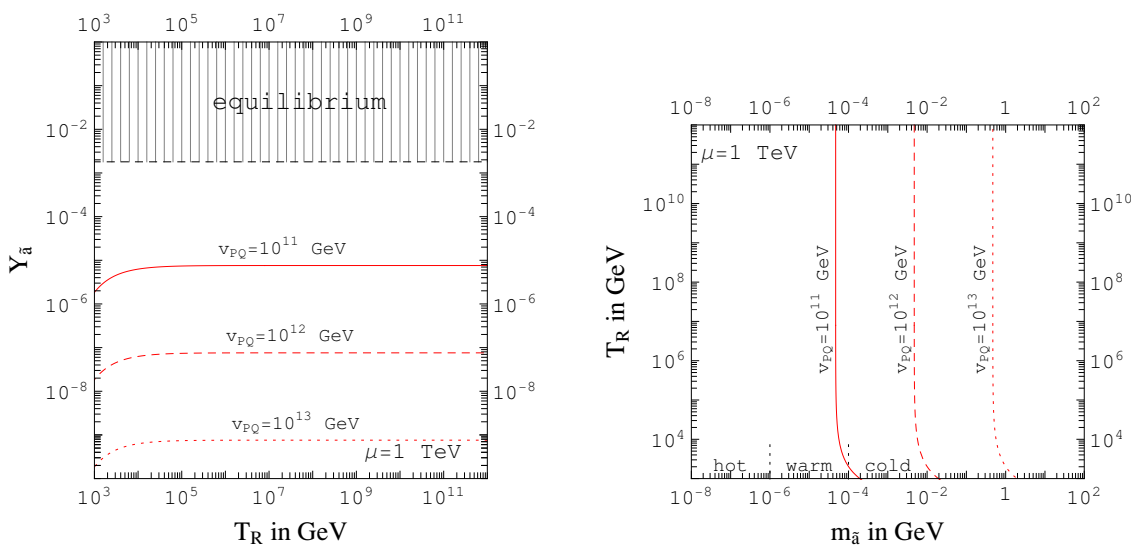


Figure 9. (Left) Axino number density vs reheat temperature T_R for $v_{PQ} = 10^{11}$ GeV (solid), 10^{12} GeV (dashed), and 10^{13} GeV (dotted). (Right) Contours giving $\Omega_{\tilde{a}} h^2 = 0.1$.

3.1 DFSZ axion model

To apply our results to the DFSZ model in which Φ, Φ^c correspond to the MSSM Higgs doublets H_u, H_d , we choose $M_{\Phi} = \mu = 10^3$ GeV, and consider the axino production processes

listed in table 1 while identifying (g, \tilde{g}) and $(\Phi, \tilde{\Phi})$ as the $SU(2)_L$ gauge supermultiplet and the Higgs supermultiplets, respectively. We use $g^2 = g_2^2(T = 10^4 \text{ GeV}) \simeq 0.5$, and the effective degrees of freedom $g_*(T > \mu) = 228.75$. The results are depicted in figure 9. In the left panel, we plot $Y_{\tilde{a}}$ as a function of the reheat temperature T_R for three different PQ scales, $v_{\text{PQ}} = 10^{11} \text{ GeV}$ (solid), 10^{12} GeV (dashed), and 10^{13} GeV (dotted). Our result shows that $Y_{\tilde{a}}$ has approximately a constant value for $T_R > 10^4 \text{ GeV}$, which is given by

$$Y_{\tilde{a}}(T_R > 10^4 \text{ GeV}) \simeq 7.6 \times 10^{-8} \left(\frac{10^{12} \text{ GeV}}{v_{\text{PQ}}} \right)^2. \quad (3.12)$$

In the figure, the black dashed line represents the axino number density when axinos were in equilibrium with the thermal bath of gauge-charged particles, which is given by

$$Y_{\tilde{a}}^{\text{eq}} = \frac{0.42}{g_*} \simeq 1.8 \times 10^{-3}. \quad (3.13)$$

As we can see from figure 9, the axino interactions with gauge-charged particles in DFSZ model are too weak for axinos to be in thermal equilibrium even for high reheat temperature comparable to v_{PQ} .

In the right panel of figure 9, we plot the contours giving the relic axino abundance which would explain the observed DM density $\Omega_{\text{DM}} h^2 \simeq 0.1$ [12]. As an approximate guideline, we can refer to hot, warm, and cold axino dark matter for axino mass in the range $m_{\tilde{a}} \lesssim 1 \text{ keV}$, $1 \text{ keV} \lesssim m_{\tilde{a}} \lesssim 100 \text{ keV}$, and $m_{\tilde{a}} \gtrsim 100 \text{ keV}$, respectively [21]. Then as shown in the figure, if $v_{\text{PQ}} \gtrsim \text{few} \times 10^{11} \text{ GeV}$, axino can provide the *cold* dark matter in our Universe with correct relic abundance. In our case, there is no upper bound on the reheat temperature coming from the relic dark matter density, and therefore we can avoid the previous conclusion that cold axino dark matter scenario is viable only when the reheat temperature is relatively low as $T_R \lesssim 10^6 \text{ GeV}$ [21]. In this analysis, we have ignored the effects of SUSY breaking and electroweak symmetry breaking on the axino effective interactions, which should be a good approximation for $T \gg 10^3 \text{ GeV}$. We note that the Higgsino (or the Higgs boson) decay into axino can be sizable if one includes SUSY breaking effect around the weak scale. If one includes the electroweak symmetry breaking effect, stop decay also can give a sizable contribution [28–30]. At any rate, as long as $T_R > M_{\Phi}$, the axino production by the decay processes does not significantly alter our result depicted in figure 9.

For axinos to be a successful cold dark matter, we should also consider the constraints from the big bang nucleosynthesis (BBN). The long-lived next lightest SUSY particle (NLSP) such as the bino-like or wino-like lightest neutralino $\tilde{\chi}$ or the lighter stau $\tilde{\tau}$ can be problematic in BBN if it destroys the light primordial elements by emitting an energetic photon or hadronic shower through the decay into axino. Neutralino decay to gauge boson and axino, $\tilde{\chi} \rightarrow \gamma/Z + \tilde{a}$ can be induced by the 1PI axino-gaugino-gauge boson amplitude (3.4), and its rate is estimated as

$$\Gamma(\tilde{\chi} \rightarrow \gamma/Z + \tilde{a}) \sim \frac{1}{16\pi} \left(\frac{g_2^2}{8\pi^2} \right)^2 \frac{M_{\tilde{\chi}}^3}{v_{\text{PQ}}^2} \sim \frac{1}{0.1 \text{ s}} \left(\frac{M_{\tilde{\chi}}}{200 \text{ GeV}} \right)^3 \left(\frac{10^{12} \text{ GeV}}{v_{\text{PQ}}} \right)^2, \quad (3.14)$$

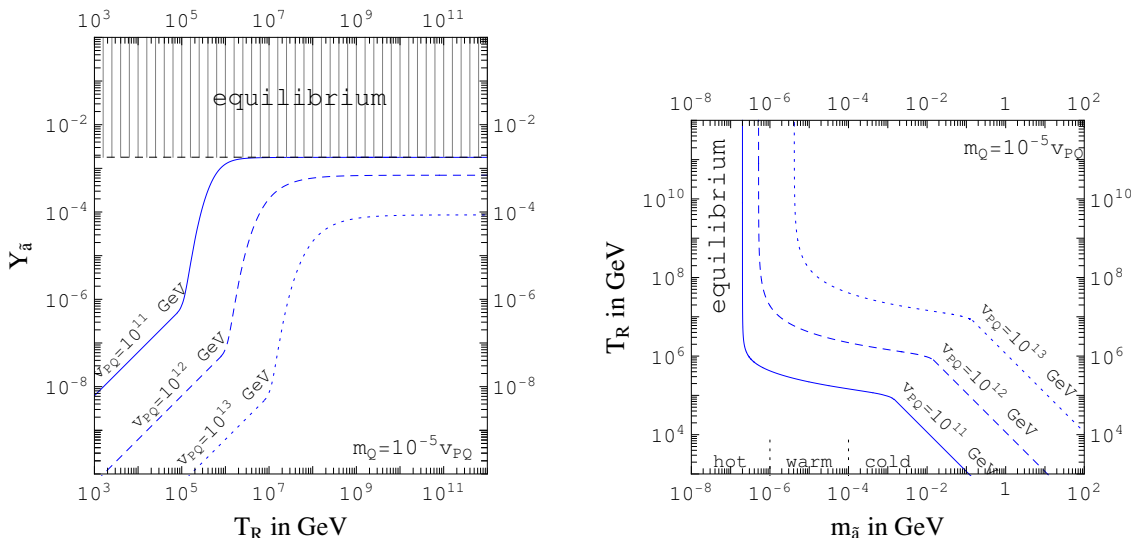


Figure 10. (Left) Axino number density vs reheat temperature T_R for $v_{PQ} = 10^{11}$ GeV (solid), 10^{12} GeV (dashed), and 10^{13} GeV (dotted) with $M_Q/v_{PQ} = 10^{-5}$. (Right) Contours giving $\Omega_{\tilde{a}} h^2 = 0.1$.

where g_2 is the $SU(2)_L$ coupling constant. On the other hand, the stau decay $\tilde{\tau} \rightarrow \tau + \tilde{a}$ is induced by tree-level process in DFSZ model since there is axino-gaugino mixing due to the electroweak symmetry breaking, and its decay rate can be estimated as [55, 56]

$$\Gamma(\tilde{\tau} \rightarrow \tau + \tilde{a}) \sim \frac{m_{\tilde{\tau}}}{16\pi} \frac{g_2^2 v_{\text{weak}}^2}{v_{PQ}^2} \frac{m_Z^2}{M_2^2} \cos^2 \theta_W \sin^4 \beta \sim \frac{1}{10^{-5} \text{ s}} \left(\frac{m_{\tilde{\tau}}}{200 \text{ GeV}} \right) \left(\frac{10^{12} \text{ GeV}}{v_{PQ}} \right)^2. \quad (3.15)$$

Hence, the NLSP neutralino lifetime is $\mathcal{O}(10^{-7}) - \mathcal{O}(10^{-1})$ s, while the NLSP stau lifetime is $\mathcal{O}(10^{-11}) - \mathcal{O}(10^{-5})$ s for $v_{PQ} = 10^9 - 10^{12}$ GeV. In order to avoid the BBN constraints, the lifetime of such long-lived particles is required to be shorter than 10^2 s [57, 58], which can be easily satisfied in DFSZ model over a reasonable parameter range of the model.

3.2 KSVZ axion model

Let us now consider the KSVZ axion model in which the heaviest PQ-charged and gauge-charged field is an exotic quark multiplet Q, Q^c with M_Q which can take any value between the weak scale and the PQ scale. Figure 8 suggests that the previous analysis of axino production based on the effective interaction $\frac{1}{32\pi^2 v_{PQ}} \int d^2\theta A W^a W^a$ alone can be applied only for $T_R \sim 0.1 M_Q$. More specifically, axinos are produced mostly by the gluon supermultiplet at $T \lesssim 0.1 M_Q$, while at higher temperature axino production is mostly due to the transition $Q \rightarrow \tilde{Q} + \tilde{a}$ (or $\tilde{Q} \rightarrow Q + \tilde{a}$). Thus our discussion in this paper can significantly alter the previous result if M_Q is well below v_{PQ} . To be specific, here we consider $M_Q = 10^{-5} v_{PQ}$, and depict the resulting relic axino density in the left panel of figure 10 for $T_R > 10^3$ GeV and $v_{PQ} = 10^{11}$ GeV (solid), 10^{12} GeV (dashed), and 10^{13} GeV (dotted).

For numerical result, we use $g^2 = g_3^2(T = 10^6 - 10^8 \text{ GeV}) \simeq 1$, and find

$$\begin{aligned}
 Y_{\tilde{a}}(T_R \gg M_Q) &\simeq 8.8 \times 10^{-4} \left(\frac{10^{12} \text{ GeV}}{v_{\text{PQ}}} \right)^2 \left(\frac{M_Q}{10^7 \text{ GeV}} \right), \\
 Y_{\tilde{a}}(T \lesssim 0.1 M_Q) &\simeq 2.2 \times 10^{-6} \left(\frac{10^{12} \text{ GeV}}{v_{\text{PQ}}} \right)^2 \left(\frac{T_R}{10^7 \text{ GeV}} \right).
 \end{aligned}
 \tag{3.16}$$

Again, the black dashed line in the figure represents the axino number density when axinos were in equilibrium with the thermal bath of gauge-charged particles. Our result shows that axinos could indeed be in thermal equilibrium if $v_{\text{PQ}} < 10^{12} \text{ GeV}$ and the reheat temperature is high enough.

In the right panel of figure 10, we plot the contours giving $\Omega_{\tilde{a}} h^2 \simeq 0.1$, which shows that the CDM constraint gives a severe upper bound on the reheat temperature depending upon the axino mass. Note that our analysis includes the axino productions by the exotic quark multiplet Q, Q^c , which in fact provide dominant production channels for $T \gtrsim 0.1 M_Q$. As a result, for certain range of the axino mass, the upper bound on T_R can be more stringent than the bound one would obtain based on the effective interaction $\frac{1}{32\pi^2 v_{\text{PQ}}} \int d^2\theta A W^a W^a$ alone. Still, one can relieve the bound by assuming that M_Q is small enough, e.g. close to the weak scale, for which the situation becomes similar to the case of DFSZ model.

As in DFSZ model, we should consider the BBN constraint on the decays of long-lived NLSP. For the neutralino decays $\tilde{\chi} \rightarrow \gamma/Z + \tilde{a}$, the rate is given by (3.14) as in DFSZ model. However, there is no tree level coupling for the stau decay $\tilde{\tau} \rightarrow \tau + \tilde{a}$ in KSVZ model, and the decay rate is suppressed by the two-loop factor as [28–30]

$$\begin{aligned}
 \Gamma(\tilde{\tau} \rightarrow \tau + \tilde{a}) &\sim \frac{1}{16\pi} \left(\frac{9\sqrt{2}\alpha_{\text{em}}^2 e_Q^2}{8\pi^2 \cos^4 \theta_W} \right)^2 \ln^2 \left(\frac{M_Q^2}{m_{\tilde{\tau}}^2} \right) \left(\frac{m_{\tilde{\tau}}^3}{2v_{\text{PQ}}^2} \right) \\
 &\sim \frac{1}{10^4 \text{ s}} \left(\frac{m_{\tilde{\tau}}}{200 \text{ GeV}} \right)^3 \left(\frac{10^{12} \text{ GeV}}{v_{\text{PQ}}} \right)^2,
 \end{aligned}
 \tag{3.17}$$

where α_{em} is the fine structure constant and $e_Q = 1/3$ is the $U(1)_{\text{em}}$ charge of Q . Hence, neutralino lifetime is the same as in the DFSZ case, while stau lifetime is $\mathcal{O}(10^{-2}) - \mathcal{O}(10^4)$ s for $v_{\text{PQ}} = 10^9 - 10^{12} \text{ GeV}$. Therefore, the neutralino NLSP is safe as before, while the stau NLSP is safe only for a relatively heavy $m_{\tilde{\tau}}$.

4 Conclusion

In this paper, we have discussed certain features of the effective interactions of axion supermultiplet, which might be important for the cosmology of supersymmetric axion model, and examined the implication to the thermal production of axinos in the early Universe. If the model has a UV completion at a fundamental scale $M_* \gg v_{\text{PQ}}$, in which the PQ symmetry is linearly realized in the standard manner and all non-renormalizable interactions are suppressed by the powers of $1/M_*$, the axion supermultiplet is decoupled from the gauge-charged fields in the limit $M_{\Phi}/v_{\text{PQ}} \rightarrow 0$ and $v_{\text{PQ}}/M_* \rightarrow 0$, where M_{Φ} is the mass of the heaviest PQ-charged and gauge-charged matter multiplet in the model. As

a result, in models with small values of M_Φ/v_{PQ} and v_{PQ}/M_* , the axino production rate at temperature $T \gg M_\Phi$ is suppressed by the powers of small M_Φ/T . Such decoupling feature is not manifest in generic form of the effective lagrangian of axion supermultiplet, however it should be imposed as a matching condition at the scale just below v_{PQ} .

Our observation is particularly important for the cosmology of supersymmetric DFSZ axion model in which M_Φ corresponds to the MSSM Higgs μ -parameter which is far below the PQ scale v_{PQ} . Cosmology of KSVZ axion model can be significantly altered also, if the PQ-charged exotic quark has a mass well below v_{PQ} . We have performed an explicit analysis of the thermal production of axinos for both the DFSZ and KSVZ axion models, and presented the resulting relic axino density as well as the bound on the reheat temperature T_R coming from the observed dark matter density in our Universe. Our analysis does not take into account the effects of SUSY breaking and electroweak symmetry breaking. For T_R close to the weak scale, a more careful analysis is required, including the NLSP decays into axino as well as the mixing due to SUSY breaking and electroweak symmetry breaking, and it will be the subject of future work [59].

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A 1PI axino-gluino-gluon amplitude

In this appendix, we provide an explicit computation of the 1PI axino-gluino-gluon amplitude for a simple, but still general enough, form of Wilsonian effective interaction of the axion superfield at a generic cutoff scale $\Lambda < v_{\text{PQ}}$, which is given by

$$\Delta_1 \mathcal{L} = - \int d^2\theta \frac{C_W}{32\pi^2} \frac{A}{v_{\text{PQ}}} W^{a\alpha} W_\alpha^a + \text{h.c.}, \tag{A.1}$$

$$\Delta_2 \mathcal{L} = \int d^4\theta \frac{(A + A^\dagger)}{v_{\text{PQ}}} \left(\tilde{y}_1 \Phi_1^\dagger \Phi_1 + \tilde{y}_1^c \Phi_1^{c\dagger} \Phi_1^c + \tilde{y}_2 \Phi_2^\dagger \Phi_2 + \tilde{y}_2^c \Phi_2^{c\dagger} \Phi_2^c \right), \tag{A.2}$$

$$\Delta_3 \mathcal{L} = - \int d^2\theta \frac{A}{v_{\text{PQ}}} \left[(\tilde{x}_1 + \tilde{x}_1^c) M_1 \Phi_1 \Phi_1^c + (\tilde{x}_2 + \tilde{x}_2^c) M_2 \Phi_2 \Phi_2^c \right] + \text{h.c.}, \tag{A.3}$$

where W_α^a are the gluon superfields, $\Phi_i + \Phi_i^c$ ($i = 1, 2$) form $3 + \bar{3}$ of $SU(3)_c$, and $M_2 \gg M_1$, but both masses are well below the cutoff scale Λ . In this effective theory, the PQ symmetry is realized as

$$A \rightarrow A + i v_{\text{PQ}} \alpha, \quad \Phi_i \rightarrow e^{i \tilde{x}_i \alpha} \Phi_i, \quad \Phi_i^c \rightarrow e^{i \tilde{x}_i^c \alpha} \Phi_i^c, \tag{A.4}$$

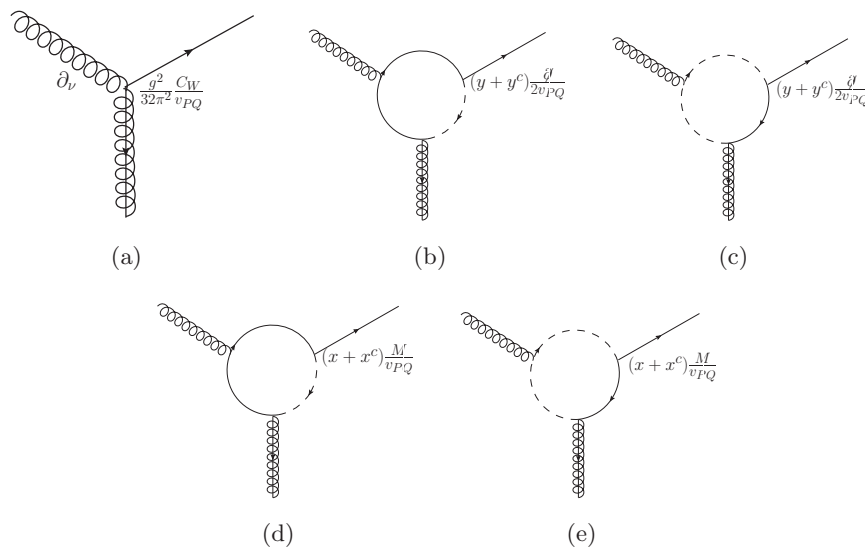


Figure 11. Axino-gluon-gluino amplitudes from the general effective interactions

and the corresponding PQ anomaly is given by

$$C_{\text{PQ}} = C_W + \sum_i (\tilde{x}_i + \tilde{x}_i^c). \quad (\text{A.5})$$

Under the field redefinition

$$\Phi_i \rightarrow e^{z_i A/v_{\text{PQ}}} \Phi_i, \quad \Phi_i^c \rightarrow e^{z_i^c A/v_{\text{PQ}}} \Phi_i^c, \quad (\text{A.6})$$

the Wilsonian couplings change as

$$C_W \rightarrow C_W + \sum_i (z_i + z_i^c),$$

$$(\tilde{y}_i, \tilde{y}_i^c, \tilde{x}_i, \tilde{x}_i^c) \rightarrow (\tilde{y}_i + z_i, \tilde{y}_i^c + z_i^c, \tilde{x}_i - z_i, \tilde{x}_i^c - z_i^c). \quad (\text{A.7})$$

It is straightforward to compute the 1PI axino-gluino-gluon amplitude from the above effective interactions, yielding (see figure 11)

$$\mathcal{A}_{\text{1PI}}(k, q, p) = -\frac{g^2}{16\pi^2 \sqrt{2} v_{\text{PQ}}} \delta^4(k + q + p) \tilde{C}_{\text{1PI}}(k, q, p) \bar{u}(k) \sigma_{\mu\nu} \gamma_5 v(q) \epsilon^\mu p^\nu, \quad (\text{A.8})$$

where k, q and p are the 4-momenta of the axino, gluino and gluon, respectively, and

$$\tilde{C}_{\text{1PI}} = C_W - \sum_i (\tilde{y}_i + \tilde{y}_i^c) + \sum_i (\tilde{y}_i + \tilde{y}_i^c + \tilde{x}_i + \tilde{x}_i^c) F(p, q; M_i) \quad (\text{A.9})$$

for

$$F(p, q; M) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{2M^2}{M^2 - [p^2 x(1-x) + q^2 y(1-y) + 2(p \cdot q)xy]}. \quad (\text{A.10})$$

Note that $\tilde{C}_{1\text{PI}}$ is invariant under the reparametrization (A.7) as it should be. For the axino production by gluon supermultiplet, the relevant kinematic situation is that axino and gluino (or gluon) are on mass-shell, i.e. $k^2 = q^2 = 0$ in the limit to ignore SUSY breaking effects, while gluon (or gluino) is off the mass-shell with $p^2 = (k + q)^2 < 0$. In such kinematic region, $F(p, q, M)$ is given by

$$F(k^2 = q^2 = 0, p^2 \neq 0) = \frac{M^2}{|p^2|} \left[\log \left(\frac{1 + \sqrt{1 - 4M^2/|p^2|}}{-1 + \sqrt{1 - 4M^2/|p^2|}} \right) \right]^2, \quad (\text{A.11})$$

which can be approximated as

$$F \simeq \frac{M^2}{|p^2|} \ln^2 \left(\frac{M^2}{|p^2|} \right) \quad \text{for } |p^2| \gg M^2 \quad (\text{A.12})$$

$$F \simeq 1 - \frac{1}{12} \frac{|p^2|}{M^2} \quad \text{for } |p^2| \ll M^2. \quad (\text{A.13})$$

We then have

$$\tilde{C}_{1\text{PI}}(k^2 = q^2 = 0, p^2 \neq 0) = C_{1\text{PI}} + \mathcal{O} \left(\frac{p^2}{M_{\text{heavy}}^2} \right) + \mathcal{O} \left(\frac{M_{\text{light}}^2}{p^2} \ln^2 \left(\frac{p^2}{M_{\text{light}}^2} \right) \right) \quad (\text{A.14})$$

with

$$\begin{aligned} C_{1\text{PI}}(M_2^2 < p^2 < \Lambda^2) &= C_W - (\tilde{y}_1 + \tilde{y}_1^c + \tilde{y}_2 + \tilde{y}_2^c), \\ C_{1\text{PI}}(M_1^2 < p^2 < M_2^2) &= C_W + (\tilde{x}_2 + \tilde{x}_2^c - \tilde{y}_1 - \tilde{y}_1^c), \\ C_{1\text{PI}}(p^2 < M_1^2) &= C_W + (\tilde{x}_1 + \tilde{x}_1^c + \tilde{x}_2 + \tilde{x}_2^c). \end{aligned} \quad (\text{A.15})$$

If $\Phi_2 + \Phi_2^c$ corresponds to the heaviest PQ-charged and gauge-charged matter superfield in the underlying model, we would have the boundary condition

$$C_{1\text{PI}}(M_2^2 < p^2 < \Lambda^2) = 0, \quad (\text{A.16})$$

which yields the following expression of $C_{1\text{PI}}$ at generic momentum scale below v_{PQ} :

$$C_{1\text{PI}}(p) = \sum_{M_i^2 > |p^2|} (\tilde{x}_i + \tilde{x}_i^c + \tilde{y}_i + \tilde{y}_i^c). \quad (\text{A.17})$$

Finally, we consider the expression of $\tilde{C}_{1\text{PI}}$ in another kinematic situation when one of the 4-momenta p, q, k is vanishing:

$$F(k = 0, p^2 = q^2 \neq 0) = \frac{2M^2/|p^2|}{\sqrt{1 + 4M^2/|p^2|}} \ln \left(\frac{1 + \sqrt{1 + 4M^2/|p^2|}}{-1 + \sqrt{1 + 4M^2/|p^2|}} \right) \quad (\text{A.18})$$

which has the limiting behavior

$$\begin{aligned} F &\simeq -\frac{2M^2}{|p^2|} \ln \left(\frac{M^2}{|p^2|} \right) \quad \text{for } |p^2| \gg M^2 \\ F &\simeq 1 - \frac{1}{6} \frac{|p^2|}{M^2} \quad \text{for } |p^2| \ll M^2, \end{aligned} \quad (\text{A.19})$$

and therefore

$$\tilde{C}_{1\text{PI}}(k=0, p^2 = q^2 \neq 0) = C_{1\text{PI}} + \mathcal{O}\left(\frac{p^2}{M_{\text{heavy}}^2}\right) + \mathcal{O}\left(\frac{M_{\text{light}}^2}{p^2} \ln\left(\frac{p^2}{M_{\text{light}}^2}\right)\right), \quad (\text{A.20})$$

where $C_{1\text{PI}}^a$ are given as (A.15).

B RG running and threshold corrections

In this appendix, we discuss the RG running of the Wilsonian and 1PI amplitudes of the axion superfield, using the method of analytic continuation into $N = 1$ superspace. For this, let us consider a generic Wilsonian effective lagrangian

$$\mathcal{L}_W(\Lambda) = \int d^4\theta Z_n(\Lambda; A + A^\dagger) \Phi_n^\dagger \Phi_n \quad (\text{B.1})$$

$$+ \int d^2\theta \left[\frac{1}{4} f_a^{\text{eff}}(\Lambda; A) W^{a\alpha} W_\alpha^a + \frac{1}{2} e^{-(\tilde{x}_m + \tilde{x}_n)A/v_{\text{PQ}}} M_{mn} \Phi_m \Phi_n + \frac{1}{6} e^{-(\tilde{x}_m + \tilde{x}_n + \tilde{x}_p)A/v_{\text{PQ}}} \lambda_{mnp} \Phi_m \Phi_n \Phi_p + \text{h.c.} \right], \quad (\text{B.2})$$

for which C_W and \tilde{y}_n can be defined as

$$C_W(\Lambda) = -8\pi^2 v_{\text{PQ}} \left. \frac{\partial f_a^{\text{eff}}}{\partial A} \right|_{A=0}, \quad \tilde{y}_n(\Lambda) = v_{\text{PQ}} \left. \frac{d \ln Z_n}{d A} \right|_{A=0}. \quad (\text{B.3})$$

We then have

$$\begin{aligned} \frac{d\tilde{x}_n}{d \ln \Lambda} &= 0, \\ \frac{dC_W}{d \ln \Lambda} &= -8\pi^2 v_{\text{PQ}} \left. \frac{\partial}{\partial A} \left(\frac{df_a^{\text{eff}}}{d \ln \Lambda} \right) \right|_{A=0} = 0, \end{aligned} \quad (\text{B.4})$$

while \tilde{y}_n can have a nontrivial RG running as

$$\frac{d\tilde{y}_n}{d \ln \Lambda} = v_{\text{PQ}} \left. \frac{\partial}{\partial A} \left(\frac{d \ln Z_n}{d \ln \Lambda} \right) \right|_{A=0}, \quad (\text{B.5})$$

where we have used that the RG running of the holomorphic gauge kinetic function f_a^{eff} is exhausted at one-loop, and therefore $df_a^{\text{eff}}/d \ln \Lambda = b_a/16\pi^2$ is an A -independent constant.

In the appendix A, we have seen that the 1PI axino-gaugino-gauge boson amplitude $\tilde{C}_{1\text{PI}}^a$ in the kinematic regime with $k = 0$ and $M_1^2 < p^2 < M_2^2$ is given by

$$\tilde{C}_{1\text{PI}} = C_{1\text{PI}} + \mathcal{O}\left(\frac{p^2}{M_2^2}\right) + \mathcal{O}\left(\frac{M_1^2}{p^2} \ln\left(\frac{p^2}{M_1^2}\right)\right), \quad (\text{B.6})$$

where $C_{1\text{PI}}$ is a constant in 1-loop approximation. Including higher loops, $C_{1\text{PI}}$ can have a logarithmic p -dependence, which can be determined by the 1PI RG equation. To examine this, let us consider the 1PI effective lagrangian at a momentum scale $p < \Lambda$, including

$$\mathcal{L}_{1\text{PI}} = \int d^4\theta \left[Z_n(p, A + A^\dagger) \Phi_n^\dagger \Phi_n + \frac{1}{16} \mathcal{F}_a(p, A + A^\dagger) \left(W^{a\alpha} \frac{D^2}{p^2} W_\alpha^a + \text{h.c.} \right) \right], \quad (\text{B.7})$$

where \mathcal{Z}_n is the 1PI wavefunction coefficient chosen to satisfy the matching condition

$$\mathcal{Z}_n(p^2 = \Lambda^2) = Z_n(\Lambda), \quad (\text{B.8})$$

where $Z_n(\Lambda)$ is the Wilsonian wavefunction coefficient at the cutoff scale Λ . We then have

$$C_{\text{1PI}} = -8\pi^2 v_{\text{PQ}} \left. \frac{\partial \mathcal{F}_a}{\partial A} \right|_{A=0}. \quad (\text{B.9})$$

One can introduce also the canonical 1PI Yukawa couplings

$$\Lambda_{mnp} \equiv \frac{e^{-(\tilde{x}_m + \tilde{x}_n + \tilde{x}_p)A/v_{\text{PQ}}} \lambda_{mnp}}{\sqrt{\mathcal{Z}_m \mathcal{Z}_n \mathcal{Z}_p}} \quad (\text{B.10})$$

which obeys

$$v_{\text{PQ}} \left. \frac{\partial |\Lambda_{mnp}|^2}{\partial A} \right|_{A=0} = -(\tilde{x}_m + \tilde{x}_n + \tilde{x}_p + \tilde{y}_m(p) + \tilde{y}_n(p) + \tilde{y}_p(p)) |\Lambda_{mnp}|^2, \quad (\text{B.11})$$

where

$$\tilde{y}_n(p) = v_{\text{PQ}} \left. \frac{\partial \ln \mathcal{Z}_n(p)}{\partial A} \right|_{A=0}. \quad (\text{B.12})$$

The 1PI gauge kinetic function obeys the Novikov-Shifman-Vainshtein-Zakhatov (NSVZ) RG equation

$$\frac{d\mathcal{F}_a}{d \ln p^2} \equiv \beta_a = \frac{1}{16\pi^2} \frac{1}{1 - \text{Tr}(T_a^2(G))/8\pi^2 \mathcal{F}_a} \left(b_a + \sum_n \text{Tr}(T_a^2(\Phi_n)) \gamma_n \right), \quad (\text{B.13})$$

where

$$\gamma_n(p) \equiv \frac{d \ln \mathcal{Z}_n}{d \ln p}. \quad (\text{B.14})$$

Both β_a and γ_n can be expressed as a function of the 1PI gauge coupling $g_a^2 = 1/\mathcal{F}_a$ and the 1PI Yukawa coupling $|\Lambda_{mnp}|^2$. We then find the 1PI RG equations for C_{1PI}^a and \tilde{y}_n are given by

$$\begin{aligned} \frac{dC_{\text{1PI}}}{d \ln p} &= -8\pi^2 v_{\text{PQ}} \left. \frac{\partial \beta_a}{\partial A} \right|_{A=0} = -8\pi^2 v_{\text{PQ}} \left(\frac{\partial \beta_a}{\partial \mathcal{F}_a} \frac{\partial \mathcal{F}_a}{\partial A} + \frac{\partial \beta_a}{\partial |\Lambda_{mnp}|^2} \frac{\partial |\Lambda_{mnp}|^2}{\partial A} \right) \Big|_{A=0} \\ &= C_{\text{1PI}}^a \frac{\partial \beta_a}{\partial \mathcal{F}_a} + 8\pi^2 (\tilde{x}_m + \tilde{x}_n + \tilde{x}_p + \tilde{y}_m + \tilde{y}_n + \tilde{y}_p) |\Lambda_{mnp}|^2 \frac{\partial \beta_{\text{1PI}}^a}{\partial |\Lambda_{mnp}|^2}, \end{aligned} \quad (\text{B.15})$$

$$\begin{aligned} \frac{d\tilde{y}_n}{d \ln p} &= v_{\text{PQ}} \left. \frac{\partial \gamma_n}{\partial A} \right|_{A=0} = v_{\text{PQ}} \left(\frac{\partial \gamma_n}{\partial \mathcal{F}_a} \frac{\partial \mathcal{F}_a}{\partial A} + \frac{\partial \gamma_n}{\partial |\Lambda_{mnp}|^2} \frac{\partial |\Lambda_{mnp}|^2}{\partial A} \right) \Big|_{A=0} \\ &= \frac{C_{\text{1PI}}^a g_a^4}{8\pi^2} \frac{\partial \gamma_n}{\partial g_a^2} - (\tilde{x}_m + \tilde{x}_n + \tilde{x}_p + \tilde{y}_m + \tilde{y}_n + \tilde{y}_p) |\Lambda_{mnp}|^2 \frac{\partial \gamma_n}{\partial |\Lambda_{mnp}|^2}. \end{aligned} \quad (\text{B.16})$$

In section 2, we noticed that the Yukawa couplings of the model are constrained by the PQ selection rule

$$(\tilde{x}_m + \tilde{x}_n + \tilde{x}_p + \tilde{y}_m + \tilde{y}_n + \tilde{y}_p) \lambda_{mnp} = \mathcal{O} \left(\frac{M_\Phi^2}{v_{\text{PQ}}^2} \right) + \mathcal{O} \left(\frac{v_{\text{PQ}}}{M_*} \right). \quad (\text{B.17})$$

With the boundary value C_{IPI}^a at $p \simeq v_{\text{PQ}}$, which is estimated as (2.73), the RG equations (B.15) and (B.16) assure that

$$C_{\text{IPI}}^a(M_\Phi < p < v_{\text{PQ}}) = \mathcal{O}\left(\frac{M_\Phi^2}{v_{\text{PQ}}^2}\right) + \mathcal{O}\left(\frac{v_{\text{PQ}}^2}{M_*^2}\right), \quad (\text{B.18})$$

and therefore the result (2.66) remains valid even after higher loop effects are included.

C Thermal axino production by matter multiplet

In this appendix, we present a calculation of the relic axino density when the axinos are produced dominantly by the matter multiplet Φ at $T \gg M_\Phi$. The Boltzmann equation of axino production is given by

$$\frac{dn_{\bar{a}}}{dt} + 3Hn_{\bar{a}} = \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle (n_I n_J - n_K n_{\bar{a}}), \quad (\text{C.1})$$

where n_I is the number density of the I -th particle species. At high temperature of our interest, all particles except axinos are in thermal equilibrium, so we can write $n_I = n_I^{\text{eq}}$, where n_I^{eq} is the equilibrium number density of particle I . From the relation of the detailed balance, we have

$$\langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle n_I^{\text{eq}} n_J^{\text{eq}} = \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle n_K^{\text{eq}} n_{\bar{a}}^{\text{eq}}. \quad (\text{C.2})$$

Defining $Y_{\bar{a}} = n_{\bar{a}}/s$, the Boltzmann equation becomes

$$\frac{dY_{\bar{a}}}{dt} = s \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle Y_K^{\text{eq}} (Y_{\bar{a}}^{\text{eq}} - Y_{\bar{a}}). \quad (\text{C.3})$$

For $\Delta_{\bar{a}} \equiv Y_{\bar{a}}^{\text{eq}} - Y_{\bar{a}}$, we have

$$\frac{d\Delta_{\bar{a}}}{\Delta_{\bar{a}}} = -s \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle Y_K^{\text{eq}} dt, \quad (\text{C.4})$$

which yields

$$\Delta_{\bar{a}}(T_0) = \Delta_{\bar{a}}(T_R) \exp \left[- \int_{T_0}^{T_R} \frac{s \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle Y_K^{\text{eq}}}{H(T)T} dT \right]. \quad (\text{C.5})$$

As long as axinos are relativistic in the temperature of interest, $Y_{\bar{a}}^{\text{eq}}$ is independent of temperature. Then we can use

$$\begin{aligned} \Delta_{\bar{a}}(T_0) &= \Delta_{\bar{a}}(T_R) \exp \left[- \frac{1}{Y_{\bar{a}}^{\text{eq}}} \int_{T_0}^{T_R} \frac{s \langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle Y_K^{\text{eq}} Y_{\bar{a}}^{\text{eq}}}{H(T)T} dT \right] \\ &= \Delta_{\bar{a}}(T_R) \exp \left[- \frac{1}{Y_{\bar{a}}^{\text{eq}}} \int_{T_0}^{T_R} \frac{\langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle n_K^{\text{eq}} n_{\bar{a}}^{\text{eq}}}{s(T)H(T)T} dT \right] \\ &= \Delta_{\bar{a}}(T_R) \exp \left[- \frac{1}{Y_{\bar{a}}^{\text{eq}}} \int_{T_0}^{T_R} \frac{\langle \sigma_{(I+J \rightarrow K+\bar{a})} v \rangle n_I^{\text{eq}} n_J^{\text{eq}}}{s(T)H(T)T} dT \right], \end{aligned} \quad (\text{C.6})$$

where the relation (C.2) is used for the last identity. Under the initial condition $\Delta_{\tilde{a}}(T_R) = Y_{\tilde{a}}^{\text{eq}} - Y_{\tilde{a}}(T_R) = Y_{\tilde{a}}^{\text{eq}}$, the above result gives

$$\begin{aligned} Y_{\tilde{a}}(T_0) &= Y_{\tilde{a}}^{\text{eq}} \left\{ 1 - \exp \left[-\frac{1}{Y_{\tilde{a}}^{\text{eq}}} \int_{T_0}^{T_R} \frac{\langle \sigma_{(I+J \rightarrow K+\tilde{a})} v \rangle n_I^{\text{eq}} n_J^{\text{eq}}}{s(T)H(T)T} dT \right] \right\} \\ &\simeq \int_{T_0}^{T_R} \frac{\langle \sigma_{(I+J \rightarrow K+\tilde{a})} v \rangle n_I^{\text{eq}} n_J^{\text{eq}}}{s(T)H(T)T} dT \\ &= \frac{\bar{g}M_{\text{Pl}}}{16\pi^4} \int_{t_R}^{\infty} dt t^3 K_1(t) \int_{(m_I+m_J)}^{tT_R} d(\sqrt{s}) \sigma(s) \left[\frac{(s - m_I^2 - m_J^2)^2 - 4m_I^2 m_J^2}{s^2} \right], \end{aligned}$$

where $\bar{g} = 135\sqrt{10}/(2\pi^3 g_*^{3/2})$, $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV, $t_R = (m_I + m_J)/T_R$, and $K_1(t)$ is the modified Bessel function [60]. The cross section $\sigma(s)$ can be obtained by

$$\left(\frac{d\sigma}{d\Omega} \right) = \frac{1}{2E_{p_1} 2E_{p_2} |v_{p_1} - v_{p_2}|} \frac{|\mathbf{k}_1|}{(2\pi)^2 4E_{\text{cm}}} |\mathcal{M}(p_1 p_2 \rightarrow k_1 k_2)|^2 \quad (\text{C.7})$$

with the amplitudes listed in table 1. Then, for $T_R \gg M_{\Phi} \gg M_{\tilde{g}}$, we find

$$\begin{aligned} Y_{\tilde{a}}^{\Delta_3 \mathcal{L}}(T_0) &\simeq \frac{\bar{g}M_{\text{Pl}}}{16\pi^4} \left(\frac{8g^2(x_{\Phi} + x_{\Phi^c})^2 M_{\Phi}^2 |T_{ij}^a|^2}{v_{\text{PQ}}^2} \right) \frac{1}{M_{\Phi}} \\ &\quad \times \left(\frac{7}{60} + \frac{3}{140} + 0.14 + \frac{11}{60} + 0.12 + 0.28 + 0.17 \right) \\ &\simeq (x_{\Phi} + x_{\Phi^c})^2 \left(\frac{\bar{g}M_{\text{Pl}}}{2\pi^4} \right) \left(\frac{g^2 M_{\Phi}}{v_{\text{PQ}}^2} \right) \left(\frac{N^2 - 1}{2} \right), \end{aligned} \quad (\text{C.8})$$

where $|T_{ij}^a|^2 = \text{Tr}(T^a T^a) = (N^2 - 1)/2$ for the $\text{SU}(N)$ gauge group, and the numbers in the first line come from the integrations for the processes C, D, E, G, I, H and J, respectively, in table 1. From the above result, the relic axino mass density can be determined as

$$\Omega_{\tilde{a}} h^2 = \rho_{\tilde{a}} h^2 / \rho_c = m_{\tilde{a}} n_{\tilde{a}} h^2 / \rho_c = m_{\tilde{a}} Y_{\tilde{a}} s(T_0) h^2 / \rho_c \simeq 2.8 \times 10^5 \left(\frac{m_{\tilde{a}}}{\text{MeV}} \right) Y_{\tilde{a}}, \quad (\text{C.9})$$

where we have used $\rho_c/[s(T_0)h^2] = 3.6 \times 10^{-9}$ GeV.

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