Volatility Swaps: Realized or Implied Volatility?

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Abstract

Volatility is a crucial variable in the trading of options. Essentially, markets for options are markets for volatilities. In practice, a trader is faced with several difficult problems when trading volatility; the trading costs and difficulty of managing a dynamically hedged position. The various volatility swaps would be excellent tools for hedging and trading the vega risk without these problems. Especially, if both volatility rates are floating, the volatility basis swap provides a wider opportunity for trading volatility. We discuss the applications of various volatility swaps in this paper and derive a simple closed-form solution for the volatility swap.

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1 Introduction

Brenner and Galai (1989) suggest the creation of exchange-traded futures and options on a volatility index. Whaley (1993) discusses how volatility derivatives can be used to hedge the volatility risk of options. In 1993, the Chicago Board Options Exchange introduced the CBOE Market Volatility Index (VIX). The VIX is calculated by averaging the implied volatilities of S&P 100 Stock Index at-the-money puts and calls. The VDax index was introduced on December 5, 1994 as a general indicator of Dax options, the implied volatility. The volatility swap agreed upon in July 1996 between Foreign & Colonial High Income Fund and NatWest Markets was based on the implied volatility in the September contract of the S&P 500 futures contract. Gross, Mezrich and Fairman (1998) described volatility swap in which Salomon Smith Barney entered into an over-the-counter agreement with an investor. At maturity, the value of the swap depends only on the realized volatility of the asset. The DTB's new Volax future on the VDax index has launched in January 1998.

Volatility is a crucial variable in the trading of options. The volatility of an underlying asset over the option's life determines the value of the option and the implied volatility reflects the price of the option. Essentially, markets for options are markets for volatilities. Index option traders have an interest in the direction of market. However, unlike in underlying markets, they are extremely sensitive to the volatility. They measure how much expensive or cheap options are in terms of an implied volatility. Judging that market conditions provide good trading opportunities, the volatility trader typically establishes position by using puts and calls in combination, and a hedge by taking an opposing position in the underlying asset market. If implied volatility ever reached his target, the trader would offset his all positions in options and underlying assets, thereby realize his expected profit without having to hold the position for time to expiration.

Here, a trader is faced with several difficult problems. First, he must know exactly the future volatility of the underlying asset. We actually could not know the future volatility over the option's life, just estimate. Second, unexpected changes in volatility over the option's life alter the value of the option. This is the vega risk on an option value. Third, unexpected movements in implied volatility result in changing trading option prices. We may call this the vega risk on an option price. Fourth, in practice, perfect replication is impossible due to transaction costs, market incompleteness, and liquidity constraint. Also, continuous hedging is impossible. Gross, Mezrich and Fairman (1998) report that there are three transaction costs incurred with each trade in a dynamically hedged strategy; a bid-offer spread, a pay commission and a clearing cost. They also indicate that the actual profit or loss of a delta-hedge position depends not only on the average volatility, but on the path of the asset.

The volatility swaps would be excellent tools for hedging and trading the vega risk without the trading costs and the difficulty of managing a dynamically hedged position. Especially, if both volatility rates are floating, the volatility basis swap provides more widened opportunity for trading volatility. We discuss the applications of various volatility swaps in this paper and derive a simple closed form solution for the value of a plain volatility swap. We organize the paper as follows. Section 2 presents the structure of volatility swap. In Section 3, we discuss the uses of a realized volatility swap, a implied volatility swap, and the other volatility swaps. Section 4 presents a simple closed-form solution to the value of the volatility swap. Section 5 summarizes the discussions.

2 Volatility Swap

The volatility swap is an agreement between two parties in which each undertakes to make periodic payments based on the volatility rates to the other in the future. The party entering into such a swap will typically agree to receive payment on a specific notional amount, for a specific term, at a floating rate determined by a particular "volatility index". The volatility index can be realized volatility or implied volatility, as in the VIX index. In exchange, it will make fixed payments on the same notional amount, for the same term, at a fixed rate. This structure is very similar to a plain vanilla interest rate swap except for the floating rate. We may make a variation of a plain vanilla volatility swap. With similar in concept to a basis swap, this may have two floating volatility rates rather than one. That is, the party entering into the swap pays the floating rate on one index, say the VIX index, and receives the floating rate on another volatility index, say the realized volatility or any other volatility index. There will ordinarily be a spread above the alternate volatility index when the base volatility index is a discount volatility index and a spread subtracted from the alternate volatility index when the base volatility index is a premium volatility index. The spread may be determined by the long-term volatility rates of two indexes and the correlations between indexes and interest rates. In the case of conventional swaps, the period of payments is usually quarterly, semiannually, or annuall. However an appropriate period for volatility swap may be monthly, bimonthly, or quarterly. This is because the time to maturity of options traded in the CBOE is usullly less than 60 days. The other important reason for this is that longer-term futures contracts may not be effective instruments for hedging volatility risk because of mean-reverting property of volatility.[1]

The key function of a swap is the transformation. A plain vanilla interest swap can have the effect of transforming a fixed-rate loan into a floating-rate loan, or vice versa. Another popular type of swap is known as a currency swap, which involves exchanging a principal and two legs of fixed or floating payments denominated in a different currency. This swap can be used to transform a loan in one currency into a loan in another currency. A differential swap is an agreement to exchange periodic interest payments associated with two different currencies. Yet the actual interest payments are denominated in a single currency. Differential swaps can be used utilized to trade the shape of the yield curve between two currencies without incurring currency exposure. A properly structured equity swap can be used to convert a volatile equity return into a stable fixed or floating income return. Volatility swaps make it possible for index option traders to change the volatility risk exposure of option portfolios. These swaps also provide a "bridge" between different option markets or underlying matkets for volatility trading.

[1] Grünbichler and Longstaff (1996) provide an excellent discussion about this property.

3 Using the Volatility Swaps

If the floating rate of the volatility swap is determined by the realized volatility of an underlying asset, the realized volatility swap may be one of the most effective derivatives available to the index option trader for hedging the volatility risk exposure of the option value. A plain vanilla volatility swap is essentially a series of volatility forward contracts. Thus, the volatility swap as in a volatility futures or forward contract provides a financial instrument for hedging against volatility risk. A plain realized volatility swap is illustrated in Figure 1. These realized volatility swaps may be used to maintain the option values at a fixed volatility level. To provide an example of a plain realized volatility swap, suppose that options are available on a certain asset with the following conditions;

Underlying asset price:	\$320
Interest rate:	8%
Time to expiration:	two monthes.

An index option trader finds that at-the-money call and put options are being offered at \$9.16 and \$4.92, respectively (i.e. the implied volatility rate is 13.2%). The trader expects a volatility of 15% over the next two months, and decides to take a long straddle position.[2] To make the profit, the trader would establish a hedge by taking an opposing position in the underlying market. If he is correct, this trade will promise a profit of \$1.82; otherwise, be less profitable. To hedge this vega risk, he simultaneously enters into a realized volatility swap agreement under which it agrees to receive a fixed volatility level (i.e. 15%) and to pay

[2] The theoritical value of a straddle is \$15.9 and the vega is 1.011.

realized volatility level.[3] Suppose that two months later the realized volatility is 14.3% (or 17%). The results of the trade are as follows:

Net swap exchange:	$1.011 \times (15\text{-}14.3\ (17)) =$ \$0.7077 (-2.022)
Buying cost of a straddle:	\$14.08
Profit of hedge position:	\$15.19 (17.92)
Total profit:	\$1.8177 (1.818).

As a realized volatility swap enables a volatility trader to maintain option values at a fixed volatility level, an implied volatility swap enables an asset portfolio manager to maintain option buying cost at a fixed implied volatility level. Figure 2 shows the structure of a plain implied volatility swap. Consider a portfolio manager who wants to eliminate an exposure to movements in asset price. Buying call or put options enable the hedger to insure himself against unexpected movements in the price. Of course, this insurance is achieved at a cost. In this case, he exposes himself to the risk of insurance costs because of changing implied volatilities even though the other variables are constant. When the portfolio manager checks option price in the marketplace, he finds that the implied volatility rate is 13.2%. Because he knows that the prices are relatively cheap, he wants to use an implied volatility swap for hedging. He would take the swap contract that pays a fixed volatility rate (i.e. 13.2%) and receives an implied volatility rate similar to the VIX index.[4] Thus, the hedger could

[3] Note that to perfectly hedge the volatility risk, the notional principal amount of the realized volatility swap must equal the vega of option value.

[4] In this case, the notional principal amount of the implied volatility swap must equal the vega of purchased option price, not it's value. In practice, we can not know the vega in advance because of changing in market conditions. Even though it is difficult to perfectly hedge the risk, the swap may be an effective hedging tool.

reduce the risk of insurance cost. Some weeks later the price of option has moved up (down) to 14.3% (11.1%). Suppose that market conditions are left to *ceteris paribus*. The effective option purchasing cost is as follows:

Net swap exchange: 0.5019×(14.3(11.1)-13.2)= \$0.552(-1.054)

Price of call option: \$9.71 (8.11)

Total cost: \$9.158(9.164).

This plain implied volatility swap could be used to trade volatility. Volatility trading is a term for derivatives-based strategies which make to profit from changes in implied volatility. Volatility traders typically use put and call options in combination, and hedge daily in the underlying assets. However, the perfect hedge is impossible even if the trader has perfect foresight on future volatility, becuase of transaction costs, market incompleteness and liquidity constraints. This means that, although the realized volatility is higher than the implied volatility, the trade may not be profitable, or at least less profitable. Still, the implied volatility swap enables a trader exploiting the opportunity to trade volatility without incurring a hedging cost. Therefore, this swap can be serve as a cheap tool with a vega of 1.0 for trading volatility.

If both volatility rates are floating, the swap provides more flexible and wide opportunities for trading volatility. One example of this volatility basis swap is illustrated in Figure 3. We first consider a volatility swap agreement in which one cash flow makes its periodic payments based upon realized volatility rates and the other based upon implied volatility rates. This swap type allows the volatility trader to execute his expectation on volatilities more completely than the plain implied volatility swap. In this case, it was the payments based implied volatility may be predetermined as it was on the last reset date before exchanged. Of the option markets of the U.S., the AMEX Major Market Index (XMI) is the most volatile index, with actual volatility readings ranging between 12% and 21%. The New York Stock Exchange (NYSE) Index is the least volatile with readings between 8% and 15%. The S&P 100 is in the middle with actual volatility readings between 10% and 18%. If the implied volatility index based on XMI options or NYSE index options is constructed like the VIX index, inter-market volatility trading will be easier. Let's call these the XMI implied volatility and the NYSE implied volatility, respectively. For instance, assuming that XMI implied volatility less spread is higher than VIX index, the trader would use the volatility basis swap to take an advantage.

Existing plain implied volatility swaps could provide an interesting application for volatility swaps that have two floating rates; this is illustrated in Figure 4. A portfolio manager to insure the cost of buying options or an index option trader to trade volatility already has entered into a plain implied volatility swap whereby it pays a fixed volatility rate and receives a VIX rate. He expects that the XMI implied volatility will rise, and so he enters into a volatility basis swap for the same dollar notional principal amount, whereby the trader agrees to pay the VIX rate and receives the XMI volatility rate less spread. The result of this transaction is an increase in the floating volatility rate receipts under the plain implied volatility swap so long as XMI volatility, adjusted for swap spread, exceeds the VIX index. Therefore, the portfolio manager could reduce the risk of insurance cost while simultaneously lowering the effective buying cost, and so the volatility trader can make more profit.

Now, let's consider the portfolio manager who has an interest in using options to insure. If he expects thw NYSE implied volatility to go down, he may utilize this opportunity to reduce the insurance cost. As shown Figure 5, he would enter the swap agreement which pays the NYSE plus a spread, and receives the VIX index. In addition to the previously described market conditions, assume that the NYSE implied volatility rate is set at 9.3%. The spread has been predetermined at 2.5%. This reduces the call option buying cost by 1.4% to implied volatility level of 12.8%. The portfolio manager could effectively convert the cost based on the VIX index into the cost based on the implied volatility calculated as the NYSE volatility plus 2.5%. If the volatility trader of the first example, who has entered the realized volatility swap, simultaneously combines his trades with this swap, he can increase profits by 1.4% to 3.2%.

Volatility basis swaps may be integrated to enhance returns from conventional volatility trading. For example, an index option trader that expected the XMI implied volatility to move up already made a volatility swap contract that pays the VIX return and receives the XMI implied volatility rate less the spread. When he checks the marketplace, the trader finds that the VIX index level is higher than the expected future volatility level. To take advantage of the situation, he would utilize a short straddle position and establish a hedge position in an underlying market. A plain realized volatility swap may be used to hedge a vega risk on option values. The VIX rate flows would be matched by the return, accruing to the trader from its short volatility in S&P 100 index option market. The overall return to the trader would be based on XMI volatility rates. Accordingly, the trader would benefit where the XMI volatility minus the spread exceeded the VIX rate over the transaction life. Figure 6 shows an example of a volatility basis swap which might be utilized by an index option trader.

4 Valuing Plain Volatility Swap

Brenner and Galai (1989) were the first to publish work on volatility derivatives, but they merely outline a heuristic valuation procedure. Whaley (1993) considered the volatility as the traded asset. In the paper, he assumed that the prices of the volatility options obey Black's (1976) future option valuation formula. Grünbichler and Longstaff (1996) presented a simple closed-form expression for volatility futures and option prices under given a risk-neutral volatility process. Up-to-date, Kim et al (1998) evaluated the derivatives on volatility in a general equilibrium framework. Werner and Roth (1998) discussed the valuation of volatility futures using a replication approach. One limitation of replication approach, however, is that the volatility index is actually a statistical artifact constructed from the implied volatilities of the eight near-the-money, nearby, and second nearby OEX option series.[5] Another limitation is that we generally can not know the forward volatility of an option.[6]

[5] The nearby OEX series is defined as the series with the shortest time expiration but with at least eight calendar days to expiration.

[6] Werner and Roth (1998) suggested that the forward volatility, σ_f, of an option, starting on t_s and expiring at t_l, can be written as:

$$\sigma_f = \sqrt{\frac{t_l \sigma_l^2 - t_s \sigma_s^2}{t_l - t_s}}$$

where σ_s is the implied at-the-money (ATM) volatility of the next quarterly option, t_s is the time to expiration, σ_s is the implied ATM volatility of the second quarterly option, and t_l is the time to expiration. For satisfying this condition, the volatility process of the underlying asset must be uncorrelated with the price process of the underlying asset. Hull and White (1987) discuss this property.

In this section, we examine the value of a volatility swap when there exist two uncertainties: the stochastic volatility and the riskless interest rate. Bailey and Stultz (1989) and Kim et al (1998) pointed out that the volatility of a market is negatively correlated with the riskless interest rate in a general equilibrium framework. We assume that the volatility and the interest rate follow mean-reverting Ornstein-Uhlenbeck processes, respectively. While a disadvantage of this process is that the volatility or the interest rate can become negative, this probabiity can be arbitrarily small. The Ornstein-Uhlenbeck process has been used by Vasicek (1977), Courtadon (1982), Scott (1987), Ravinovitch (1989), Chen (1992), and Wei (1994). We derive a simple closed form solution for the value of a plain volatility swap. Our framework is similar to those in Grünbichler and Longstaff (1996) and in Wei (1994).

Suppose an index option trader enters a volatility swap to hedge the vega risk of an options portfolio. He would receive periodic payments based on floating volatility rates and make payments based on fixed volatility rates. To find the value of the volatility swap, the trader has to discount back the payments from the two cash flows. The net difference of the present values of the two cash flows is the swap's value to the trader. Let $t_0 = 0 < t_1 < \cdots < t_n = T$ be $n+1$ dates. The trader enters the volatility swap on the trade date $t_0 = 0$. There are n cash flows to be paid periodically at times $t_1, \cdots, t_n$. The contract terminates after the final payment $t_n = T$.

We assume that the stochastic volatility is given by;

$$dV = \alpha\,(V^* - V)\,dt + \sigma_V dZ_1, \tag{1}$$

where V is the instantaneous volatility, α, V^* and σ_V are positive constraints, and Z_1 is a standard Wiener process. The stochastic volatility is expected to drift toward the long-run average level, V^*, with a speed of adjustment α and a standard deviation σ_V. The stochastic process of the riskless interest rate is given by:

$$dr = \beta\left(r^* - r\right)dt + \sigma_r dZ_2, \tag{2}$$

where β, r^* and σ_r are positive constants and Z_2 is a standard Wiener process whose instantaneous correlation with Z_1 is ρ. We further assume that the market prices of the volatility risk and the interest rate risk, λ_V and λ_r, are constants. Then, the risk-neutral processes of Equations (1) and (2) could be written as, respectively,

$$dV = \kappa\left(\bar{V} - V\right)dt + \sigma_V dZ_1 \tag{3}$$

and

$$dr = \beta\left(\bar{r} - r\right)dt + \sigma_r dZ_2, \tag{4}$$

where $\kappa = \alpha + \lambda_V$, $\bar{V} = \frac{\alpha V^*}{\alpha + \lambda_V}$, and $\bar{r} = r^* + \frac{\lambda_r \sigma_r}{\beta}$. Note that these assumptions are similar to the implications of the general equilibrium framework of Cox, Ingersoll and Ross (1985).

It is straightforward to discount the cash flow on the fixed rate leg. If F is the fixed rate of volatility (usually called the swap coupon), the present value of the fixed leg for \$1 of principal is

$$PV_F = F\sum_{i=1}^{n} P\left(r_{t_0}, t_i\right), \tag{5}$$

where $P\left(r_0, t_i\right)$ denotes the price at time t_0 of a default-free discount bond yielding \$1 at time t_i. It is well known that $P\left(r_0, t_i\right) = \hat{E}_0\left[\exp\left(-R(t_0, t_i)\right)\right]$ where $\hat{E}_0$ is the conditional expectation on time t_0 in the risk-neutral world and $R_{0,i} = \int_{t_0}^{t_i} r(t)dt$ is normally distributed. Thus, we have

$$P(t_0, t_i) = \exp\left(-\mu_{R_{0,i}} + \frac{1}{2}\sigma^2_{R_{0,i}}\right),$$

where

$$\mu_{R_{0,i}} = r_0 \frac{1 - e^{-\beta(t_i - t_0)}}{\beta} + \bar{r}\left(t_i - t_0 - \frac{1 - e^{-\beta(t_i - t_0)}}{\beta}\right)$$

and

$$\sigma^2_{R_{0,i}} = \frac{\sigma_r^2}{2\beta^3}\left(2\beta(t_i - t_0) - 3 + 4e^{-\beta(t_i - t_0)} - e^{-2\beta(t_i - t_0)}\right).$$

Consider next the floating rate of volatility based some index. If PV_V^i is the present value,[7]

$$PV_V^i = P(r_0, t_i)\left(\mu_{V_{0,i}} - \sigma_{V_{0,i}, R_{0,i}}\right), \tag{6}$$

where

$$\mu_{V_{0,i}} = e^{-\kappa(t_i - t_0)} V_0 + \bar{V}\left(1 - e^{-\kappa(t_i - t_0)}\right)$$

and

$$\sigma_{V_{0,i}, R_{0,i}} = \frac{\sigma_V \sigma_r \rho}{\kappa + \beta}\left(\frac{1 - e^{-\kappa(t_i - t_0)}}{\kappa} - \frac{e^{-k(t_i - t_0)} - e^{-(\kappa + \beta)(t_i - t_0)}}{\beta}\right).$$

Thus, the present value of the floating leg is

$$PV_V = \sum_{i=1}^{n} PV_V^i. \tag{7}$$

The volatility swap agreement has an initial value of the difference between Equation (5) and (7). To determine the swap coupon rate, the swap is required to have zero initial value.

[7] The present value of the cash flow at time t_i is $PV_V^i = \hat{E}_0\left[\exp(-R_{0,i}) V_{t_i}\right]$. $R_{0,i}$ and $V_{0,i}$ have a bivariate normal distribution.

We conclude that the swap coupon rate should be:

$$F = \frac{\sum_{i=1}^{n} PV_V^i}{\sum_{i=1}^{n} P(r_0, t_i)}. \tag{8}$$

In Figure 7, swap coupon rates are plotted for varying correlation coefficient between the volatility index and the interest rate. The parameter values are as follows:

$$\begin{aligned}
&V_0 = 0.15, \quad r_0 = 0.05 \\
&\bar{V} = 0.15, \quad \bar{r} = 0.08 \\
&\kappa = 4, \qquad \beta = 0.2 \\
&\sigma_V = 0.4, \quad \sigma_r = 0.02
\end{aligned}$$

the period of payments : quarterly

swap's life : one year.

Swap coupon F is a complex expression dependent upon the correlation between the volatility index and the interest rate, as well as other factors. The effect of the correlation coefficient on the swap coupon is found by differentiating Equation (7) with respect to ρ. This is given by;

$$\frac{\partial F}{\partial \rho} = -\frac{\sigma_V \sigma_r}{\kappa + \beta} \times \frac{\sum_{i=1}^{n} P(r_0, t_i)\, c_{0,i}}{\sum_{i=1}^{n} P(r_0, t_i)} < 0,$$

where $c_{0,i} = \frac{1-e^{-\kappa(t_i-t_0)}}{\kappa} - e^{-\kappa(t_i-t_0)}\frac{1-e^{-\beta(t_i-t_0)}}{\beta} > 0$ for $\forall i$. As the correlation between the volatility index and the interest rate increases, the volatility swap coupon linearly decreases. To understand why the correlation affects the swap coupon, we first assume that the correlation is positive. When payments based on the volatility index are high, they tend to get discounted at a high interest rate; when they are low, they tend to get discounted at a low interest rate. The overall effect is to the reduction of the present value of the volatility index cash flow relative to the situation where the correlation is zero. Similarly, when the volatility index and the interest rate are negatively correlated, the effect is the increase of the present value of the volatility index cash flow relative to the situation when the correlation is zero.

5 Conclusion

Volatility swap transactions have two major functions: to allow greater efficiency in the management of volatility exposure for option portfolios and to provide new instruments for volatility trading. A plain realized volatility swap is an effective tool for hedging the vega risk of option values. An implied volatility swap enables an asset portfolio manager to maintain option buying costs at a fixed implied volatility level. As such, this swap is an excellent tool for trading volatility. If both volatility rates are floating, the volatility basis swap provides flexible opportunities for trading or hedging the vega risk. Combinations of volatility swaps and integration with conventional volatility trading are used to seek to enhance returns. We derive a simple closed form for the value of a plain volatility swap.

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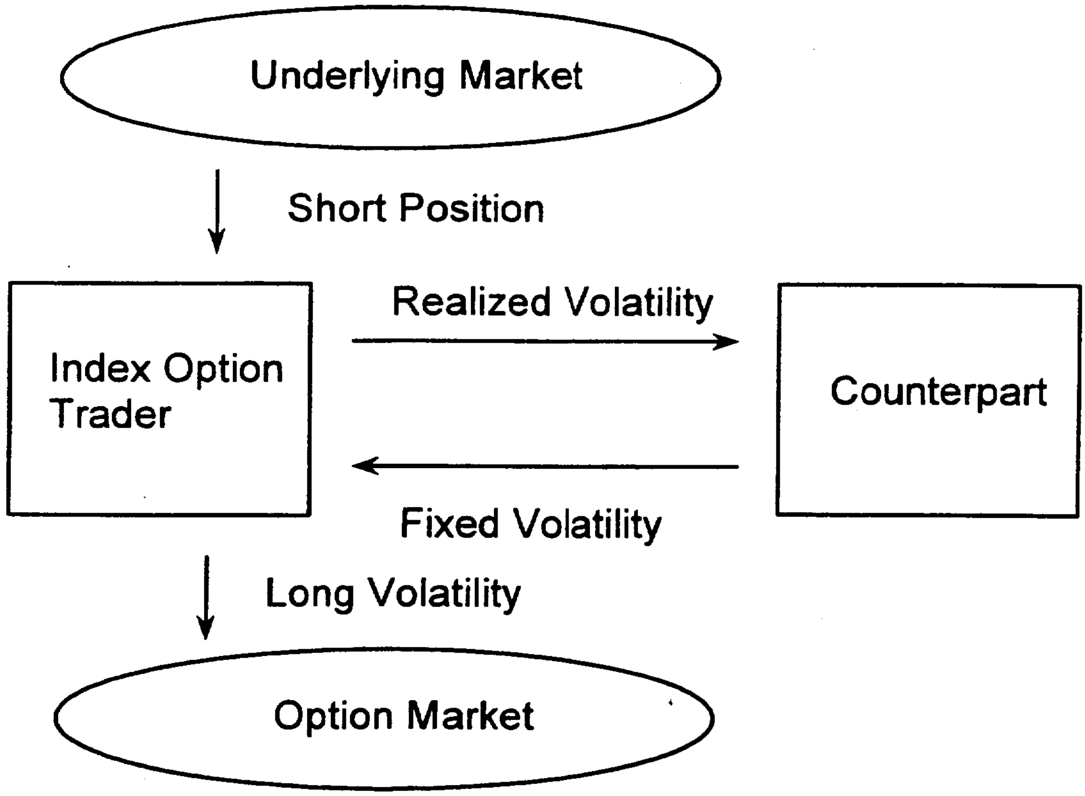

Figure 1: Hedging the Volatility Risk of Option Value

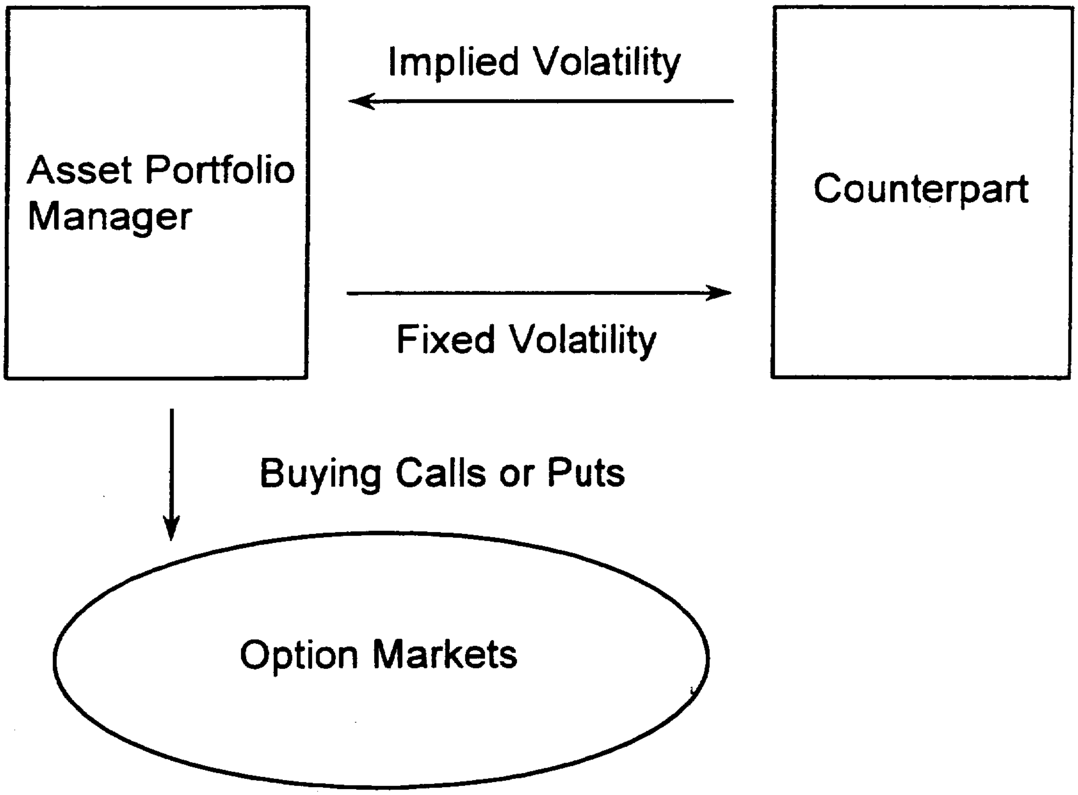

Figure 2: Hedging the Volatility Risk of Option Price

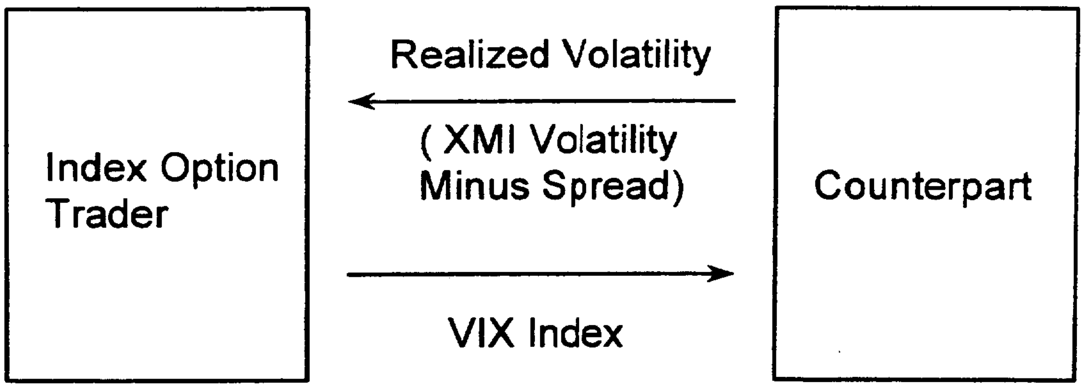

Figure 3: Basis Swap for Volatility Trading

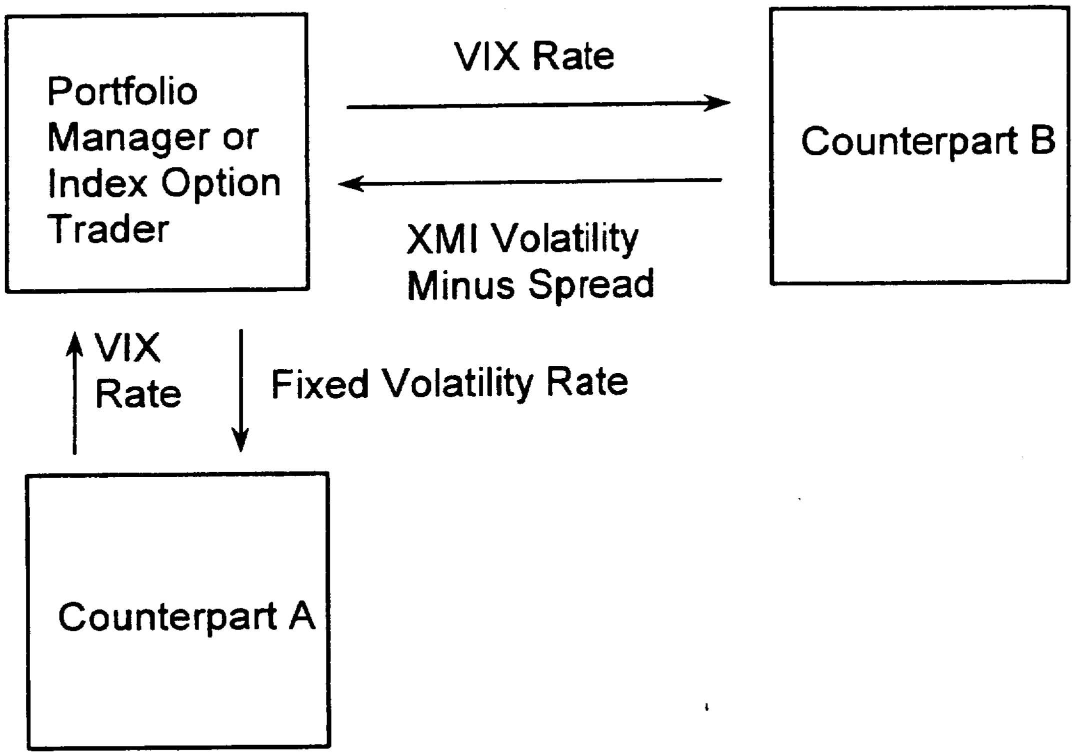

Figure 4: Combination of Volatility Swaps

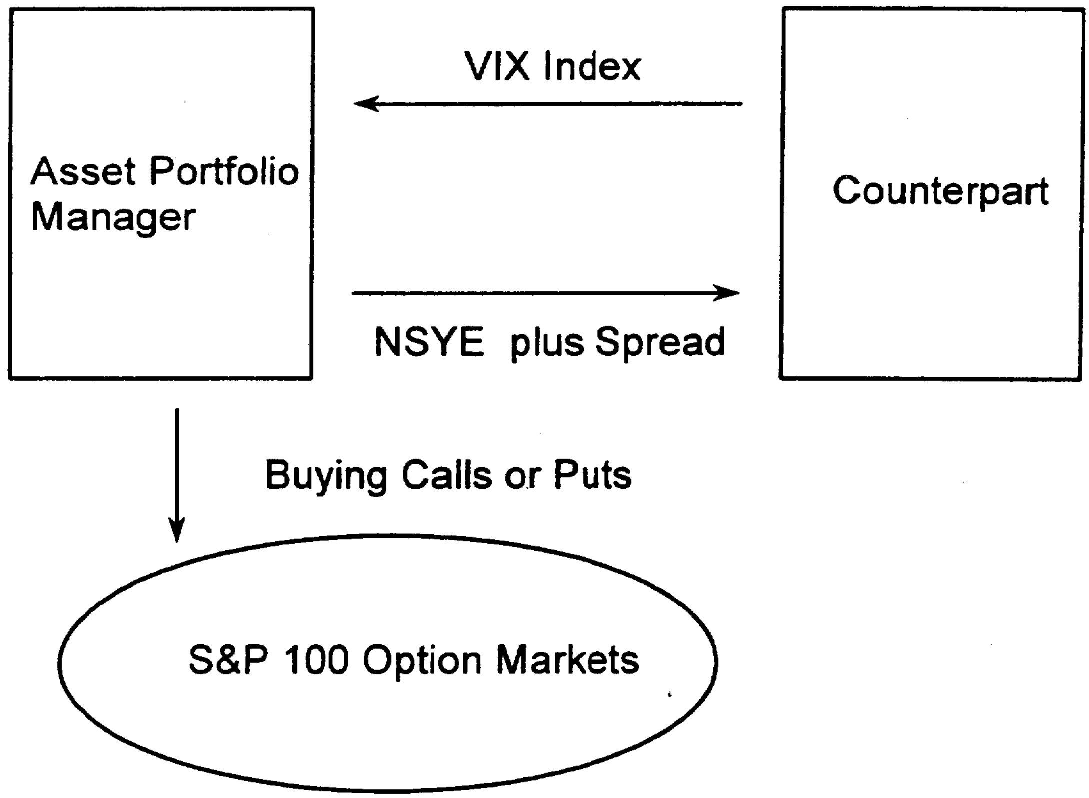

Figure 5: Reducing the Buying Cost

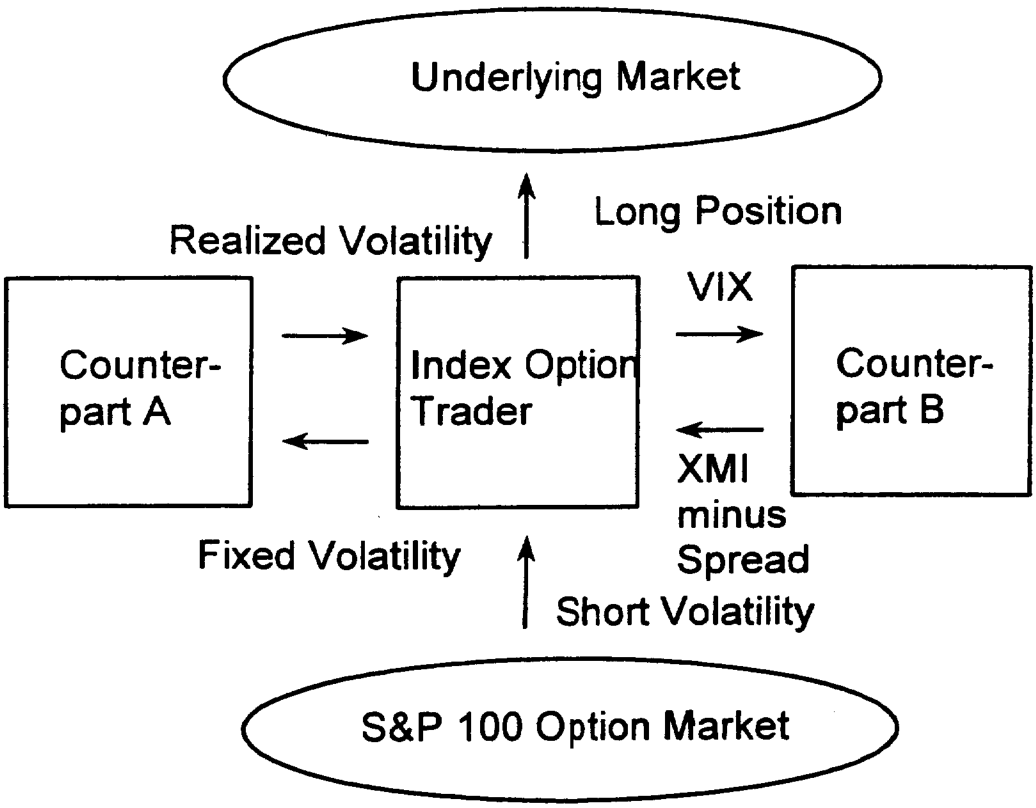

Figure 6: Integration with Volatility Trading

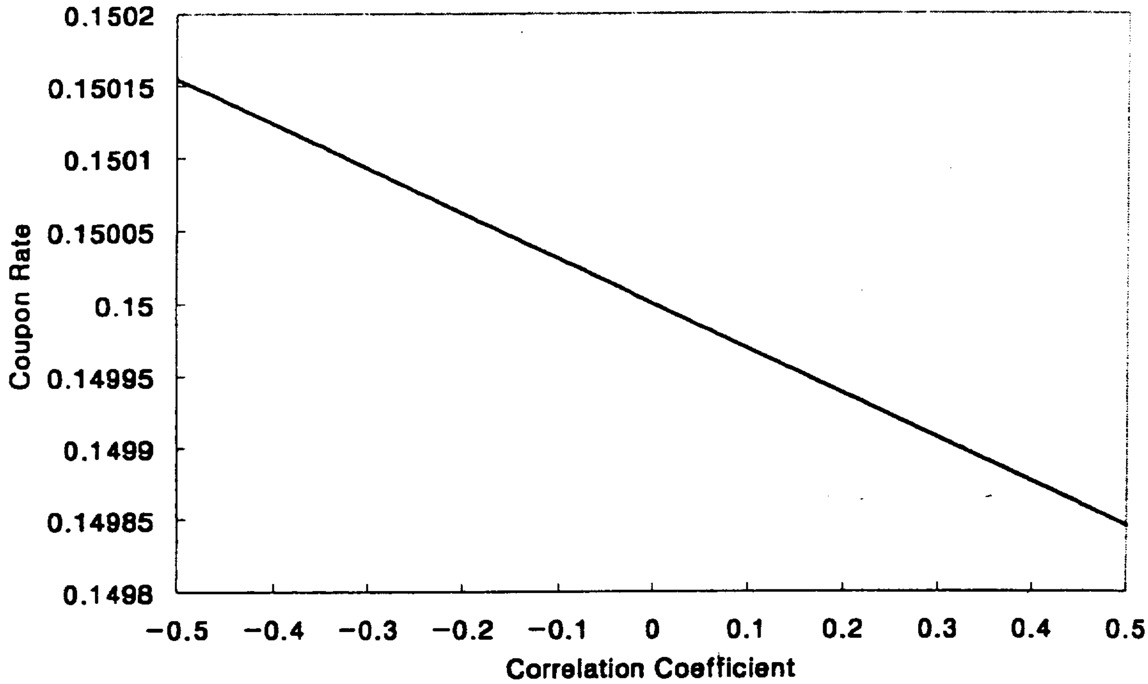

Figure 7: The Effet of Correlation Coefficient between Volatility Index and Interest rate