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A Novel PAPR Reduction Scheme for OFDM Systems: Selective Mapping of Partial Tones (SMOPT)

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Abstract — *A novel scheme called the selective mapping of partial tones (SMOPT) is addressed for the reduction of the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. The SMOPT first produces a set of modified OFDM symbols with reduced PAPRs by adding a set of mapping symbols designed on peak reduction tones (PRT) to the original OFDM symbol. The SMOPT then selects the symbol with the lowest PAPR for transmission among the modified OFDM symbols. When simulated under the IEEE 802.11a wireless local area network and ETSI EN 300 401 digital audio broadcasting physical layer models, the SMOPT is observed to have lower sensitivity to PRT positions, lower complexity, and more design flexibility than the tone reservation scheme*[1].

Index Terms — OFDM, peak-to-average power ratio, tone reservation scheme, selective mapping of partial tones.

I. INTRODUCTION

Known to be robust against multipath fading, the orthogonal frequency division multiplexing (OFDM) system can transmit high speed data using a number of orthogonal sub-carriers. The OFDM system, however, exhibits a relatively large peak-to-average power ratio (PAPR) when the sub-carriers are added with the same phase, which would reduce the efficiency of the analog-to-digital and digital-to-analog converters and radio frequency power amplifier [1], [2].

Various techniques have been proposed for reducing the PAPR [3]-[7]. The techniques can generally be divided into two classes, one using all the sub-carriers and the other using a subset of the sub-carriers called the peak reduction tones (PRT). In this paper, we are concerned with the latter.

Among the schemes using the PRT, the most widely used is the tone reservation (TR) scheme [8], [9]. The TR scheme shows very good PAPR reduction performance when operating with optimized PRT positions. However, it is not easy to find the optimal PRT positions, and the performance of the TR scheme degrades with non-optimal PRT positions. In addition, although the TR scheme can be implemented with the iterative gradient algorithm (IGA), it could take a long time to yield a converged result. In this paper, to overcome these drawbacks of the TR scheme, we propose a novel PAPR reduction scheme called the selective mapping of partial tones (SMOPT).

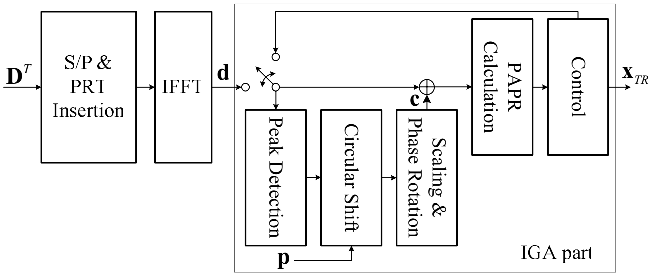

Fig. 1. The OFDM transmitter with the TR scheme.

The remainder of this paper is organized as follows. In Section II, the PAPR is formulated and the complementary cumulative distribution function (CCDF) of the PAPR is derived. The CCDF is the most general performance measure for PAPR reduction. The TR scheme and its weaknesses are also described in this section. Section III proposes the SMOPT and depicts its structure and characteristics. In Section IV, the performance of the TR scheme and SMOPT is simulated and compared in terms of the sensitivity to PRT positions and multiplication complexity. Finally, Section V concludes this paper.

II. THE CONVENTIONAL SCHEME

A. The Complementary Cumulative Distribution Function (CCDF) of PAPR

This subsection formulates the PAPR of an OFDM symbol and derives the CCDF of the PAPR, which gives the probability that the PAPR exceeds a given threshold. Let us consider an OFDM system implemented by the inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT), each of size N for modulation and demodulation. The OFDM samples d_n at the outputs of the IFFT are given by

$$d_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} D_k e^{j2\pi nk/N} , \qquad (1)$$

for $n = 0, 1, \cdots, N-1$, where D_k is the quadrature phase shift keying (QPSK) or quadrature amplitude modulation (QAM) data symbol transmitted through the k th sub-carrier and $j = \sqrt{-1}$.

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The PAPR of an OFDM symbol is defined as the ratio of the maximum to the average power of the OFDM symbol and can be expressed as the ratio [1]

$$PAPR(\mathbf{d}) = \frac{\max\limits_{0\le n\le N-1} |d_n|^2}{E\left[|\mathbf{d}|^2\right]}, \tag{2}$$

where $PAPR(\square)$ represents the PAPR calculation, $\mathbf{d} = (d_0, d_1, \cdots, d_{N-1})^T$ with $(\cdot)^T$ denoting the transpose operation, and $E[\cdot]$ denotes the expectation.

Let us assume that $D_k \in \{1, -1, j, -j\}$ (i.e., D_k is assumed to be a QPSK data symbol). From the central limit theorem, the real and imaginary values of d_n become Gaussian distributed for $N >> 1$, each with mean zero and variance $1/2$. Thus, the amplitude of d_n has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom: the cumulative distribution function (CDF) $F(x)$ of power is given by

$$F(x) = P\left(|d_n|^2 < x\right) = 1 - e^{-x}. \tag{3}$$

Finally, assuming that $\{d_n\}_{n=0}^{N-1}$ are mutually uncorrelated, we can obtain the CCDF of the PAPR as

$$CCDF_{PAPR(\mathbf{d})}(PAPR_0) = 1 - F^N(PAPR_0) = 1 - (1 - e^{-PAPR_0})^N, \tag{4}$$

where $PAPR_0$ is a given threshold.

B. Tone Reservation (TR) Scheme

The structure of the TR scheme is shown in Fig. 1. The structure is composed of three parts: PRT reservation, IFFT, and IGA [8]. First, in the PRT reservation part, L sub-carriers out of N sub-carriers are reserved for PAPR reduction. Subsequently, the data and null symbols are modulated by the IFFT on $N-L$ and L sub-carriers, respectively. Finally, the signal $\mathbf{x}_{TR}$ with the PAPR reduced through the IGA, is transmitted.

Details of the operation of the IGA are as follows. First, the index of the OFDM sample with the maximum absolute amplitude (i.e., $\arg\left[\max\limits_{0\le n\le N-1} |d_n|\right]$) is detected, and then a pre-determined peak reduction kernel (PRK), denoted by $\mathbf{p}$, is circularly shifted by the detected index. The PRK is an impulse-like time domain signal and can be obtained by setting all the PRTs to a constant. The circularly shifted PRK is scaled by a given step size and by the difference between the maximum absolute amplitude of the OFDM sample and a desired threshold. The PRK is then phase-rotated in such a way that the phase of the PRK is the opposite to that of the OFDM sample with the maximum absolute amplitude. Finally, the modified PRK $\mathbf{c}$ is added to the OFDM symbol $\mathbf{d}$ and the PAPR of the modified OFDM symbol $\mathbf{d} + \mathbf{c}$ is calculated. If the PAPR is smaller than the desired threshold, $\mathbf{x}_{TR}$ is transmitted; otherwise the above procedure is repeated.

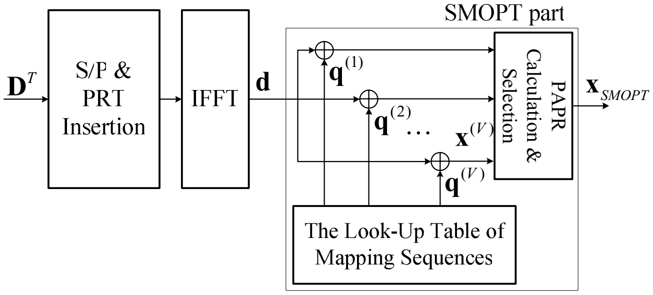

Fig. 2. The OFDM transmitter with the SMOPT scheme.

The IGA requires long iteration times, generally 30-40 iterations, in yielding a converged result, and thus, the TR scheme employing the IGA may not be suitable for real time implementations. Moreover, the TR scheme is sensitive to the PRT positions, because how the PRK is impulse-like strongly depends on the PRT positions.

III. SELECTIVE MAPPING OF PARTIAL TONES (SMOPT) SCHEME

In this section, a novel PAPR reduction scheme called the SMOPT is proposed to overcome the drawbacks of the TR scheme. The structure of the SMOPT is shown in Fig. 2. The operation through the IFFT is same as that of the TR scheme. Therefore, we focus on the SMOPT part. The SMOPT part is composed of three sub-parts: a look-up table that stores V mapping symbols each of size $N \times 1$ for peak reduction, complex adders, and a selector that selects the one with the lowest PAPR among V modified OFDM symbols. The transmit signal $\mathbf{x}_{SMOPT}$ can be expressed as

$$\mathbf{x}_{SMOPT} = \mathbf{x}^{(\kappa)}, \tag{5}$$

where $\kappa = \arg[\min\limits_{0\le v\le V-1}\{PAPR(\mathbf{x}^{(v)})\}] = \arg[\min\limits_{0\le v\le V-1}\{PAPR(\mathbf{d}+\mathbf{q}^{(v)})\}]$, $\mathbf{x}^{(v)}$ is the v th modified OFDM symbol, and $\mathbf{q}^{(v)}$ is the v th mapping symbol defined as the IFFT of

$$\mathbf{Q}^{(v)} = \begin{cases} 0, & k \notin \{i_1, \cdots, i_L\} \\ A_k^{(v)}, & k \in \{i_1, \cdots, i_L\} \end{cases}. \tag{6}$$

In (6), $\{i_n\}_{n=1}^{L}$ denote the indices of the sub-carriers assigned for the L PRT, and $A_k^{(v)} = \alpha \exp(j\theta_k^{(v)})$, where α is the amplitude of $A_k^{(v)}$ decided subject to a desired PAPR and $\theta_k^{(v)}$ is the random phase distributed uniformly over $[0, 2\pi)$.

Since the SMOPT always selects the symbol with the lowest PAPR among a set of sufficiently different candidates, the performance of the SMOPT is robust to the PRT positions, whereas the PRT positions for the TR scheme have to be optimized for high PAPR reduction performance. Consequently, the SMOPT exhibits lower sensitivity to PRT positions than the TR scheme.

In addition, once the PRT positions and the number of mapping symbols are specified, the mapping symbols can be stored in a look-up table in advance, resulting in a complexity decrease in the implementation of the SMOPT. The SMOPT also offers more design flexibility when compared with the TR scheme using the IGA since the mapping and PAPR calculation processes in the SMOPT can be implemented both in serial and parallel.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, the performance of the TR scheme and SMOPT is simulated and compared in terms of the performance sensitivity to PRT positions and multiplication complexity.

The IEEE 802.11a wireless local area network (WLAN) [10] and ETSI EN 300 401 digital audio broadcasting (DAB) [11] physical layer models are used for the simulations. The CCDF of the PAPR is used as the performance measure, and data symbols are assumed to be QPSK modulated.

A. Sensitivity to PRT Positions

We consider two sets of PRT positions used widely: the continuous sub-carrier and randomly optimized sets. In the former, the PRT is placed continuously, and in the latter a set of PRT positions are obtained by first generating 10^5 random sets and then selecting the best [9].

Fig. 3 shows the PAPR reduction performance of the SMOPT and TR scheme for the two sets of PRT positions, where I denotes the iteration number used in the IGA for the TR scheme. It is clearly observed that the PAPR reduction performance of the TR scheme is highly sensitive to the variation of the PRT positions. On the other hand, the performance of the SMOPT is quite robust to such a variation.

B. Multiplication Complexity

Since the operation through the IFFT is the same in the SMOPT and TR schemes, we focus on the multiplication complexity after the IFFT. Specifically, the multiplication complexities of the SMOPT and TR schemes are

$$\gamma_{SMOPT} = N+V, \tag{7}$$

and

$$\gamma_{TR} = I(7N+1), \tag{8}$$

respectively.

Fig. 4 shows the PAPR reduction performance of the SMOPT and TR scheme when the two schemes have similar multiplication complexity. In Fig. 4, 'Case 1' and 'Case 2' represent that the simulation is based on the IEEE 802.11a WLAN and ETSI EN 300 401 DAB physical layer models, respectively. The continuous sub-carrier and randomly optimized sets are used for the PRT positions of the SMOPT and TR scheme, respectively.

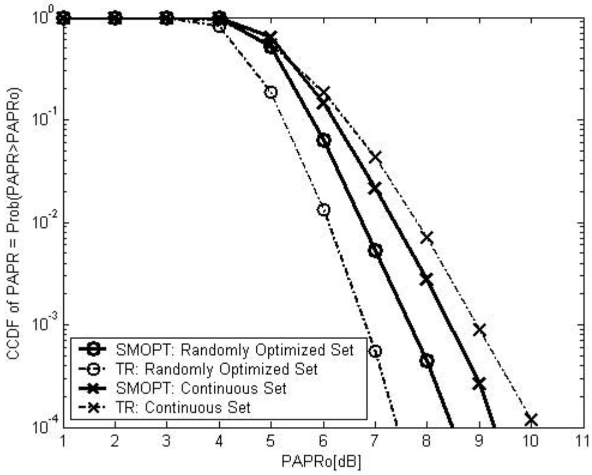

Fig. 3. The PAPR reduction performance of the SMOPT and TR scheme for continuous sub-carrier and randomly optimized sets of PRT positions (N=64, L=11, V=64, and I=40).

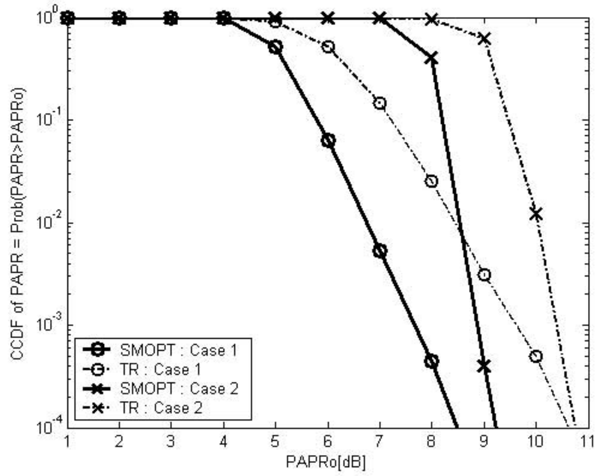

Fig. 4. PAPR reduction performances of the SMOPT and TR schemes with similar multiplication complexity (Case1: N=64, L=11, V=4096, and I=10, Case2: N=2048, L=64, V=2^{17}, and I=10).

From Fig. 4, we can clearly see that, in both models, the SMOPT has better PAPR reduction performance than the TR scheme when the two schemes have similar multiplication complexity. Another important observation is that, even if the SMOPT uses the non-optimized PRT set (the continuous sub-carrier set), it outperforms the TR scheme with the optimized PRT set.

V. CONCLUSION

In this paper, we have proposed a PAPR reduction scheme. The SMOPT has the following advantages over the conventional TR scheme. First, the SMOPT is robust to the variation of the PRT positions. Secondly, once the PRT positions and the number of mapping symbols are given as system design specifications, the mapping symbols can be stored in a look-up table in advance, resulting in a complexity decrease in the implementation of the SMOPT. Lastly, since the mapping and PAPR calculation processes can be implemented in serial or parallel, the SMOPT offers more design flexibility than the TR scheme employing the IGA.

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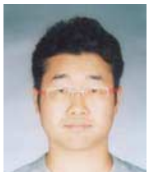

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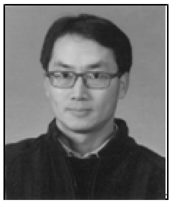

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