Title no. 93-M24

Prediction of Shear Strength of Reinforced Concrete Beams without Web Reinforcement


by Jin-Keun Kim and Yon-Dong Park

A rational and mechanics-based equation is proposed for the prediction of shear strength of reinforced concrete beams without web reinforcement. This prediction is based on basic shear transfer mechanisms, a modified Bazant's size effect law, and numerous published experimental data, including high-strength concrete beams with compressive strengths of concrete up to 100 MPa (14,500 psi). Comparisons with experimental data indicate that the proposed equation estimates properly the effects of primary factors, such as concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth. It is shown that the proposed equation is considered to be better than the other equations compared in this study with respect to accuracy and estimation of primary factors. A simplified design equation is also derived within the limited range of effective depth for practical purposes.

Keywords: aggregate interlock; beams; dowel action; failure mode; shear strength; size effect.

According to the shear span-to-depth ratio (*a*/*d*), shear failure of a reinforced concrete beam without web reinforcement is divided into two modes, as shown in Fig. 1. For *a*/*d* greater than 2.0 ~ 3.0, the inclined cracking load exceeds the shear compression failure load. With the formation of the inclined crack, a beam without web reinforcement becomes unstable and fails. This type of failure is usually called "diagonal tension failure." For *a*/*d* less than 2.0 ~ 3.0, however, the failure load exceeds the inclined cracking load. If sufficient anchorage length is provided, after the inclined crack develops, failure may occur by concrete crushing in the upper end, and this type of failure is called "shear compression failure."

Shear force in reinforced concrete member is transferred in various ways. For slender beams where *a*/*d* is greater than 2.0 ~ 3.0, shear force is carried by the shear resistance of uncracked concrete in the compression zone, the interlocking action of aggregates along the rough concrete surfaces on each side of the crack, and the dowel action of the longitudinal reinforcement. For relatively short beams, however, after the breakdown of beam action, shear force is resisted mainly by arch action.

Test results have shown that shear strength of reinforced concrete beams without web reinforcement depends mainly on concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth. Of course, factors such as maximum aggregate size, diameter of the bars, and spacing of the flexural cracks show some minor contribution. A part or all of these primary factors are included in the existing shear strength prediction models, but the effects of these factors are estimated differently according to the models.

Recently, high-strength concrete has been increasingly used in practice. With the development of concrete technology and the introduction of superplasticizers and silica fume, the compressive strength of concrete in the field of ready-mixed concrete reached 100 MPa (14,300 psi) and higher. Since the mechanical properties of concrete are changed in high-strength concrete, a reevaluation of the prediction model

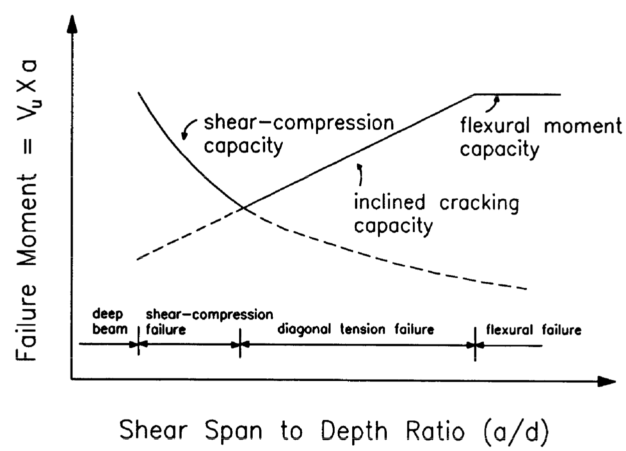

Fig. 1—Effect of a/d *on shear failure mode*

ACI Materials Journal, V. 93, No. 3, May-June 1996.
Received Feb. 8, 1994, and reviewed under Institute publication policies. Pertinent discussion will be published in the March-April 1997 *ACI Materials Journal* if received by Dec. 1, 1996.

Jin-Keun Kim, *MACI, is a professor of civil engineering at the Korea Advanced Institute of Science and Technology (KAIST), Taejon, Korea. He obtained his BS and MS degrees from Seoul National University, and his PhD from Northwestern University. His research interests include the application of fracture mechanics to concrete structures and inelastic analysis of concrete structures.*

Yon-Dong Park *is a senior research engineer at Tong Yang Central Laboratories, Korea. He obtained his PhD from KAIST. His research interests include production and mechanical properties of high-strength and high-performance concrete and shear behavior of reinforced concrete members.*

is necessary to reliably estimate the shear strength of beams made with high-strength concrete. Moreover, because of the wider range of concrete strength used, more accurate predictions of shear strength of reinforced concrete members are required.

The shear failure of reinforced concrete beams without web reinforcement has been known to be a typical case of brittle failure and indicates significant size effect. In 1981, Reinhardt[1] introduced fracture mechanics in the prediction of shear strength. He analyzed limited test data for shear failure based on linear elastic fracture mechanics. Subsequently, it was established that the size effect implied by linear elastic fracture mechanics is too strong in the case of concrete, and that brittle failures of concrete structures are better described by nonlinear fracture mechanics. Meanwhile, a simple and approximate size effect law on the basis of nonlinear fracture mechanics was proposed by Bazant.[2] Several studies[3-6] have shown that Bazant's size effect law is in good agreement with test results. However, there is some discrepancy between the prediction by Bazant's law and the test data, particularly for large-sized specimens. Recently, Kim and Eo[7] proposed a modified Bazant's size effect law to reduce the discrepancy.

In the present study, a simple and accurate equation predicting the shear strength of reinforced concrete beams without web reinforcement is proposed based on basic mechanisms of shear transfer and a modified Bazant's size effect law deduced by Kim and Eo, and it was verified by the published test data. In addition, a simplified equation is also proposed for practical design purposes. These equations that include the effects of all the factors previously mentioned are supported by test results and are compared with other prediction equations for the shear strength of beams without web reinforcement.

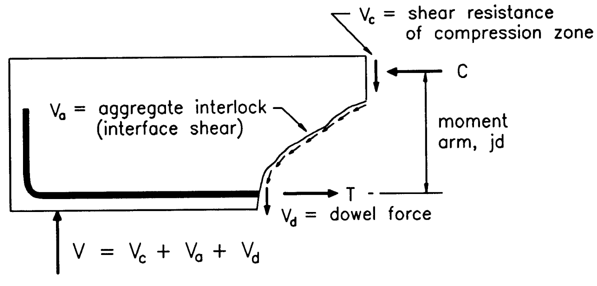

Fig. 2—Shear transfer mechanisms of slender beams

RESEARCH SIGNIFICANCE

Research reported in this paper provides a rational and accurate equation for the prediction of shear strength of reinforced concrete beams without web reinforcement. The results show that the proposed equation predicts the existing experimental data more accurately than the other equations in this study. A simplified design equation is also proposed for practical purposes without a significant accuracy reduction compared with the original equation.

BASIC SHEAR TRANSFER MECHANISMS

For slender beams where a/d is greater than 2.0 ~ 3.0, the shear force in a cracked section of a reinforced concrete beam is mainly resisted by the shear resistance of compression zone, interlocking action of aggregates, and dowel action, as shown in Fig. 2. For rectangular beams, after an inclined crack has formed, the proportion of the shear force transferred by the various mechanisms is as follows:[8] 20 to 40 percent by the uncracked concrete of compression zone; 33 to 50 percent by interlocking action of aggregates; and 15 to 25 percent by dowel action. Meanwhile, in a relatively short beam, the load is transferred directly from the loading points to supports owing to arch action.

Shear resistance of uncracked concrete

In a reinforced concrete beam, after the development of flexural cracks, a certain amount of shear is carried by the concrete in the compression zone. It is clear that the shear failure in the uncracked concrete is recognized as the failure under combined compression and shear[9] and the area of uncracked zone. The position of the neutral axis in a beam after flexural cracking is mainly dependent on the elastic modulus of concrete and the longitudinal steel ratio because elastic modulus of steel is nearly constant. Therefore, the shear force carried by the uncracked concrete in the compression zone can be represented by the compressive strength of concrete and the longitudinal steel ratio, since the concrete strength in the biaxial state and elastic modulus of concrete is the function of the compressive strength of concrete.

Interlocking action of aggregates

Previous experimental studies[8-10] have shown that a large portion of the total shear force on the beam without web reinforcement is carried across the cracks by aggregate interlocking. Among many variables, the width of the crack and the concrete strength are likely to be the most important factors.

Since the flexural crack width is approximately proportional to the strain of tension reinforcement, the crack width at failure becomes smaller as the longitudinal steel ratio is increased. Also, with increasing a/d, the strain of tension reinforcement at failure is increased. Meanwhile, it is naturally expected that the interlocking force will be increased when the strength of concrete is high.

Dowel action

When shear displacement occurs along the cracks, a certain amount of shear force is transferred by means of dowel action of the longitudinal bars. Although there is some contribution in dowel action by the number and arrangement of longitudinal bars, spacing of flexural cracks, and the amount of concrete cover, etc., the main factors influencing this action are flexural rigidity of longitudinal bars and the strength of the surrounding concrete.

Arch action

In relatively short beams, applied loads are transferred directly to the supports by arch action. The main factors influencing this action are the span-to-height ratio of the analogous arch and the strength of the compression strut. The span-to-height ratio of the analogous arch is approximately equal to the shear span-to-depth ratio. The strength of the compression strut is closely related to the compressive strength of concrete and the area of tension reinforcement.

DEVELOPMENT OF EQUATION

As previously discussed, shear resistance of uncracked concrete in the compression zone can be expressed as

$$V_c = c_1 (f'_c)^l (bkd) \tag{1}$$

in which f'_c is the compressive strength of concrete, b is the width of the beam, kd is the neutral axis depth, and c_1 and l are empirical constants. The constant l depends on the ratio of shear stress to compressive stress in an uncracked zone, and it is in the range of 0.5 to 1.0. According to the classical bending theory of reinforced concrete beams with only tensile reinforcement and a negligible tensile capacity of concrete, we would have

$$k = (n^2\rho^2 + 2n\rho)^{1/2} - n\rho \tag{2}$$

in which $n = E_s/E_c$ = ratio of elastic moduli of steel and concrete. Eq. (2) is, however, unnecessarily complicated and may be replaced by the following simpler expression

$$k = c_2 (n\rho)^m \tag{3}$$

in which c_2 and m are certain constants. Within the practical range, i.e., $5 \le n \le 10$ and $0.005 \le \rho \le 0.05$; consequently, $0.025 \le n\rho \le 0.5$. These constants can be chosen so that the values given by Eq. (2) and (3) are almost undistinguishable. Fig. 3 shows the optimum values of c_2 and m.

The elastic modulus of normal-weight concrete can be represented as

$$E_c = c_3 (f'_c)^{0.5} \tag{4}$$

in which f'_c is the compressive strength of concrete and c_3 is a certain constant. Therefore, substituting with Eq. (3), where c_2 and m are the values shown in Fig. 3, and Eq. (4) into Eq. (1), we obtain

$$V_c = c_5 (f'_c)^{(l-0.18)} \rho^{0.36} bd \tag{5}$$

in which c_5 is a constant.

According to Reineck,[11] the maximum value of shear stress along the crack surface due to aggregate interlock depends on the tensile strength of concrete and is decreased linearly with increasing crack width Δn as follows

$$\tau_{fu} = 0.45 f'_t \left(1 - \frac{\Delta n}{\Delta n_u}\right) \text{ with } \Delta n_u = 0.9\text{mm} \tag{6}$$

Crack width is resolved as $\Delta n = 0.71\varepsilon_s s_{cr}$, in which ε_s is the strain of the longitudinal reinforcement and s_{cr} is the spacing of the primary cracks. From theoretical considerations and experimental comparisons, the spacing of the primary cracks was also derived by Reineck as $s_{cr} = 0.7(d - kd)$. Substituting the values of Δn and s_{cr} into Eq. (6) and assuming that $k = 0.3$ for the usual simplification and $d = 500$ mm as a reference size

$$\tau_{fu} = 0.45 f'_t (1 - 193.3\varepsilon_s) \tag{7}$$

Eq. (7) can be replaced approximately by the following simpler expression within the practical range of ε_s, i.e., $0.0008 \le \varepsilon_s \le 0.003$.

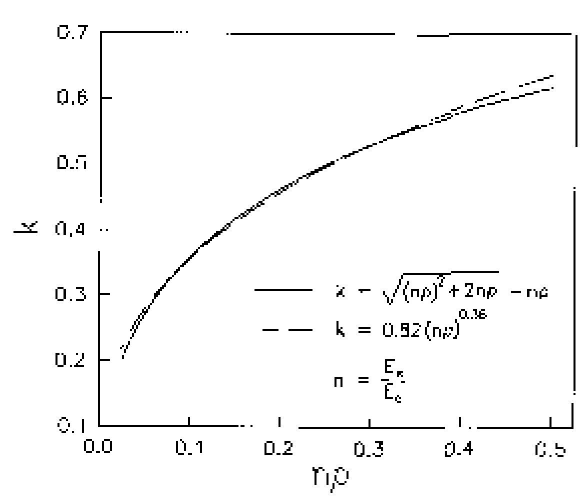

Fig. 3—Comparison of two equations on neutral axis

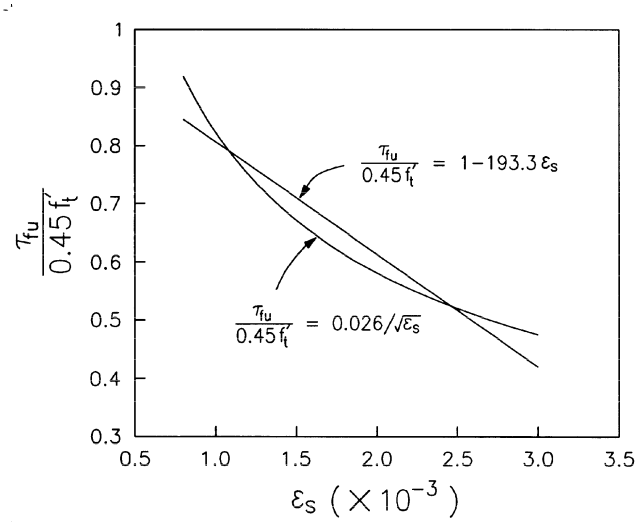

Fig. 4—Comparison of two equations on aggregate interlocking stress

$$\tau_{fu} = 0.012 f_t' / \sqrt{\varepsilon_s} \qquad (8)$$

The comparison of Eq. (7) and Eq. (8) is shown in Fig. 4. In reinforced concrete beams, as long as the tension reinforcement has not yielded, the strain of the longitudinal reinforcement at the ultimate state can be approximately expressed as

$$\varepsilon_s = \frac{V_u a_{eff}}{E_s A_s j d} \qquad (9)$$

in which V_u is ultimate shear force and a_{eff} is effective length of shear span. Substituting Eq. (9) into Eq. (8) and tentatively assuming that shear strength of reinforced concrete beams is approximately proportional to 1/3 power of ($f_c' \times \rho \times {}^d/a$) suggested by Zsutty, the shear force carried by the aggregate interlock [= $\tau_{fu} b(d - kd)$] is expressed as

$$V_a = c_8 (f_c' \rho {}^d/a)^{1/3} bd \qquad (10)$$

in which c_8 is some constant.

According to Vintzeleou and Tassios,[12] the dowel force is represented as

$$V_d = c_9 b_{ct} d_b f_t' \qquad (11)$$

in which c_9 is a certain constant, d_b is diameter of the bar, and b_{ct} is net width of the section (= $b - qd_b$, q being the number of bars). Eq. (11) can only be used when the number and diameter of the bar are known. Assuming the number of the bar is not changed, the dowel force can be expressed approximately by more general terms ρ as follows

$$V_d = c_{10} (f_c')^{0.5} \rho^r b d \qquad (12)$$

in which c_{10} is some constant and r, varying from 0.3 to 0.5 in a practical range, is a parameter that is dependent on spacing of the reinforcement.

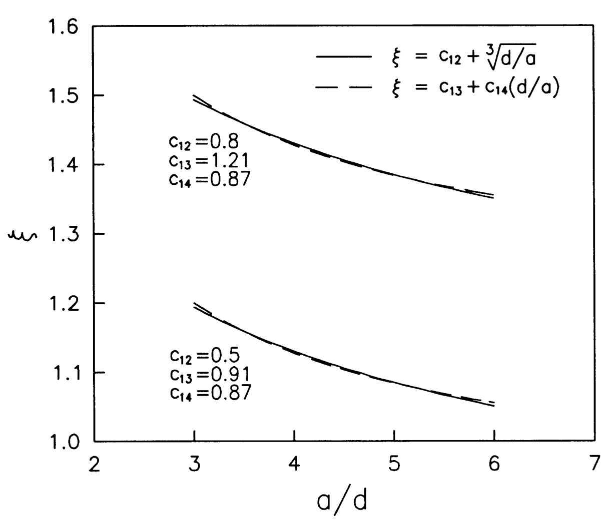

Fig. 5—Comparison of two equations on ξ

As previously discussed, the shear capacity of slender beams may be represented as Eq. (13), although the primary shear transfer mechanisms are coupled

$$V_u = V_c + V_a + V_d \qquad (13)$$

Substituting Eq. (5), (10), and (12) into Eq. (13), we obtain

$$V_u = [c_5 (f_c')^{(l - 0.18)} \rho^{0.36} + c_8 (f_c' \rho d/a)^{0.33} + c_{10} (f_c')^{0.5} \rho^r] b d \qquad (14)$$

It is noticed in Eq. (14) that the powers of f_c' have similar values. In practice, uncracked parts of reinforced concrete beams are stresed in two directions (biaxal stress condition). The power of f_c' in the first term of Eq. (14), therefore, may be varied in some range. Considering this fact and the minor contribution of dowel action (approximately 20 percent of total shear resistance), it can be assumed for simplicity that the power of f_c' is identical for all three terms in Eq. (14). In addition, the values of power of ρ in all three terms of Eq. (14) are very similar to each other. Consequently, Eq. (14) is then expressed as

$$V_u = c_{11} (f_c')^s \rho^t [c_{12} + ({}^d/a)^{1/3}] bd \qquad (15)$$

in which c_{11}, c_{12}, s, and t are certain constants. Because the term $[c_{12} + ({}^d/a)^{1/3}]$ can be replaced by $[c_{13} + c_{14}({}^d/a)]$ without a large difference (in which c_{13} and c_{14} are certain constants), as shown in Fig. 5, the shear capacity of reinforced concrete beams with constant effective depth can be expressed as

$$V_u = c_{15} (f_c')^s \rho^t (c_{16} + {}^d/a) bd \qquad (16)$$

in which c_{15} and c_{16} are certain constants.

To reduce the discrepancy between the prediction by Bazant's size effect law and the experimental data, especially for very large-sized specimens, Kim and Eo[7] recently suggested a modified Bazant's size effect law based on the concept of dissimilar initial cracks. The formula is as follows

$$\sigma_N = \frac{k_1 \sigma_r}{(1 + k_3 \beta)^{1/2}} + k_2 \sigma_r \qquad (17)$$

in which σ_N is the nominal strength of a specimen at failure, σ_r is the nominal strength of the specimen with reference size, and k_1, k_2, and k_3 are empirical constants. β is brittleness number, expressed as

$$\beta = \frac{(f_t')^2 D}{E_c G_f} \qquad (18)$$

in which f_t' is tensile strength, E_c is modulus of elasticity, G_f is fracture energy, and D is the characteristic size of the structure. f'_t and E_c are mainly dependent on compressive strength, and G_f is dependent on maximum aggregate size (d_a) and compressive strength. G_f is increased with d_a and compressive strength, although the increase is minimal in the higher strength range. For reinforced concrete beams, D is effective depth. Consequently, β can be expressed as a function of f'_c, d_a, and d. If stirrups are used, G_f is increased because crack localization is prevented, to some extent, by stirrups. Thus, the effect of stirrups in the size effect of reinforced concrete beams in shear can be considered. Meanwhile, since the effects of factors influencing G_f are not clear and the ranges of f'_c and d_a are limited in the construction field, experimenal data have shown that the capacity of beams is not significantly influenced by d_a; therefore, it is rational to assume that $(f'_t)^2/E_c G_f$ is a constant. Therefore, Eq. (17) can be expressed as

$$\sigma_N = \frac{k_1 \sigma_r}{(1 + k_4 d)^{1/2}} + k_2 \sigma_r \tag{19}$$

in which k_4 is an empirical constant. Eq. (16) does not take into account the size effect. According to Eq. (19), we finally obtain

$$v_u = c_{17} (f'_c)^s \rho^t (d/a + c_{16}) \left(\frac{1}{(1 + k_4 d)^{1/2}} + k_5 \right) \tag{20}$$

in which $c_{17} = c_{15} k_1$ and $k_5 = k_2/k_1$. Eq. (20) has been compared to all other important experimental data, with regard to not only the effects of concrete strength, longitudinal steel ratio, and shear span-to-depth ratio, but also to the effect of effective depth.

From the statistical analysis of existing experimental data,[13-44] the values of all six parameters in Eq. (20) have been determined. The experimental data used in the analysis are only three or four point test results of simple beams with rectangular sections except for Shioya's test results,[37] which applied uniform load. Moreover, test results of beams with small d [less than 100 mm (3.94 in.)] or small maximum aggregate size [less than 5 mm (0.2 in.)] were excluded for practical purposes. Cubic compressive strength of concrete reported in some tests was converted to the cylindrical compressive strength according to Neville's empirical relation.[45] Finally, the following equation is proposed for the mean nominal shear strength of reinforced concrete beams with a/d equal to or greater than 3.0

$$v_u = 3.5 f_c'^{1/3} \rho^{3/8} (0.4 + d/a) \lambda(d) \tag{21}$$

in which

$$\lambda(d) = \frac{1}{\sqrt{1 + 0.008d}} + 0.18$$

f'_c and v_u are in MPa and d is in mm. Fig. 6 shows the ratio of shear strength tested to predicted with $a/d \geq 3.0$ for each

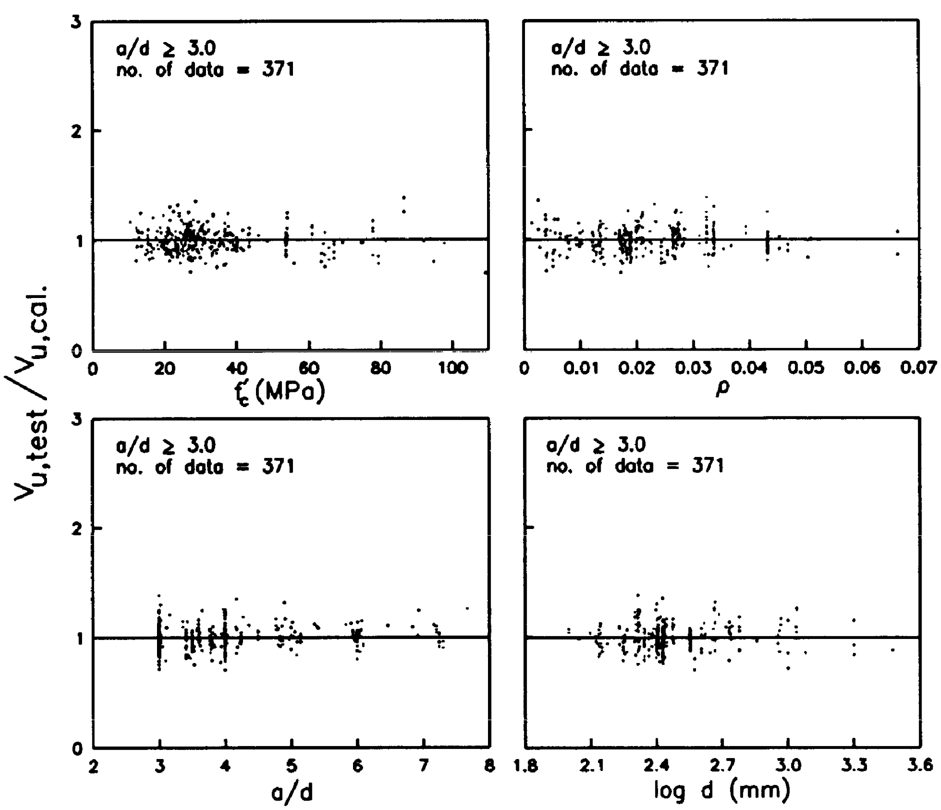

Fig. 6—Ratio of shear strength for each primary factor

factor by the proposed equation. As shown in the figure, it can be concluded that the proposed equation estimates properly the effects of primary factors.

For relatively short beams, the failure mode is changed gradually from diagonal tension to extreme shear compression. Consequently, the effect of concrete strength on the shear strength is also gradually increased with decreasing a/d. Previous experimental work[32] has shown that the effect of concrete strength on the shear strength of reinforced concrete beams becomes more significant with decreasing a/d. If we roughly assume that the effects of ρ and d are not changed with a change of failure mode, the shear strength of relatively short beams can be expressed as

$$\nu_u = 3.5 f_c'^{\alpha/3} \rho^{3/8} (0.4 + d/a) \lambda(d) \tag{22}$$

where α is the failure mode index that is dependent on the a/d ratio. From the compatibility condition, which is $\alpha = 1$ at $a/d = 3.0$, and the statistical analysis of existing experimental data, α is determined as

$$\alpha = 2 - \frac{a/d}{3} \quad \text{for } 1 \le a/d < 3 \tag{23}$$

EVALUATION OF PROPOSED EQUATION

Many equations have already been proposed to estimate the shear strength of reinforced concrete beams. To evaluate the proposed model, three well-known equations are selected for comparison: 1) Zsutty's equation[46] deduced by multiple regression analysis; 2) Bazant's equation[5] derived based on Bazant's size effect law; and 3) the ACI Code equation.[47] These equations are as follows:

ACI Code equation

$$\nu_c = 0.1578\sqrt{f_c'} + 17.25\rho_w \frac{V_u d}{M_u} (\text{MPa}) (a/d \ge 2.5) \tag{24}$$

$$\nu_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right) \times [\text{Eq. (24)}] \quad (a/d < 2.5) \tag{25}$$

Zsutty's equation

$$\nu_u = 2.1746\left(f_c' \rho \frac{d}{a}\right)^{1/3} \quad (\text{MPa}) \quad (a/d \ge 2.5) \tag{26}$$

$$\nu_u = \left(2.5\frac{d}{a}\right) \times [\text{Eq. (26)}] \quad (a/d < 2.5) \tag{27}$$

Bazant's equation

$$\nu_u = 0.54\sqrt[3]{\rho}\left(\sqrt{f'_c} + 249\sqrt{\frac{\rho}{(a/d)^5}}\right) \times \frac{1 + \sqrt{5.08/d_a}}{\sqrt{1 + d/(25 d_a)}} \quad (\text{MPa}) \tag{28}$$

The ratios of shear strength (tested to predicted) with the bulk of existing experimental data determined by three other equations and the proposed equation are shown in Fig. 7. For the ACI Code equation, the scatter is much larger than those of the other equations. It is shown that the ratio of shear strength obtained by Zsutty's equation increases gradually with increasing shear strength. This phenomenon is mainly due to the fact that Zsutty's equation ignores size effect and slightly underestimates the effect of a/d when a/d is less than 2.5. Bazant's equation predicts the shear strength of experimental data comparatively well, although the scatter is somewhat larger than that of the proposed equation. As shown in the figure, the proposed equation, which has a standard deviation of 0.162 with respect to average ratio of shear strength and a correlation coefficient of 0.926, predicts more accurately than the other equations.

Fig. 8 shows the ratios of tested-to-calculated shear strength of existing test data failed by diagonal tension ($a/d \ge 3.0$) with variation of d. The ratios of shear strength obtained from the ACI Code equation and Zsutty's equation are decreased with increasing d due to the disregard of size effect, whereas Bazant's equation and the proposed equation have nearly constant value. Although Bazant's equation reflects not only the effect of d but also the effect of maximum aggregate size, the scatter is somewhat larger than that of the proposed equation.

For relatively short beams failing in shear compression, the scatter of test data is larger than that of slender beams failing in diagonal tension. As shown in Fig. 9, the standard deviation of the ratio of shear strength for the ACI Code equation is more than twice that of the proposed equation. Although scatters of the ratio of shear strength for Zsutty's equation and Bazant's equation are lower than that of the ACI Code equation, they are larger than that of the proposed equation, to some extent. Similar to the case of slender beams, the proposed equation with a standard deviation of 0.230 and a correlation coefficient of 0.901 is considered to be better than the other equations in terms of accuracy and estimation of primary factors.

PROPOSED SIMPLE DESIGN EQUATION

From a practical point of view, design equations should be as simple as possible within a range of acceptable accuracy. Although the proposed equation is relatively simple and accurate, and estimates the effects of the primary factors well, it is somewhat complicated to use in practical design, especially regarding the size factor $\lambda(d)$. The effective depth of reinforced concrete beams in construction is usually limited within the range of $d \ge 250$ mm (9.84 in.). Since small-sized structures that conform to the strength criterion are not presented, the term $\lambda(d)$, which indicates size effect, can be approximately represented using a simpler expression as follows

$$\lambda(d) = \frac{k_6}{\sqrt{d}} + k_7 \tag{29}$$

in which k_6 and k_7 are some constants. From the regression analysis of existing experimental data, the values of k_6 and k_7 have been determined. As a result, the simplified size factor

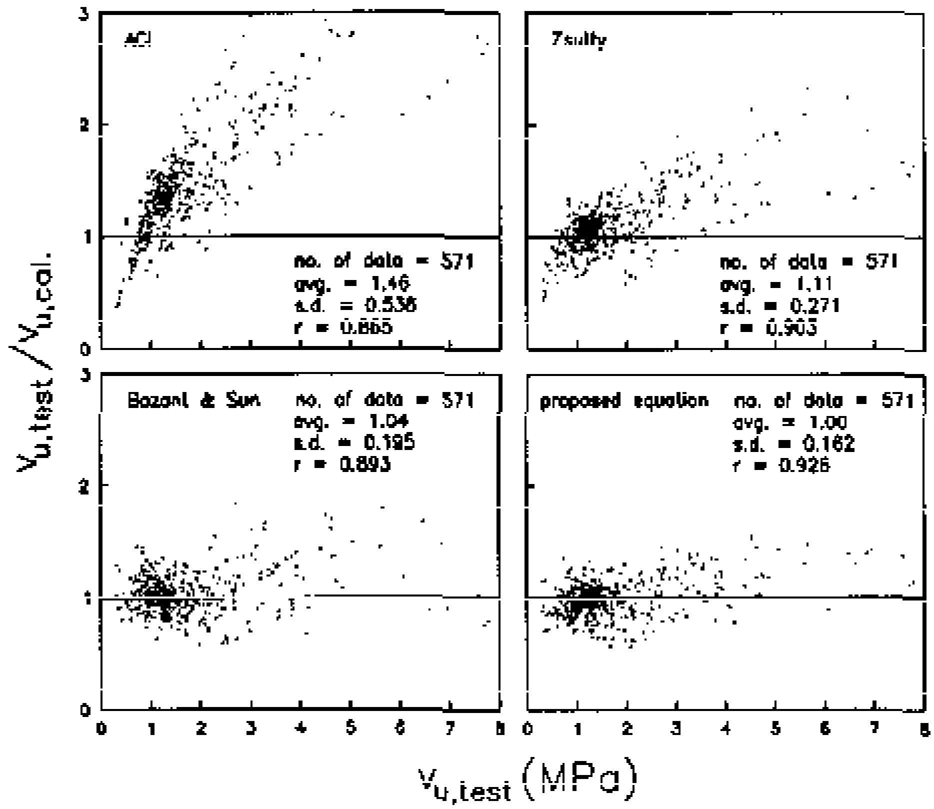

Fig. 7—Comparison of various equations with bulk of existing data

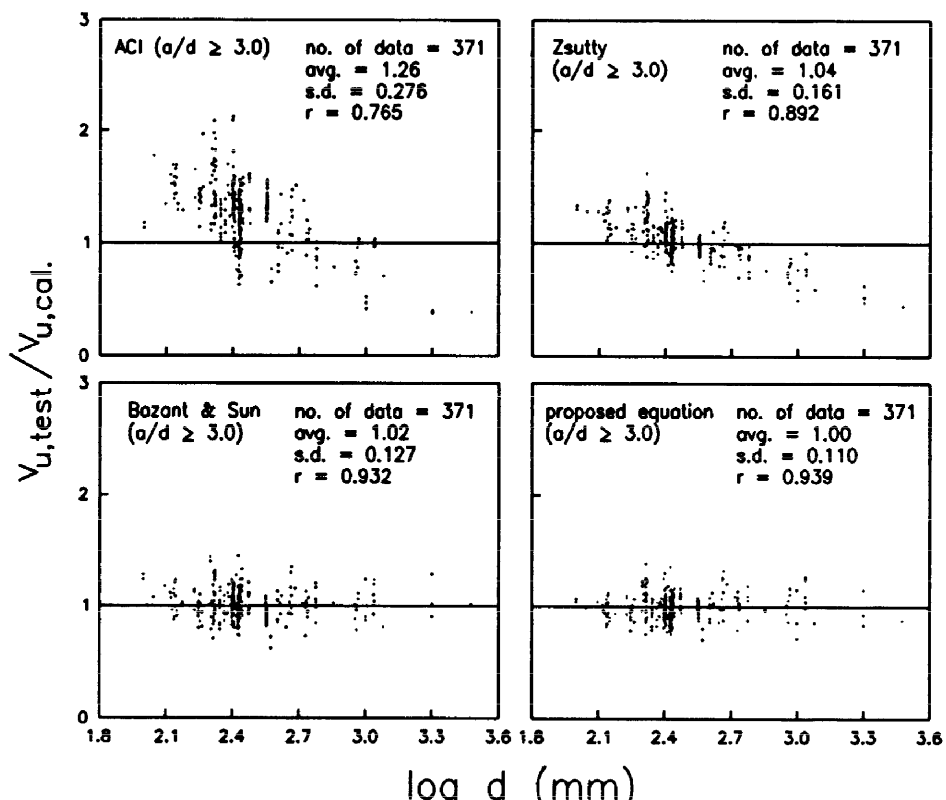

Fig. 8—Comparison of various equations with increasing d (a/d ≥ *3.0)*

is nearly identical with the one in the CEB-FIP model code.[48] Consequently, the following simplified equation is derived for the mean nominal shear strength

$$\nu_u = 19.4 f_c'^{\alpha/3} \rho^{3/8} (0.4 + d/a)\left(\frac{1}{\sqrt{d}} + 0.07\right) \quad \text{for } d \geq 250 \text{ mm (9.84 in.)} \tag{30}$$

in which $\alpha = 1$ for $a/d \geq 3.0$ and $2 - (a/d)/3$ for $1.0 \leq a/d < 3.0$.

Fig. 10 shows the comparison of results obtained from the original equation and the simplified equation. As shown in the figure, the simplified equation has nearly identical accuracy compared with the original equation, although some approximation has been made.

Assuming a normal distribution of the ratios of shear strength, the design equation for the shear strength of reinforced concrete beams with 90 percent reliability is represented as

$$\nu_{u,\,d} = 15.5 f_c'^{\alpha/3} \rho^{3/8} (0.4 + d/a)\left(\frac{1}{\sqrt{d}} + 0.07\right) \tag{31}$$

This equation is shown by the straight line in Fig. 11. As shown, only a few data points lie below the equation, and no data points fall significantly below this equation.

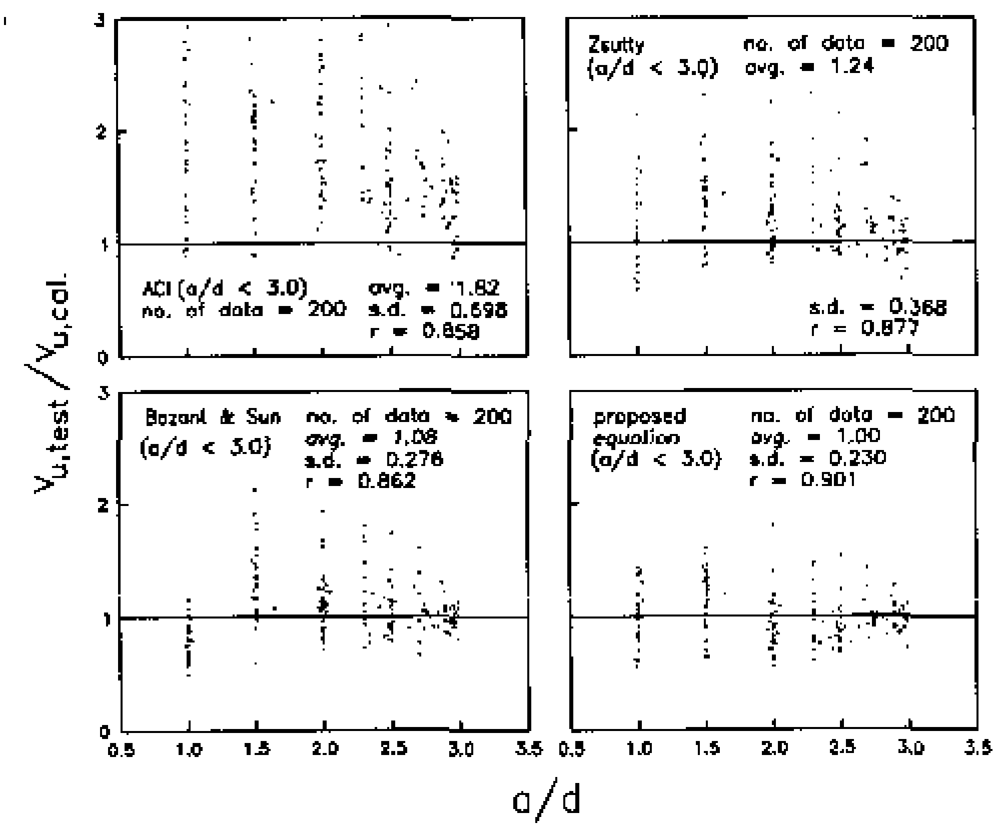

Fig. 9—Comparison of various equations with increasing a/d (a/d < *3.0)*

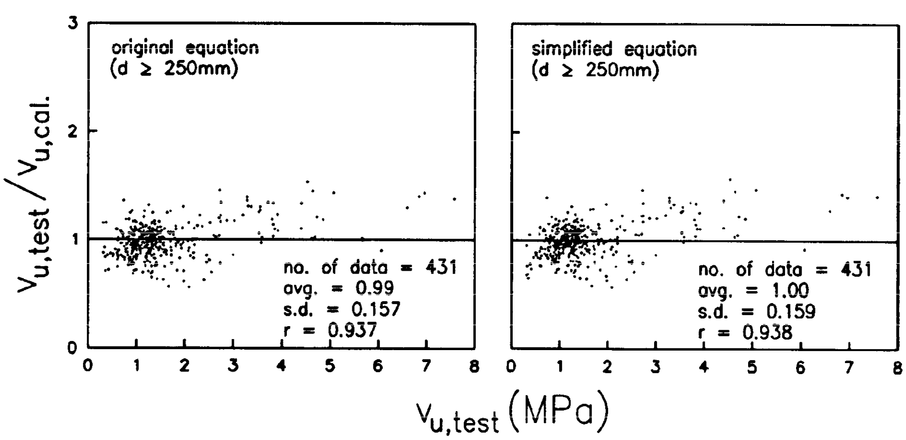

Fig. 10—Comparison between original equation and simplified equation

CONCLUSIONS

On the basis of the results obtained in this study, the following conclusions may be drawn.

1. Based on shear transfer mechanisms, modified size effect law, and numerous published experimental data, the following equation is derived for the nominal shear strength of reinforced concrete beams without web reinforcement

$$\nu_u = 3.5 f_c'^{\alpha/3} \rho^{3/8} (0.4 + d/a) \left(\frac{1}{\sqrt{1 + 0.008d}} + 0.18 \right)$$

in which $\alpha = 1$ for $a/d \geq 3$, $2 - (a/d)/3$ for $1 \leq a/d < 3$.

2. Comparison with published experimental data indicates that the proposed equation estimates properly the effects of primary factors, such as concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth.

3. As a result of the comparison with the ACI Code equation, Zsutty's equation, and Bazant's equation, the accuracy of the proposed equation is better than that of any other equation.

4. For practical design, a simple and accurate equation deduced from the simplification of the originally proposed equation is derived as

$$\nu_{u,d} = 15.5 f_c'^{\alpha/3} \rho^{3/8} (0.4 + d/a) \left(\frac{1}{\sqrt{d}} + 0.07 \right)$$

for $d \geq 250$ mm (9.84 in.)

The accuracy of the simplified equation is nearly identical compared with the original equation and is considered to be better than those of the other equations compared in this study.

5. The effect of concrete strength on the shear strength of beams with $a/d < 3.0$ is estimated well by introducing failure mode index α.

ACKNOWLEDGMENTS

Support for this research was partially provided by the Engineering Research Center (ERC) on Advanced Structure Research (STRESS) of KOSEF. The STRESS is gratefully acknowledged.

NOTATION

a_{eff} = effective length of shear span
A_s = area of tension reinforcement
a/d = shear span-to-depth ratio
b = width of beam
b_{ct} = net width of beam section
$c_1, c_2, ..., c_{17}$ = constants
D = characteristic size of structure
d = effective depth of beam
d_a = maximum aggregate size
d_b = diameter of reinforcing bar
E_c = elastic modulus of concrete
E_s = elastic modulus of steel
f_c' = compressive strength of concrete
f_t' = tensile strength of concrete
G_f = fracture energy
jd = internal lever arm
$k_1, k_2, ..., k_7$ = constants
kd = neutral axis depth
l, m, p, s, t = constants
M_u = ultimate moment
n = ratio of elastic moduli of steel and concrete
s_{cr} = spacing of primary cracks
V_a = shear force carried by aggregate interlock
V_c = shear resistance of compression zone
V_d = dowel force
V_u = shear capacity of beams
v_u = shear strength of beams
α = failure mode index
β = brittleness number
Δn = crack width
Δn_u = ultimate crack width
ε_s = strain of tension reinforcement
$\lambda(d)$ = size factor
ρ, ρ_w = longitudinal steel ratio
σ_N = nominal strength of specimen at failure
σ_r = nominal strength of specimen with reference size
τ_{fu} = maximum shear stress along crack surface

REFERENCES

1. Reinhardt, H. W., "Masstabseinfluss bei Schubversuchen im Licht der Bruchmechanik," *Beton-und Stahlbetonbau* (Berlin), No. 1, 1981, pp. 19-21.

2. Bazant, Z. P., "Size Effect in Blunt Fracture: Concrete, Rock, Metal," *Journal of Engineering Mechanics*, ASCE, V. 110, EM4, 1984, pp. 518-535.

3. Bazant, Z. P., and Kim, J. K., "Size Effect in Shear Failure of Longitudinally Reinforced Beams," ACI JOURNAL, *Proceedings* V. 81, No. 5, Sept.-Oct. 1984, pp. 456-468.

4. Bazant, Z. P., and Cao, Z., "Size Effect of Shear Failure in Prestressed Concrete Beams," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 260-268.

5. Bazant, Z. P., and Sun, H. H., "Size Effect in Diagonal Shear Failure: Influence of Aggregate Size and Stirrups," *ACI Materials Journal*, V. 84, No. 4, July-Aug. 1987, pp. 259-272.

6. Bazant, Z. P., and Kezmi, M. T., "Size Effect on Diagonal Shear Failure of Beams without Stirrups," *ACI Structural Journal*, V. 88, No. 3, May-June 1991, pp. 268-276.

7. Kim, J. K., and Eo, S. H., "Size Effect in Concrete Specimens with Dissimilar Initial Cracks," *Magazine of Concrete Research* (London), V. 42, No. 153, Dec. 1990, pp. 233-238.

8. Taylor, H. P. J., "Investigation of Forces Carried Across Cracks in Reinforced Concrete Beams in Shear by Interlock of Aggregate," *Technical Report* No. 42.447, Cement and Concrete Association, London, Nov. 1970, 22 pp.

9. Guralnick, S. A., "Shear Strength of Reinforced Concrete Beams," *Proceedings*, ASCE, V. 85, No. ST1, Jan. 1959, pp. 1-42.

10. Fenwick, R. C., and Paulay, T., "Mechanisms of Shear Resistance of Concrete Beams," *Proceedings*, ASCE, V. 94, No. ST10, Oct. 1968, pp. 2325-2350.

11. Reinock, K. H., "Ultimate Shear Force of Structural Concrete Members without Transverse Reinforcemt Derived from a Mechanical Model,"

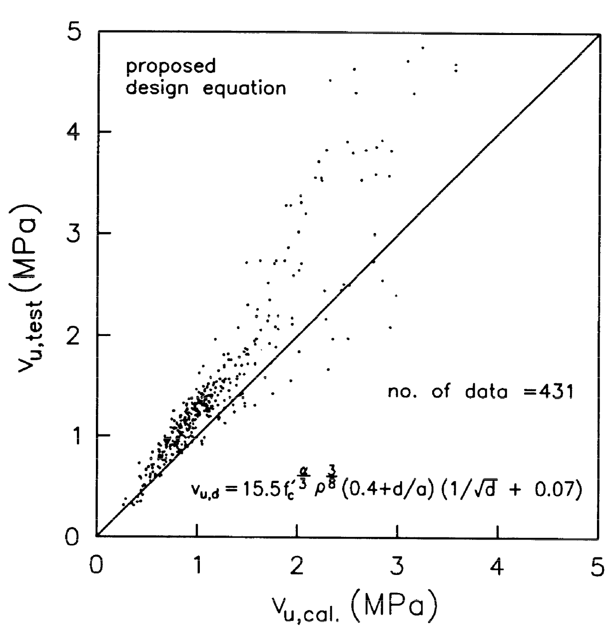

Fig. 11—Comparison between experimental data and calculated values by using design equation

ACI Structural Journal, V. 88, No. 5, Sept.-Oct. 1991, pp. 592-602.

12. Vintzeieou, E. N., and Tassios, T. P., "Mathematical Models for Dowel Action under Monotic and Cyclic Conditions," *Magazine of Concrete Research* (London), V. 38, No. 134, Mar. 1986, pp. 13-22.

13. Alimad, S. H.; Khaloo, A. R.; and Poveda, R., "Shear Capacity of Reinforced High-Strength Concrete Beams," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 297-305.

14. Akiyama et al., "FIexural and Shear Behavior of Beams with High-Strength Concrete and High-Strength Reinforcement," *Research Report*, Institute of Kajima Consruction Technology, Japan, No. 38, Oct. 1990, pp. 75-84.

15. Bial, N. S., "Uber den Einfluss der Balkenhohe auf Schubtragfahigkeit von einfledrigen Stahlbetonbalken mit und ohne Schubberwehrung," dissertation, Universitat Stuttgart, 1968, 124 pp.

16. Bresler, B., and Sedelis, A. C., "Shear Strength of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 60, No. 1, Jan. 1963, pp. 51-74.

17. Chana, P. S., "Some Aspects of Modeling the Behavior of Reinforced Concrete under Shear Loading," *Technical Report* No. 543, Cement and Concrete Association, July 1981, 22 pp.

18. Diaz de Cossio, R., and Siess, C. P., "Behavior and Strength in Shear of Beams and Frames without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 56, No. 8, Feb. 1960, pp. 695-735.

19. Elzanty, A. H.; Nilson, A. H.; and Slate, F. O., "Shear Capacity of Reinforced Concrete Beams Using High-Strength Concrete," ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar-Apr. 1986, pp. 290-296.

20. Gaston, J. R.; Siess, C. P.; and Newmark, N. M., "An Investigation of Load Determination Characteristics of Reinforced Concrete Beams Up to the Point of Failure," Civil Engineering Studies, *Structural Research Series* No. 40, University of Illinois, Urbana, Dec. 1952.

21. Ikeda, S., and Uji, K., "Studies on the Effect of Bond on the Shear Behavior of Reinforced Concrete Beams," *Proceedings*, JSCE, No. 293, Jan. 1980, pp. 101-109.

22. Johnson, M. K., and Ramirez, J. A., "Minimum Shear Reinforcement in Beams with Higher Strength Concrete," *ACI Structural Journal*, V. 86, No. 4, July-Aug. 1989, pp. 376-382.

23. Kani, G. N. J., "Basic Facts Concerning Shear Failure," ACI JOURNAL, *Proceedings* V. 63, No. 6, June 1966, pp. 675-692.

24. Kani, G. N. J., "How Safe Are Our Large Reinforced Concrete Beams?" ACI JOURNAL, *Proceedings* V. 64, No. 3, Mar. 1967, pp. 128-141.

25. Kim, J. K., and Park, Y. D., "Shear Strength of Reinforced High-Strength Concrete Beams without Web Reinforcement," *Magazine of Concrete Research* (London), V. 46, No. 166, Mar. 1994, pp. 7-16.

26. Ko, K. I., and Kim, W., "Shear Strength of High- Strength Reinforced Concrete Beams," *Proceedings*, KSCE, V. 9, No. 4, Dec. 1989, pp. 63-71.

27. Krefeld, W. J., and Thurston, C. W., "Studies of Shear and Diagonal Tension Strength of Simply Supported Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 63, No. 4, Apr. 1966, pp. 451-476.

28. Leonhardt, F., and Walther, R., "Beitrage zur Behandlung der Schubprobleme in Stahlebtonbau," Beton-und Stahlbetonbau (Berlin), Mar. 1962, pp. 32-44, 54-64.

29. Mathey, R. G., and Watstein, D., "Shear Strength of Beams without Web Reinforcement," ACI JOURNAL, *Proceedings* V. 60, No. 2, Feb. 1963, pp. 183-208.

30. Mattock, A. H., "Diagonal Tension Cracking in Concrete Beams with Axial Forces," *Proceedings*, ASCE, V. 95, No. ST9, Sept. 1969, pp. 1887-1900.

31. Moody, K. G.; Viest, I. M.; Elstner, R. C.; and Hognestad, E., "Shear Strength of Reinforced Concrete Beams, Part 1—Tests of Simple Beams," ACI JOURNAL, *Proceedings* V. 51, No. 3, Nov. 1954, pp. 317-332.

32. Mphonde, A. G., and Frantz, G. C., "Shear Tests of High- and Low-Strength Concrete Beams without Stirrups," ACI JOURNAL, *Proceedings* V. 81, No. 4, July-Aug. 1984, pp. 350-357.

33. Niwa, J.; Yamada, K.; Yokozawa, K.; and Okamura, H., "Re-Evaluation of the Equation for Shear Strength of Reinforced Concrete Beams without Web Reinforcement," *Proceedings*, JSCE, No. 372, Aug. 1986, pp. 167-176.

34. Placas, A., and Regan, P. E., "Shear Failure of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 68, No. 10, Oct. 1971, pp. 763-773.

35. Rajagopalan, K. S., and Ferguson, P. M., "Exploratory Shear Tests Emphasizing Percentage of Longitudinal Steel," ACI JOURNAL, *Proceedings* V. 65, No. 8, Aug. 1968, pp. 634-638.

36. Rusch, H.; Haugli, F. R.; and Mayer, H., "Schubversuche an Stahlbeton-Rechteckbalken mit gleichmassing verteilter Belastung," *Bulletin* No. 145, Deutcher Ausschuss fur Stahlbeton, Berlin, 1962, pp. 4-30.

37. Shioya, T., "Shear Properties of Large Reinforced Concrete Members," *Report* No. 25, Institute of Technology of Shimizu Corporation, Japan, 1989, 198 pp.

38. Taylor, H. P. J., "Shear Strength of Large Beams," *Proceedings*, ASCE, V. 98, No. ST1 1, Nov. 1972, pp. 2473-2490.

39. Taylor, R., "Some Shear Tests on Reinforced Concrete Beams without Shear Reinforcement," *Magazine of Concrete Research* (London), V. 12, No. 36, Nov. 1960, pp. 145-154.

40. Taylor, R., and Brewer, R. S., "Effect of the Type of Aggregate on Diagonal Cracking of Reinforced Concrete Beams," *Magazine of Concrete Research* (London), V. 15, No. 44, July 1963, pp. 87-92.

41. Thorenfeldt, E., and Drangsholt, G., "Shear Capacity of Reinforced High-Strength Concrete Beams," *High-Strength Concrete—Second International Symposium*, SP-121, American Concrete Institute, Detroit, 1990, pp. 129-154.

42. van den Berg, F. J., "Shear Strength of Reinforced Concrete Beams without Web Reinforcement, Part 2—Factors Affecting Load at Diagonal Cracking," ACI JOURNAL, *Proceedings* V. 59, No. 11, Nov. 1962, pp. 1587-1600.

43. Walraven, J. C., "Influence of Depth on Shear Strength of Lightweight Concrete Beams without Shear Reinforcement," *Stevin Laboratory Report* No. 5-78-4, Delft University of Technology, 1972, 36 pp.

44. Walraven, J. C., "Size Effect: Their Nature and Their Recognition in Building Codes,"*Proceedings of JCI International Workshop on Size Effect in Concrete Structures*, Japan, 1993, pp. 295-314.

45. Neville, A. M., "General Relation for Strengths of Concrete Specimens of Different Shapes and Sizes," ACI JOURNAL, *Proceedings* V. 63, No. 10, Oct. 1966, pp. 1095-1109.

46. Zsutty, T. C., "Shear Strength Prediction for Separate Categories of Simple Beam Tests," ACI JOURNAL, *Proceedings* V. 68, No. 2, Feb. 1971, pp. 138-143.

47. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-89)," American Concrete Institute, Detroit, 1989, 353 pp.

48. Comite Euro-International du Beton (CEB), "CEB-FIP Model Code 1990," Switzerland, July 1991.