THE GRAPHICAL D-Q TRANSFORMATION OF GENERAL POWER SWITCHING CONVERTERS

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Abstract- New circuit D-Q transformation concept is introduced to analyze AC converters such as inverters, rectifiers and cyclo-converters with ease. The equivalent linear time invariant circuit is obtained by substituting switches with equivalent turn-ratio variable transformers and changing balanced AC reactors into equivalent DC reactors combined by gyrators. This circuit enables us to utilize the powerful linear system analysis techniques such as Laplace transform otherwise which could not be applied to the time varying switching systems. Direct substitution of switches of DC converters with transformers is shown as a preliminary. Then the modeling procedure is shown for a controlled rectifier-inverter circuit. Finally an analysis example is proposed for a buck-boost inverter and the result is compared with the conventional approach. This approach is applicable to all AC converter families to determine the AC transfer functions and the DC operating points. It is identified that the switching systems are equivalent to the RLC filter circuits with transformers and gyrators.

I. INTRODUCTION

It has been a great problem in the power electronics to deal with the switches since they make the linear time invariant systems be non-linear time-varying systems [1]. Simplifying or eliminating the switches is a great issue in the modeling of switching systems. A successful method is the state-space averaging technique [2]. This approach is well established for DC-DC converters. A trial for DC-AC converters is found in [3]. This eliminates the time-varying nature of the switching systems by the equational D-Q transformation. The equational approach is, however, limited to the system having less than 4-5 reactive elements since the manipulation of matrices whose order is higher than 4 is formidable to handle in practice.

The graphical modeling method is very easy to apply and reduces the effort to obtain the model. Also it gives much physical insight in comparison with the equational approach. In this paper a circuit D-Q transformation method for AC converters is suggested. This approach adopts the D-Q transformation and an equivalent circuit is finally obtained without including the switches and AC reactors. This makes it very easy to understand and analyze the switching systems. This approach eliminates all cumbersome equations and reduces the effort drastically. What to do is only to draw an equivalent circuit from the original circuit and write the results from it. The modeling and analysis procedures are shown for two cases: rectifier-inverter and buck-boost inverter, respectively.

II. SWITCHES AS EQUIVALENT TRANSFORMERS

A. Switches in the DC-DC Converters

It is known that conventional DC-DC converters have a unified model which consists of filters and transformers [2]. This result is obtained by several equational or graphical manipulations. In this proposed modeling, direct substituting of switches with a transformer is done. So the procedure is drastically simplified. Also the DC operating points are determined without any computation.

As an example, the buck converter shown in Fig. 1(a) is selected. The two switches constitute a switch set. This switch set is equivalent to a switched time-varying auto-transformer whose turn-ratio is defined to be the dual value of the switch set as shown in Fig. 1(b). This can be understood as follows: When the switch S_1 is turned on, that is $d(t)=1$, the upper turn-ratio is set to zero, that is $n(t)=0$, then the configurations of the two circuits become identical, that is, points A and C are connected and points B and C are disconnected. Since the magnatizing inductance of an ideal transformer is infinite, points B and C are disconnected. When the switch S_1 is turned off, the lower turn-ratio is set to zero whereas the upper one is set to unity. There has been no approximation so a switch set is exactly equivalent to an auto-transformer. The only approximation is the averaging of the turn-ratio. If the switching frequency is much higher than the cut-off frequency of the filter, then the switching harmonics are negligible compared with the fundamental component. Under this condition the switched transformer can be regarded as the conventional ideal transformer whose turn-ratio is the fundamental component of $n(t)$ as shown in Fig. 1(c). The DC model is obtained by simply eliminating the reactive elements as shown in Fig. 1(d) since there is no time-varying switching element.

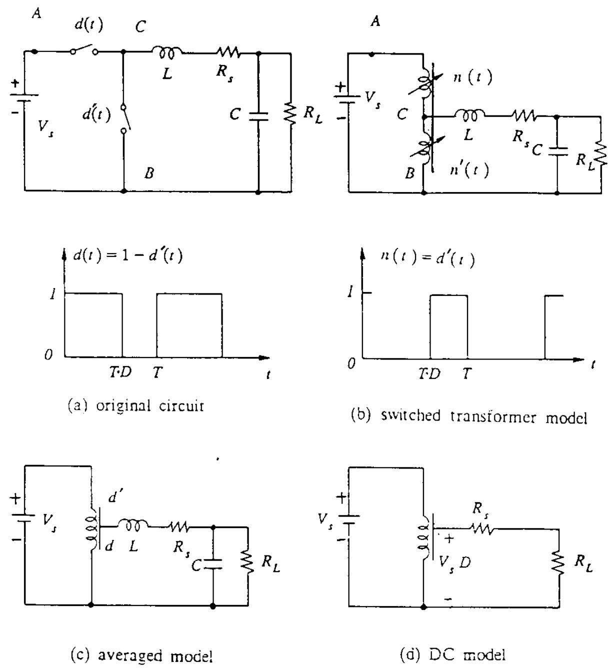


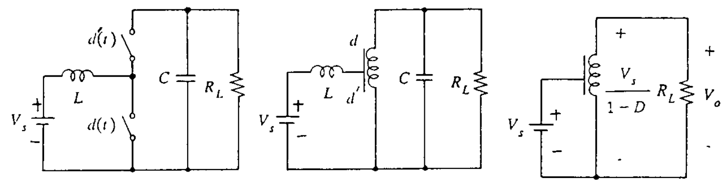
Fig. 1 Buck converter example .

The procedures for the boost, buck-boost and Ćuk converters are the same as those discussed above and the resultant equivalent circuits are shown in Fig. 2, Fig. 3 and Fig. 4, respectively. Note that the equivalent DC circuits of the buck-boost and Ćuk converter are just identical.

It is not necessary to use any equations when one is get used to this procedure. This useful feature is to be extended to the AC converters.

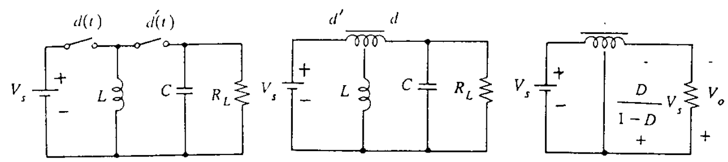
B. Switches in DC-AC (or AC-DC) Converters

The fact that a certain switching converter acts as a transformer which changes frequency, voltage, current, phase and power factor is known in [4]. In this paper this property of switches



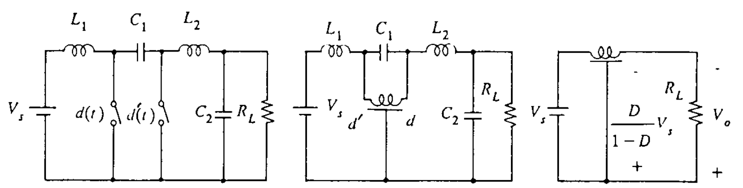
(a) original circuit (b) averaged circuit (c) DC circuit

Fig. 2 Boost converter.



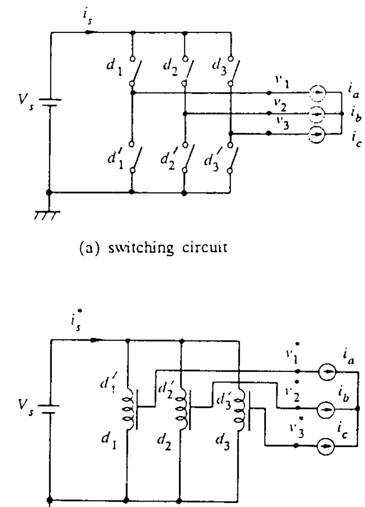
(a) original circuit (b) averaged circuit (c) DC circuit

Fig. 3 Buck-boost converter.



(a) original circuit (b) averaged circuit (c) DC circuit

Fig. 4 Ćuk converter.



(a) switching circuit

(b) transformer circuit

Fig. 5 Switches as transformers of VSI.

as transformers is proved in general and extended to all switching converters. The proof of the equivalence of the two circuits can be done by proving that the outer characteristics of both circuits are identical.

Those of Fig. 5(a) are

$$\begin{aligned} i_s &= i_a d_1 + i_b d_2 + i_c d_3 \\ v_1 &= V_s d_1 \\ v_2 &= V_s d_2 \\ v_3 &= V_s d_3 \quad . \end{aligned} \tag{1}$$

And those of Fig. 5(b) are

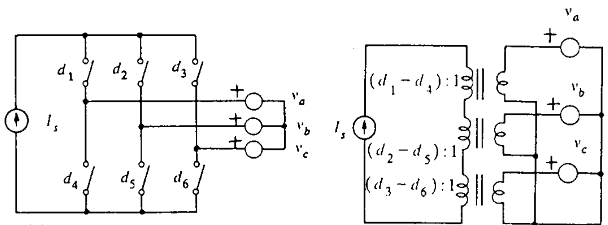
$$\begin{aligned} i_s^* &= i_a d_1 + i_b d_2 + i_c d_3 \\ v_1^* &= V_s d_1 \\ v_2^* &= V_s d_2 \\ v_3^* &= V_s d_3 \quad . \end{aligned} \tag{2}$$

By comparing (1) and (2), it is found that

$$\begin{aligned} i_s^* &= i_s \\ v_1^* &= v_1 \\ v_2^* &= v_2 \\ v_3^* &= v_3 \end{aligned} \tag{3}$$

which implies that the two circuits are equivalent. By similar procedure, the equivalent circuit for a CSI is obtained as depicted in Fig. 6.

C. Switches in AC-AC Converters



(a) switching circuit (b) transformer circuit

Fig. 6 Switches as transformers of CSI

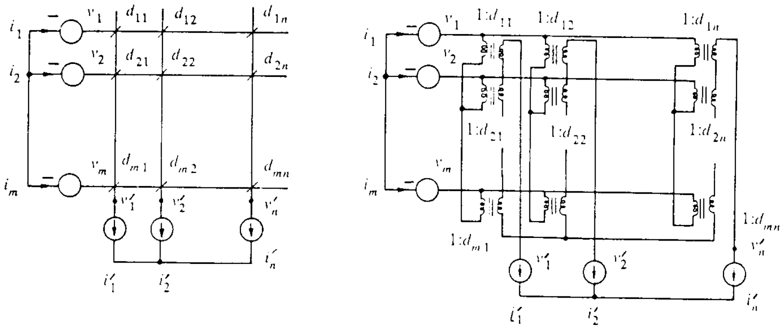
A general switching system is shown in Fig. 7(a) which has m inputs and n-outputs. The equivalent transformer circuit is Fig. 7(b). The proof is straightforward. Applying the following relationships to both circuits, it is easily verified that both of them satisfy the relationships.

$$\begin{aligned} \mathbf{V}' &= \mathbf{D}^T \mathbf{V} \\ \mathbf{I} &= \mathbf{D}\mathbf{I}' \end{aligned} \tag{4}$$

where

$$\begin{aligned} \mathbf{V} &= [\ v_k\ ]_{m \times 1} \\ \mathbf{V}' &= [\ v_k'\ ]_{n \times 1} \\ \mathbf{I} &= [\ i_k\ ]_{m \times 1} \\ \mathbf{I}' &= [\ i_k'\ ]_{n \times 1} \\ \mathbf{D} &= [\ d_{jk}\ ]_{m \times n} \quad . \end{aligned} \tag{5}$$

It can be claimed that the switch sets consist of the time-varying transformers. How to change the time-varying feature of the transformer into the time-invariant one is the important problem to be solved.



(a) general switching system

(b) general time-varying transformer

Fig. 7 General switches as transformers.

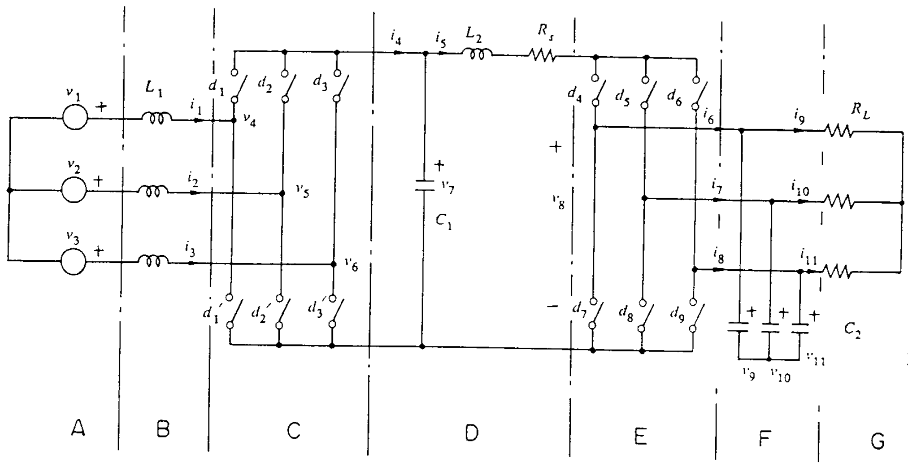
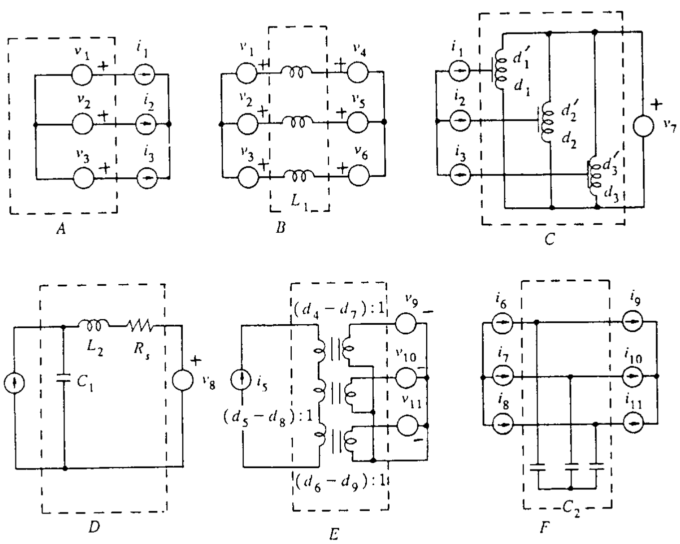


Fig. 8 3-phase rectifier-inverter circuit.



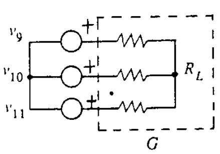
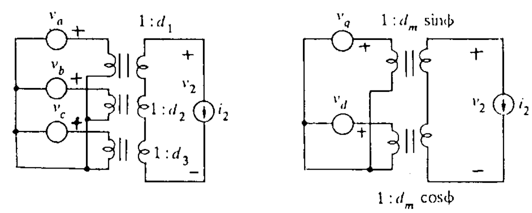


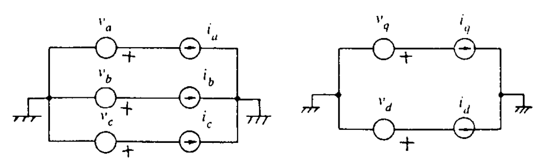
Fig. 9 Partitioned circuits.



(a)original circuit

(b) D-Q transformed circuit

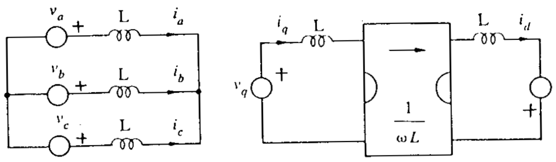
Fig. 10 D-Q transformation of transformer set.



(a) original circuit

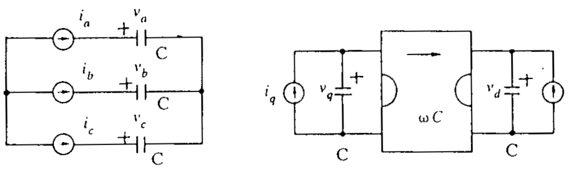
(b) D-Q transformed circuit

Fig. 11 D-Q transformation of voltage (or current) source set.



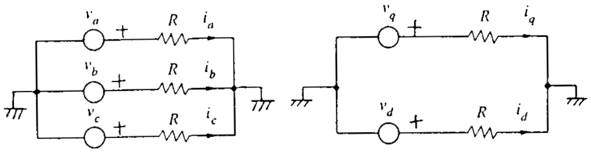
(a) original circuit (b) D-Q transformed circuit

Fig. 12 D-Q transformation of inductor set.



(a) original circuit (b) D-Q transformed circuit

Fig. 13 D-Q transformation of capacitor set.



(a) original circuit (b) D-Q transformed circuit

Fig. 14 D-Q transformation of resistor set.

III. CIRCUIT D-Q TRANSFORMATION
(3-Phase Rectifier-inverter Example)

The modeling procedure is shown for a selected converter as shown in Fig. 8. The analysis procedure will be shown in the succeeding session for a simpler case for illustration purpose.

A. Circuit Partitioning

The first step is to divide the original circuit into several sub-circuits for convenience. Then the independent partitioned circuits are obtained regarding the adjacent voltage or current as the external sources as shown in Fig. 9

B. Sub-circuit D-Q Transformation

There are five kinds of subcircuits. Now the equivalent D-Q transformed circuits are obtained respectively assuming all elements are balanced.

1) Time-varying Transformer Set: The D-Q transformed circuit is shown in Fig. 10. It is assumed that all harmonics are negligible.

< Proof >

$$\begin{aligned} v_2 &= \mathbf{D}^T v_{abc} = \mathbf{D}^T \mathbf{K}^{-1} v_{qdo} \\ i_{abc} &= \mathbf{D} i_2 = \mathbf{K}^{-1} i_{qdo} \end{aligned} \tag{6}$$

where

$$\mathbf{D} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \sqrt{\frac{2}{3}}\, d_m \begin{bmatrix} \sin(\omega t + \phi) \\ \sin(\omega t - \frac{2}{3}\pi + \phi) \\ \sin(\omega t + \frac{2}{3}\pi + \phi) \end{bmatrix},$$

$$v_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \quad v_{qdo} = \begin{bmatrix} v_q \\ v_d \\ v_o \end{bmatrix}, \quad i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\mathbf{K} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega t) & \cos(\omega t - \frac{2}{3}\pi) & \cos(\omega t + \frac{2}{3}\pi) \\ \sin(\omega t) & \sin(\omega t - \frac{2}{3}\pi) & \sin(\omega t + \frac{2}{3}\pi) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\mathbf{K}^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 1 \\ \cos(\omega t - \frac{2}{3}\pi) & \sin(\omega t - \frac{2}{3}\pi) & 1 \\ \cos(\omega t + \frac{2}{3}\pi) & \sin(\omega t + \frac{2}{3}\pi) & 1 \end{bmatrix}. \tag{7}$$

Applying (7) to (6), it is found that

$$\begin{aligned} v_2 &= d_m (v_q \sin\phi + v_d \cos\phi) \\ i_q &= d_m \sin\phi\, i_2 \\ i_d &= d_m \cos\phi\, i_2 \end{aligned} \tag{8}$$

The circuit reconstruction of (8) is Fig. 10(b). Note that the time-varying nature of the transformer is removed by the D-Q transformation.

2) 3-phase Voltage (or Current) Source: The D-Q transformed circuit is shown in Fig. 11. The elimination of 0-axis term is proved.

< Proof >

$$\begin{aligned} v_{qdo} &= \mathbf{K} v_{abc} = V_m \begin{bmatrix} \sin\phi \\ \cos\phi \\ 0 \end{bmatrix} \\ i_{qdo} &= \mathbf{K} i_{abc} = I_m \begin{bmatrix} \sin\phi' \\ \cos\phi' \\ 0 \end{bmatrix} \end{aligned} \tag{9}$$

where

$$v_{abc} = \sqrt{\frac{2}{3}}\, V_m \begin{bmatrix} \sin(\omega t + \phi) \\ \sin(\omega t - \frac{2}{3}\pi + \phi) \\ \sin(\omega t + \frac{2}{3}\pi + \phi) \end{bmatrix},$$

$$i_{abc} = \sqrt{\frac{2}{3}}\, I_m \begin{bmatrix} \sin(\omega t + \phi') \\ \sin(\omega t - \frac{2}{3}\pi + \phi') \\ \sin(\omega t + \frac{2}{3}\pi + \phi') \end{bmatrix} \tag{10}$$

3) 3-phase Inductor Set: The D-Q transformed circuit is shown in Fig. 12.

< Proof >

$$L\, \dot{i}_{abc} = v_{abc} \tag{11}$$

Then

$$L\left[(\dot{\mathbf{K}}^{-1})\, i_{qdo} + \mathbf{K}^{-1} \dot{i}_{qdo}\right] = v_{abc} \tag{12}$$

or

$$L\, \dot{i}_{qdo} = -L\, \mathbf{K} (\dot{\mathbf{K}}^{-1})\, i_{qdo} + \mathbf{K} v_{abc}$$

$$= -L\omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} i_{qdo} + v_{qdo}. \tag{13}$$

This means

$$L\dot{i}_q = -\omega L\, i_d + v_q$$
$$L\dot{i}_d = \omega L\, i_q + v_d$$
$$L\dot{i}_o = v_o = 0 \text{ ; balanced condition .} \tag{14}$$

The circuit reconstruction of (14) is Fig. 12(b).

4) 3-phase Capacitor Set: The D-Q transformed circuit is shown in Fig. 13.

< Proof >

$$C\,\dot{\mathbf{v}}_{abc} = \mathbf{i}_{abc} \tag{15}$$

Then

$$C\left[(\dot{\mathbf{K}}^{-1})\,\mathbf{v}_{qdo} + \mathbf{K}^{-1}\dot{\mathbf{v}}_{qdo}\right] = \mathbf{i}_{abc} \tag{16}$$

or

$$C\,\dot{\mathbf{v}}_{qdo} = -C\,\mathbf{K}(\dot{\mathbf{K}}^{-1})\,\mathbf{v}_{qdo} + \mathbf{K}\,\mathbf{i}_{abc}$$
$$= -C\omega\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{v}_{qdo} + \mathbf{i}_{qdo} \; . \tag{17}$$

$$C\dot{v}_o = i_o = 0 \text{ ; balanced condition .} \tag{18}$$

The circuit reconstruction of (18) is Fig. 13(b).

5) 3-phase Resistor Set: The D-Q transformed circuit is shown in Fig. 14.

< Proof >

$$\mathbf{v}_{qdo} = \mathbf{K}\,\mathbf{v}_{abc} = \mathbf{K}R\,\mathbf{i}_{abc} = R\,\mathbf{i}_{qdo} \tag{19}$$

C. Circuit Reconstruction

The D-Q transformed subcircuits of Fig. 8 is shown in Fig. 15. The reconstructions are done by just connecting the adjacent circuits as shown in Fig. 16. Note that DC side subcircuit D is unchanged.

D. Circuit Perturbation

The transformer turn-ratio and the gyrator frequency are the control signal in the power electronics. To determine the AC transfer function, it is necessary to obtain a perturbed circuit. Those who want to know the circuit perturbation in detail see the reference [2].

1) Transformer Perturbation: It is neglected the bilinear term in Fig. 17(d).

2) Gyrator Perturbation: There are two kinds of gyrators as shown in Fig. 18 and Fig. 19. By applying these perturbed transformer and gyrator to the D-Q transformed circuit of Fig. 16, the AC transfer function can be calculated.

E. DC Analysis

It's possible to determine the DC operating point from Fig. 20 by shorting inductors and opening capacitors.

IV. ANALYSIS EXAMPLE

To illustrate the analysis procedure, the buck boost inverter as shown in Fig. 21, which was analyzed in [3], is analyzed here again to compare the two approaches.

The graphically D-Q transformed circuit is briefly obtained as shown in Fig. 22. The perturbed circuit is Fig. 23.

The transfer function of $G_d(s)$ is obtained for example ; that is

$$G_d(s) = \frac{\hat{v}_d(s)}{\hat{d}_m(s)} = \frac{G_1(s)}{G_2(s)} \tag{20}$$

where

$$G_1(s) = \frac{1}{D_m^{\,2}}(1 + sCR_L)\,[\,sL\,I_L - V_d D_m\,]$$

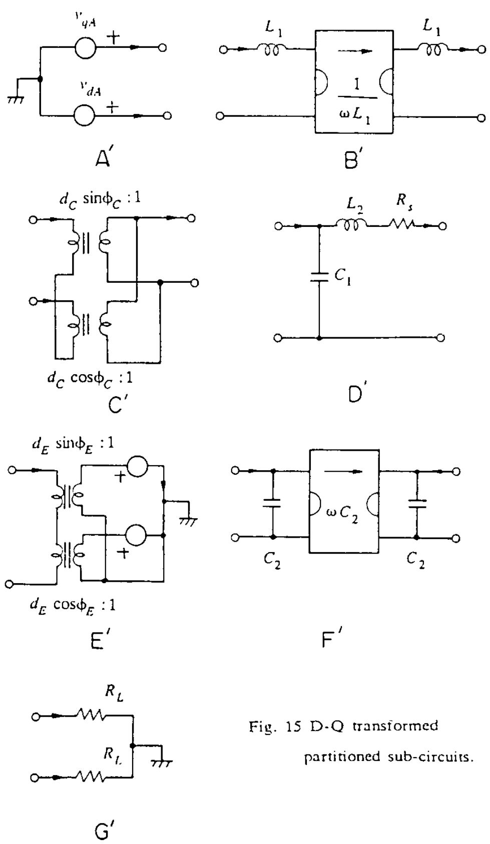


Fig. 15 D-Q transformed partitioned sub-circuits.

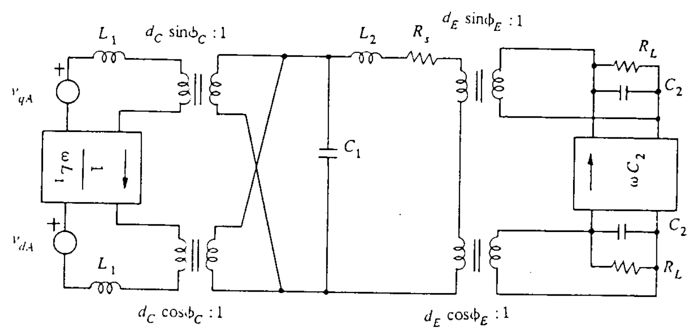


Fig. 16 Reconstruction of the partitioned circuits.

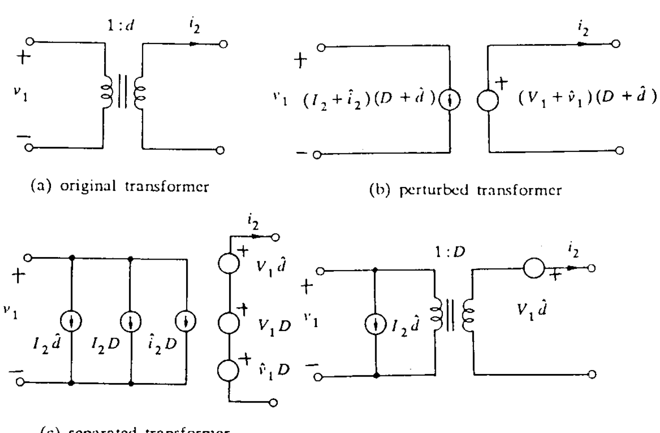
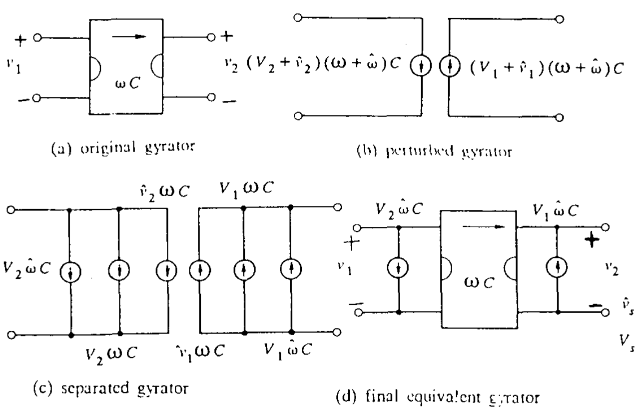


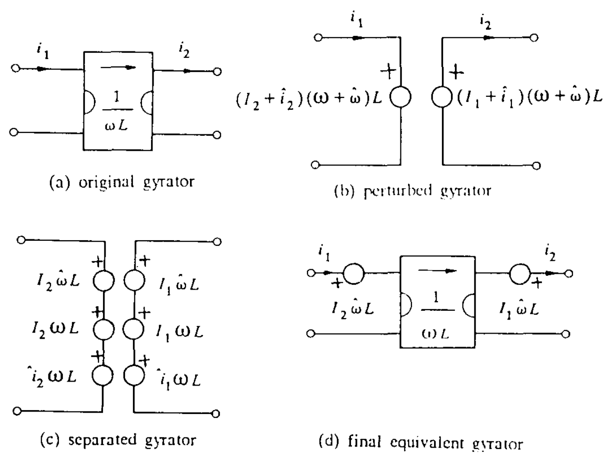
Fig. 17 Perturbed transformer.



(a) original gyrator (b) perturbed gyrator

(c) separated gyrator (d) final equivalent gyrator

Fig. 18 Perturbed gyrator (capacitor).



(a) original gyrator (b) perturbed gyrator

(c) separated gyrator (d) final equivalent gyrator

Fig. 19 Perturbed gyrator (inductor)

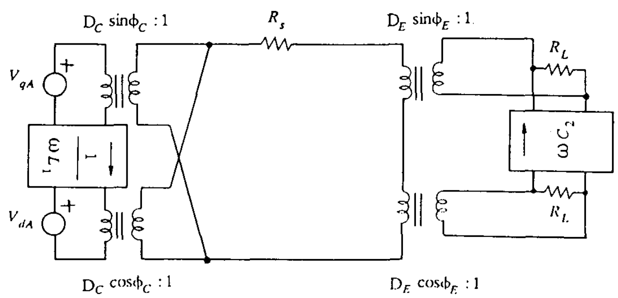


Fig. 20 DC circuit of rectifier-inverter.

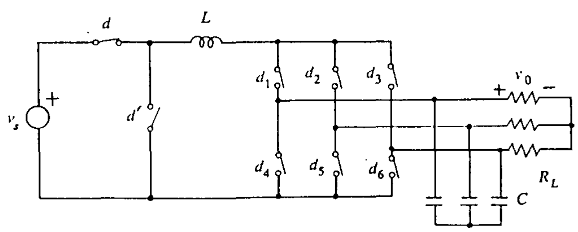


Fig. 21 A buck-boost inverter.

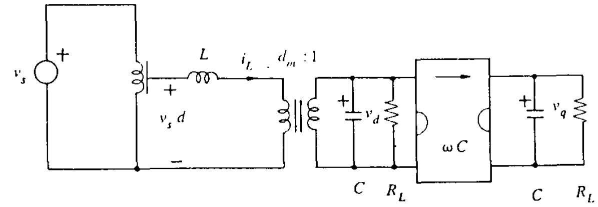


Fig. 22 D-Q transformed buck-boost inverter.

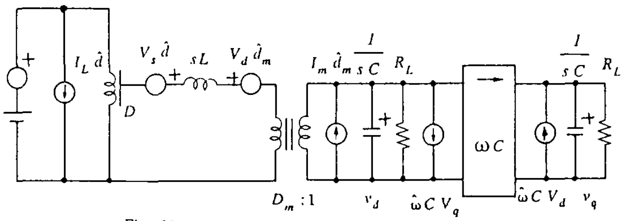


Fig. 23 Perturbed buck-boost inverter

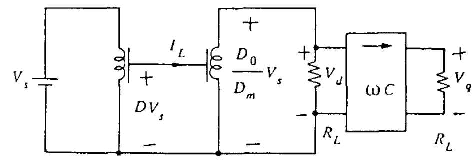


Fig. 24 DC circuit of buck-boost inverter.

$$G_2(s) = 1 + (R_L C + \frac{\omega^2 L C^2 R_L}{D_m^{\ 2}} + \frac{L}{D_m^2 R_L})\, s + \frac{2LC}{D_m^{\ 2}} s^2 + \frac{LC^2 R_L}{D_m^{\ 2}} s^3 \ . \tag{21}$$

Comparing this result with that in [3], it is found that the characteristic equation is the same each other. The DC circuit is obtained as depicted in Fig. 24. Then the DC voltage gain becomes

$$G_v = \frac{V_o}{V_s} = \frac{\sqrt{V_d^2 + V_q^2}}{V_s} = \frac{D_2}{D_m} \cos\phi \sqrt{1 + (\omega C R_L)^2} \tag{22}$$

This result is also the same as that in [3]. The modeling procedure is, however, considerably simplified by this new approach. The physical insight or equivalent circuit are easily obtained by this modeling.

V. CONCLUSION

The graphical D-Q transformation technique is identified to be general, simple and useful in practice. The switches in the power electronics are found to be the time-varying transformers and the time-varying feature is removed by the D-Q transformation. And it is found that the gyrators are introduced when D-Q transformation is taken to the reactors at AC side. Somewhat curious operation of AC converters is due to the gyrators.

In general, the switching systems are equivalent to RLC filters with transformers and gyrators.

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