Proceedings of the 1999 IEEE/RSJ
International Conference on
Intelligent Robots and Systems

Optimization of the Antagonistic Stiffness Characteristic of a Five-bar Mechanism with Redundant Actuation

Sang Heon Lee*, Byung-Ju Yi**, Soo Hyun Kim*, and Yoon Keun Kwak*

* Dept. of Mechanical Engineering, KAIST,
373-1, Kusong-dong, Yusong-gu, Taejon 305-701, Korea

** Dept. of Control & Instrumentation Engineering, Hanyang University,
1271 sa-1 dong, Ansan, Kyunggi-do 425-791, Korea

Abstract

A closed-chain mechanism having redundancy in force domain can produce spring effect by proper internal load distribution. The so-called antagonistic stiffness is provided by redundant actuation in conjunction with nonlinear geometric constraints. In this work, an optimal structure of five-bar mechanism that can maximize efficiency in generation of antagonistic stiffness is evaluated and analyzed. A stiffness modulation index that represents isotropic characteristics in antagonistic stiffness generation is proposed. Gradient design index that shows rate of change of the isotropic index is also employed to distribute the isotropy of stiffness uniformly throughout the workspace. To deal with multi-criteria based design, a composite design index based on max-min principle of fuzzy theory is used as an objective function. Two optimization results are obtained. One is optimizing the X-directional stiffness and the other corresponds to optimizing the Y-directional stiffness. The result of the former design is found suitable for antagonistic stiffness generation as well as for first-order kinematic performances.

1. Introduction

Mobility of a system is defined as the number of independent variables that must be specified in order to locate its elements relative to another. When the mobility of a system is greater than the degree-of-freedom, the system is called a kinematically redundant system, On the other hand, when the number of actuation input is greater than the mobility, the system is called a redundantly actuated system. Structure of a system with redundant actuation usually has closed-chain mechanism. The redundantly actuated system can produce proper internal load by distributing the loads of the actuating input. Then, spring like effect can be produced by antagonistic internal load with conjunction with complex nonlinear geometry of the mechanism. The antagonistic stiffness can be modulated arbitrarily by controlling actuation inputs without feedback control [1]. A concept of antagonistic stiffness has been addressed by Hogan [2], Tong [3], and Yokoi [4]. Yi and Freeman[1] modeled and analyzed antagonistic stiffness by redundant actuation. Maekawa *et al.* [5] analyzed negative stiffness due to the motion of an object grasped by mutifingered hand. The stiffness also came from internal load but it could not be controlled because of inherent structure of the hand mechanism.

Besides a system that can actively vary its own natural frequency [6], application of antagonistic stiffness effect can expand to the field of impact reduction. However, real application of the antagonistic stiffness has been rare. Thus, in this work, we investigate an optimal kinematic structure of a five-bar mechanism that efficiently generates antagonistic stiffness via redundant actuation. To do this, the modeling of antagonistic stiffness is performed and a stiffness modulation index, which represents the isotropy of the transformation matrix between the actuation efforts and the antagonistic stiffness, is proposed. The transform matrix is the second derivative of the kinematic constraint [1,6]. Also in order to obtain the global isotropic characteristic, a global design index is employed which is defined as the average value of the isotropic indices calculated over the workspace. And a gradient design index is proposed to obtain uniform distribution of the isotropic index over the workspace. To cope with these design indices in optimization, composite design index based on max-min principle of fuzzy theory [7] is defined and used as objective function. The optimization is performed in two paths. One is optimizing X-directional stiffness modulation, and the other is corresponding to Y-directional stiffness. The result from optimization is suitable for generating stiffness more effectively as well as general force/velocity transmission.

2. Kinematic Modeling

2.1 Open-chain Kinematics

The five-bar mechanism shown in Fig. 1 has one closed-kinematic chain which consists of two open-chains by cutting the middle joint. It is identical with cooperating two robots handling a common object. The end-effector positions $\boldsymbol{u}$ of the two open-chains are identical, and they are described by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{pmatrix} = \begin{pmatrix} l_5 + l_4 c_5 + l_3 c_{45} \\ l_4 s_5 + l_3 s_{45} \end{pmatrix}. \quad (1)$$

And the orientation angle of link 2 is

$$\varphi = \theta_1 + \theta_2 = \theta_3 + \theta_4 + \theta_5 + \pi . \quad (2)$$

Adopting the standard Jacobian representation for the velocity of dependent output vector $\mathbf{u} \in \Re^N$ in terms of independent input coordinates ${}_r\dot{\phi} \in \Re^M$ of rth open-chain, one has

$$\dot{\boldsymbol{u}} = [{}_r\boldsymbol{G}_\phi^u]\, {}_r\dot{\phi} . \quad (3)$$

Here,

$$[{}_r\boldsymbol{G}_\phi^u] = \left[\frac{\partial \boldsymbol{u}}{\partial_r \phi} \right] \in \Re^{N\times M} \quad (4)$$

is the first-order Kinematic Influence Coefficient (KIC) relating $\dot{\boldsymbol{u}}$ to ${}_r\dot{\phi}$.

Acceleration vector $\ddot{\boldsymbol{u}}$ also can be represented in terms of input coordinates [1]

$$\ddot{\boldsymbol{u}} = [{}_r\boldsymbol{G}_\phi^u]\, {}_r\ddot{\phi} + {}_r\dot{\phi}^T [{}_r\boldsymbol{H}_{\phi\phi}^u]\, {}_r\dot{\phi} , \quad (5)$$

where the element of the second-order KIC array $[{}_r\boldsymbol{H}_{\phi\phi}^u] \in \Re^{N\times M\times M}$ is defined as

$$[{}_r\boldsymbol{H}_{\phi\phi}^u]_{ijk} = \frac{\partial^2 u_i}{\partial_r \phi_j \, \partial_r \phi_k} . \quad (6)$$

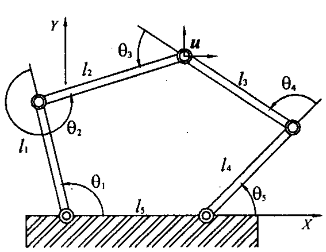


Figure 1. Five-bar mechanism

2.2 Internal Kinematics

In redundantly actuated systems, the independent coordinates can be selected arbitrary. Since the mobility of the five-bar mechanism is 2, at least two actuation inputs are required to operate the system. In this work, the minimum independent (input) coordinates and dependent coordinates are selected as

$$\phi_a = (\theta_1, \theta_5)^T \quad (7)$$

$$\phi_p = (\theta_2, \theta_3, \theta_4)^T . \quad (8)$$

From the equivalent velocity relationship of the two open-chains

$$\dot{\boldsymbol{u}} = [{}_1\boldsymbol{G}_\phi^u]\, {}_1\dot{\phi} = [{}_2\boldsymbol{G}_\phi^u]\, {}_2\dot{\phi} , \quad (9)$$

the velocity relation between independent and dependent coordinate set can be obtained by rearranging the above equation :

$$[\boldsymbol{A}]\dot{\phi}_p = [\boldsymbol{B}]\dot{\phi}_a , \quad (10)$$

where

$$[\boldsymbol{A}] = \left[[{}_1G_\phi^u]_{:2}, -[{}_2G_\phi^u]_{:1}, -[{}_2G_\phi^u]_{:2} \right], \quad (11)$$

$$[\boldsymbol{B}] = \left[-[{}_1G_\phi^u]_{:1}, [{}_2G_\phi^u]_{:3} \right]. \quad (12)$$

In Eq. (11), $[{}_rG_\phi^u]_{:i}$ is ith column vector of the matrix. The velocity vector for the dependent coordinate set is obtained by premultiplying the inverse of $[\boldsymbol{A}]$ to Eq. (10) :

$$\dot{\phi}_p = [\boldsymbol{A}]^{-1}[\boldsymbol{B}]\dot{\phi}_a = [\boldsymbol{G}_a^p]\dot{\phi}_a , \quad (13)$$

and the velocity vector of total system's joints can be expressed in terms of the velocity vector of the independent joint set by

$$\dot{\phi} = [\boldsymbol{G}_a^\phi]\dot{\phi}_a = \begin{bmatrix} [I] \\ [\boldsymbol{G}_a^p] \end{bmatrix} \dot{\phi}_a . \quad (14)$$

The second order KIC array $[\boldsymbol{H}_{aa}^p]$ relating $\ddot{\phi}_p$ to $\dot{\phi}_a$ can be easily obtained in a similar manner [1].

2.3 Forward Kinematics

Since the joints of the rth open-chain is composed of some of the independent and dependent joints, ${}_r\dot{\phi}$ can be expressed in terms of the total system's independent joints by

$${}_r\dot{\phi} = [{}^r\boldsymbol{G}_a^\phi]\dot{\phi}_a , \quad (15)$$

where $[{}^rG_a^\phi]$ is formed by rearranging rows of $[G_a^\phi]$. Thus the forward kinematics for the common object is obtained by embedding the first-order KIC into one of the rth pseudo open-chain kinematic expression as follows :

$$\dot{u} = [{}_rG_\phi^u]\,{}_r\dot{\phi} = [{}_rG_\phi^u][{}^rG_a^\phi]\dot{\phi}_a \\ = [G_a^u]\dot{\phi}_a \tag{16}$$

Using the same augmentation method employed in Eq. (16) evaluation of the second-order forward kinematic array $[H_{aa}^u]$ is also straightforward. It is given by

$$[H_{aa}^u] = [{}_rG_\phi^u]\circ[{}^rH_{aa}^\phi] \\ +[{}^rG_a^\phi]^T[{}_rH_{\phi\phi}^u][{}^rG_a^\phi] \tag{17}$$

where '$\circ$' means a generalized scalar dot product [8].

2.4 Statics of the Five-bar Mechanism

According to the duality existing between the velocity and force vector, the force relation between the independent and dependent joints can be described. So the effective load referenced to the independent joints is given by

$$T_a^* = T_a + [G_a^p]^T T_p = [G_a^\phi]^T T_\phi \tag{18}$$

where T_a, T_p, and T_ϕ are force or torque at the independent joints, the dependent joints, and the total system's joints, respectively.

In static equilibrium, the torque at the independent coordinates can be expressed as

$$T_a^* = [G_a^A]^T T_A = 0, \tag{19}$$

where T_A is the torque at the actuated joints ϕ_A. $[G_a^A]$ is the first-order KIC relating $\dot{\phi}_A$ and $\dot{\phi}_a$ obtained by rearranging the rows of $[G_a^\phi]$.

3. Modeling of Antagonistic Stiffness

Given a disturbance to the system under static equilibrium, a spring-like behavior occurs to the system. Assuming that the magnitude of T_A remains constant, the effective stiffness matrix $[K_{aa}]$ with respect to the independent coordinates is obtained by differentiating Eq. (19) with respect to the independent coordinate set ϕ_a.

$$[K_{aa}] = -\frac{\partial T_a^*}{\partial \phi_a} = (-T_A)^T \circ [H_{aa}^A] \tag{20}$$

The stiffness relationship between the output coordinates and the independent coordinates is given by [9]

$$[K_{uu}] = [G_u^a]^T[K_{aa}][G_u^a]. \tag{21}$$

where $[\mathbf{G}_u^a]$ denotes the inverse of $[\mathbf{G}_a^u]$ given in Eq. (16). Substituting Eq. (20) into Eq. (21) yields the following stiffness matrix expressed with respect to the output space.

$$[K_{uu}] = (-T_A)^T \circ [H_{uu}^A] \tag{22}$$

where $[H_{uu}^A] = [G_u^a]^T[H_{aa}^A][G_u^a]$ is obtained by plane by plane multiplication [1].

An alternative form of Eq. (22) given in a matrix form is described by

$$K_u = -[H_u^A]T_A, \tag{23}$$

where K_u is consists of the upper diagonal elements of $[K_{uu}]$ and $[H_u^A]$ is also obtained by collecting the upper diagonal columns of the three-dimensional array $[H_{uu}^A]$, which are defined as follows:

$$K_u = \left(K_{xx}, K_{xy}, K_{yy}\right)^T \tag{24}$$

$$[H_u^A] = \begin{bmatrix} [H_{uu}^A]_{;xx} \\ [H_{uu}^A]_{;xy} \\ [H_{uu}^A]_{;yy} \end{bmatrix}. \tag{25}$$

Necessary (but not sufficient) conditions for full stiffness generation are derived in [1] : A closed-chain is capable of full stiffness generation only if it satisfies

$$N_A \geq D + N \tag{26}$$

where N is DOF (system mobility), D is number of independent stiffness elements, and N_A is number of actuated joints and

$$NC = (IC - LC) \geq D \tag{27}$$

where IC is number of independent constraint equations, NC is number of independent nonlinear constraint equations, and LC is number of independent linear equations.

The five-bar mechanism has only one independent closed-chain and thus it has two independent nonlinear constraint equations, so only two elements of K_u can be independently modulated. According to the condition of Eq. (26), at least four actuators are needed to modulate two stiffness elements. Four joints except the third joint are actuated in the given five-bar.

4. Stiffness Modulation Index

4.1 Local Design Index

The static equilibrium can be also expressed as

$$T_u^* = [G_u^A]^T T_A = 0 \tag{28}$$

where

$$[G_u^A] = [G_a^A][G_u^a]$$

and T_u^* is the effective load at the end-effector coordinates. Eq. (28) represents the primary condition for antagonistic stiffness generation. For Eq. (28), the general solution for T_A is obtained as

$$T_A = \left([I] - \left([G_u^A]^T\right)^+ [G_u^A]^T\right)\varepsilon_1, \tag{29}$$

where ε_1 is an arbitrary vector and the superscript '+' means pseudo-inverse. Eq. (29) denotes the secondary subtask. Substituting Eq. (29) into Eq. (23) yields

$$K_u = -[H_u^A]\left([I] - \left([G_u^A]^T\right)^+ [G_u^A]^T\right)\varepsilon_1 \tag{30}$$

and

$$[\hat{H}]\varepsilon_1 = K_u, \tag{31}$$

where

$$[\hat{H}] = -[H_u^A]\left([I] - \left([G_u^A]^T\right)^+ [G_u^A]^T\right). \tag{32}$$

If the exact solution of ε_1 exists for Eq. (31), it implies the second subtask can be realized. The general solution of ε_1 is

$$\varepsilon_1 = [\hat{H}]^+ K_u + \left([I] - [\hat{H}]^+[\hat{H}]\right)\varepsilon_2, \tag{33}$$

where ε_2 is an arbitrary vector. Then the solution of T_A is represented as

$$\begin{aligned} T_A &= \left([I] - ([G_u^A]^T)^+ [G_u^A]^T\right)[\hat{H}]^+ K_u \\ &+ \left([I] - ([G_u^A]^T)^+ [G_u^A]^T\right)\left([I] - [\hat{H}]^+[\hat{H}]\right)\varepsilon_2 \end{aligned} \tag{34}$$

The first term of right-hand side of the above equation is reduces to $[\hat{H}]^+ K_u$ [10], by using condition of the symmetry and idempotency of $\left([I] - \left([G_u^A]^T\right)^+ [G_u^A]^T\right)$. We can realize another subtask (the third-priority subtask) by ε_2 if the system has more redundancy. Assuming that there is no other subtask ($\varepsilon_2 = 0$), the solution becomes

$$T_A = [\hat{H}]^+ K_u. \tag{35}$$

So the relationship between K_u and T_A which satisfies Eqs. (23) and (28) is

$$K_u = -[H_u^A]\left([I] - \left([G_u^A]^T\right)^+ [G_u^A]^T\right)T_A. \tag{36}$$

Now, the stiffness modulation index is defined by isotropic index of $[\hat{H}]$, which shows isotropy characteristic in the stiffness modulation process :

$$\kappa = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{37}$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are the minimum and maximum singular values of the matrix $[\hat{H}]$, respectively.

4.2 Global Design Index

The design index κ defined in Eq. (37) is a local index that varies as the pose of the five-bar mechanism changes, so it can not represent overall characteristics of the mechanism throughout the workspace. Therefore a design index which represents the global feature should be defined and used in optimization.

The global design index which represents an average value of κ over the workspace is defined as

$$\mathrm{K} = \frac{\int_W \kappa \, dW}{\int_W dW}, \tag{38}$$

where W denotes workspace.

Since the value of the local design index changes as the configuration of mechanism changes, gradient design index is also considered so that final design can have even distribution of the local design indices over the workspace. The gradient design index represents the change rate of the local design index [11]. The gradient design index is defined as the maximum of the differences among the local design indices calculated at adjacent points throughout the workspace. The smaller the global gradient design index, the more uniformly distributed the design index over the workspace, therefore the gradient design index should be minimized.

4.3 Composite Design Index

The design of a mechanism can be made based on any particular criterion. However, the single criterion-based design does not provide sufficient control on the range of the design parameters involved. Therefore, multi-criteria based design has been proposed. But various design indices are usually incommensurate concepts due to differences in unit and physical meanings, and therefore should not be combined with normalization and weighting functions unless they are transferred into a common domain. In other words, quantitative combination should be avoided. Instead, these design indices should be combined qualitatively. To consider these facts, we employ a composite design index [12].

As an initial step to this process, preferential information should be given to each design parameter and design index. Then, each design index is transferred to a common preference design domain which ranges from zero to one. Here, the preference given to each design criterion is subjective to the designer. Preference can be given to each criterion by weighting. This provides flexibility in design.

For K, the best preference is given the maximum value, and the least preference was given the minimum value of the criterion. Then the design index is transferred into common preference design domain as below

$$\widetilde{\mathrm{K}} = \frac{\mathrm{K} - \mathrm{K}_{\min}}{\mathrm{K}_{\max} - \mathrm{K}_{\min}}, \tag{39}$$

where '~' implies the index which is transferred into common preference design domain. Reversely, if the best preference is given the minimum value, and the least preference is given the maximum value of the criterion, like the gradient design index K^d, the design index transferred into common preference design domain is given by

$$\widetilde{\mathrm{K}}^d = \frac{\mathrm{K}^d_{\max} - \mathrm{K}^d}{\mathrm{K}^d_{\max} - \mathrm{K}^d_{\min}}. \tag{40}$$

Note that each transferred design index is constructed such that a large value represents a better design.

A set of optimal design parameters was obtained based on max-min principle of fuzzy theory [7]. Initially, minimum values among the design indices for all set of design parameters are obtained, and then a set of design parameters, which has the maximum value of the minimum values already obtained, is chosen as the optimal set of design parameters. Based on this principle, the composite design index(*CDI*) is given by

$$CDI = \min\left(\widetilde{\mathrm{K}}^{\alpha}, \left(\widetilde{\mathrm{K}}^{d}\right)^{\beta}\right). \tag{41}$$

The upper Greek letter is the degree of weighting, and large value implies large weighting. The weightings are all set to ones in this work.

5. Optimization

Optimization problem is formulated as follows: The objective is to evaluate optimal link lengths which maximizes *CDI*. The design variables are the ratios of link lengths to the base link length l_5, and they are defined by

$$x_i = l_i / l_5, \quad i = 1,2,3,4. \tag{42}$$

Constraints to the design variables are

$$0.5 \le x_i \le 1.5 \tag{43}$$

and

$$l_5 = 0.2m \tag{44}$$

Two objective functions are defined and used in optimization. One noted as CDI_x corresponds to the modulation of the X-directional stiffness. It contribute to enhance the isotropic characteristic for K_{xx} and K_{xy}. The other noted as CDI_y corresponds to modulation of the Y-directional stiffness. It contribute to enhance the isotropic characteristic for K_{xy} and K_{yy}.

Three numerical methods are used to deal with multivariable nonlinear optimization problem with constraints. The exterior penalty function method is employed to transform the constrained optimization problem into an unconstrained problem. Powell's method is applied to obtain an optimal solution for the unconstrained multivariable problem, and quadratic interpolation method is used for unidirectional optimization [12].

Optimization result for modulation of the X-directional stiffness is given as

$$l_1/l_5 = 0.697434392$$
$$l_2/l_5 = 1.250000000$$
$$l_3/l_5 = 1.487984717$$
$$l_4/l_5 = 0.729829688$$

and the shape of optimized mechanism (Design 1) is shown in Fig. 2.

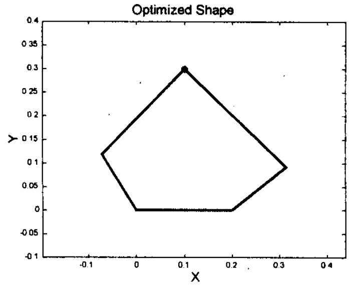


Figure. 2 Optimized shape of Design 1

Optimization result for modulation of the Y-directional stiffness is given as

$$l_1/l_5 = 0.750000100$$
$$l_2/l_5 = 0.750000050$$
$$l_3/l_5 = 0.693700249$$
$$l_4/l_5 = 1.499999810$$

and the shape of optimized mechanism (Design 2) is shown in Fig. 3

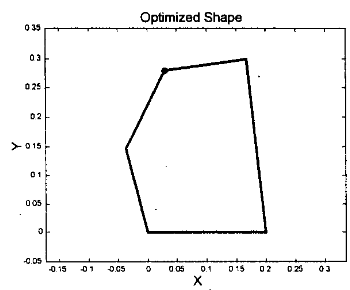
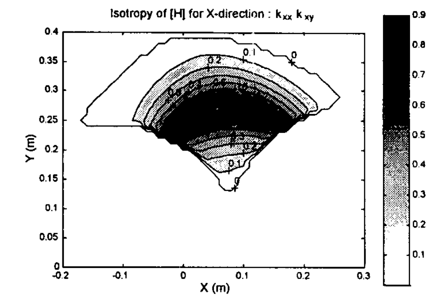


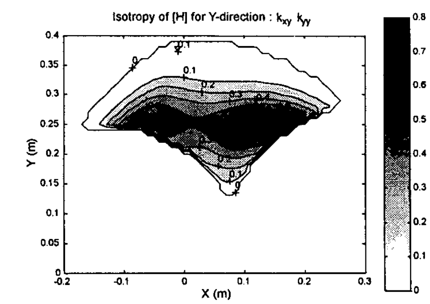
Figure 3. Optimized shape of Design 2

As shown in Fig. 2, the length of distal floating link is longer than that of the link attached to the base. This structure is suitable to translate the torque given at the two base joints to the forces at the end-effector. In Fig. 3, conversely, the length of right connecting link shows extreme value, and this represents bad force transmission from base actuator to end-effector. By intuition, the shape of the optimized five-bar should be symmetric when solely considering the isotropy in optimization. However, both optimization results show unsymmetrical configurations. Its is remarked that the unsymmetry is caused by consideration of the gradient characteristic of the isotropic index in optimization.

The contours of stiffness modulation index for Design 1 are shown in Fig. 4. As shown in Fig. 4, κ_x is high at the center of workspace. Isotropies for Y-directional stiffness modulation are also high over most workspace. This factor is advantageous to operate the mechanism in both x and y directions.



(a) Distribution of κ_x

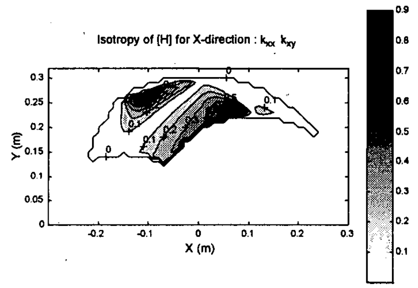


(b) Distribution of κ_y

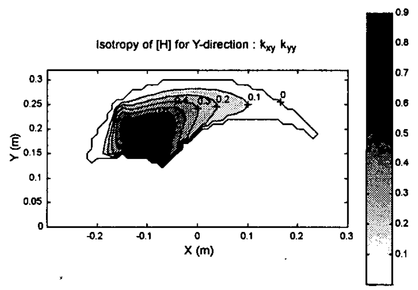
Figure 4. Isotropic index of Design 1

Figure 5 shows the contours of stiffness modulation index for Design 2. The workspace is narrow and the peak is not on the center of workspace. Therefore this design can not be said to be useful. Contrastively the workspace of Design 1 has shape of a fan and is close to a diamond, and it is another advantage. As shown in Fig. 5 (a), very low isotropy is detected on the middle of the workspace. Although Design 2 is optimized to maximize Y-directional stiffness modulation effect only not to optimize X-directional stiffness modulation, but this design is unfit for general purpose.

Shape of optimization result for maximizing isotropy of Jacobian $[G_u^a]$ and minimizing the gradient of its isotropy also shows like that of Design 1, though the result is not represented here. Therefore, Design 1 is suitable for both general operational purpose and antagonistic stiffness generation.



(a) Distribution of κ_x



(b) Distribution of κ_y

Figure 5. Isotropic index of Design 2

6. Conclusion

The purpose of this work is to propose an optimal kinematic architecture of five-bar mechanism that can modulate antagonistic stiffness by using redundant actuators more effectively. The model of antagonistic stiffness generation is constructed and the stiffness modulation index is proposed. The global design index that can represent whole isotropic characteristic of the mechanism is defined, and the gradient design index is also defined and employed in optimization to obtain uniform isotropic characteristic. Composite design index integrating several design indices into one index systematically is defined to deal with two design indices.

Two optimization results are obtained based on two objective functions. One corresponds to stiffness modulation in the X-direction, and the other to the Y-direction. It is observed that the first design can modulate both of X-directional and Y-directional stiffness effectively, but the second design is not appropriate. Besides this, it is found that the result of the first design coincides with that of the optimal design to enhance the first-order kinematic characteristics such as velocity and force transmission ratios.

Currently, we are developing five-bar mechanism for the purpose of useful utilization of the antagonistic stiffness. The first objective is precise force control. The second objective is impact minimization. Both applications utilize the soft spring effect created by redundant actuation.

References

[1] B-J. Yi, and R.A. Freeman, "Geometric analysis of antagonistic stiffness in redundantly actuated parallel mechanisms", *Journal of Robotic Systems*, Vol. 10, No. 5, pp. 581-603, 1993.

[2] N. Hogan, "Impedance control: an approach to manipulation, Part II: Implementation", *ASME J. of Dynamic Systems, Measurement, and Control*, Vol. 107, pp. 8-16, 1985.

[3] J. Tong, and J. Somerset, "Control, performance, and application of antagonistic manipulator joints", *Proc. of American Control Conf.*, pp. 63-64, 1985.

[4] K. Yokoi, M. Kaneko, and K. Tanie, "A compliance control method suggested by muscle networks in human arms", *Proc. of IEEE/RSJ Conf. On Intelligent Robots and Systems*, pp. 385-390, 1988.

[5] H. Maekawa, K. Tanie, and K. Komoriya, "Kinematics, statics and stiffness effect of 3D grasp by multifingered hand with rolling contact at the fingertip", *IEEE Proc. on Robotics and Automation Conf.*, pp. 78-85, 1997.

[6] B-J. Yi, I.H. Suh, and S-R. Oh, "Analysis of a fivebar finger mechanism having redundant actuators with applications to stiffness and frequency modulation". *IEEE Proc. on Robotics and Automation Conf.*, pp. 759-765, 1997.

[7] G. J. Klir and T. A. Folger, *Fuzzy sets, uncertainty and information*, Prentice Hall, 1988.

[8] R. A. Freeman, and D. Tesar, "Dynamic modeling of serial and parallel mechanisms/ robotic systems, Part I: Methodology, Part II: Applications", *Proc. ASME Biennial Mechanisms Conf.*, DE-Vol. 15-3, pp. 7-27, 1988.

[9] Salisbury, *Kinematic and force analysis of articulated hands*, Ph.D. dissertation, Dept. of Mechanical Eng., Stanford University, 1982.

[10] A. A. Maciejewski, and C. A. Klein, "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments", *Int. Journal of Robotics Research*, Vol. 4, No. 3, pp. 109-117, 1985.

[11] S. H. Lee, B-J. Yi, and Y. K. Kwak, "Optimal kinematic design of an anthropomorphic robot module with redundant actuators", *Mechatronics*, Vol. 7, No. 5, pp. 443-464, 1997.

[12] D. G. Luenberger, *Linear and nonlinear programming*, Addison Wesley, 2nd Ed., 1984.