# The Relationship between R&D and Market Share: The Schumpeterian Hypothesis Revisited and Implications

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**Abstract**

This paper presents a simple model to explore the revisited Schumpeterian hypothesis concerning the relationship between R&D expenditure and market share. The paper generalizes existing works in two ways. First, the model includes both process R&D as a cost-reducing investment and product R&D as a quality-enhancing investment. Second, market share in the model depends on the product quality relative to price, rather than product quality alone. An open-loop Nash equilibrium is derived on the basis of the optimal control method. This paper shows the relationship between market share and each type of R&D, and discusses how the composition of total R&D changes over the industry life cycle.

JEL Classification: L10, O30

Key words: Market share; Product R&D; Process R&D; Schumpeterian hypothesis; Composition of R&D

## 1. Introduction

This paper aims to provide a perspective on the revisited Schumpeterian hypothesis regarding the relationship between innovation and market share, through dynamic models of R&D. Originally, one of the long-debated Schumpeterian hypotheses[1] has been concerned with the relationship between innovation and firm size, and absolute firm size in terms of the number of employees or total sales has been employed in the previous studies on the hypothesis. On the contrary, this paper discusses the relationship between R&D and market share. Market share has been generally accepted as a measure of market dominance of a firm over its industry (Caves and Porter, 1978; Davies and Geroski, 1997) and as a measure of relative firm size within an industry. Therefore, this approach to the revisited Schumpeterian hypothesis aims to explain the relationship between R&D and market dominance and so moves beyond the usual interpretation of the Schumpeterian hypothesis concerning R&D and firm size.

This paper shows that the relationship between R&D expenditure and market share depends on whether price has an effect upon the competition among firms. When firms compete only through the perceived quality of products, the relationship between market share and product R&D expenditure is inverted U-shaped. However, product R&D expenditure decreases with market share when the perceived quality relative to price determines the market shares of firms. Regardless of the effects of price on market competition, the relationship between process R&D expenditure and market share is positive but less than proportional.

Since the seminal paper of Nerlove and Arrow (1962), many studies have discussed the dynamic model of firms' behavior and advertising expenditures. Nakao (1982) extended the

[1] Another Schumpeterian hypothesis has been concerned with the relationship between innovation and market structure. For summaries of previous studies and debates on the Schumpeterian hypothesis concerning firm size and R&D, see Cohen (1995), Cohen and Klepper (1996a), and Lee and Sung (2005).

Nerlove-Arrow model to explain R&D activity of firms under oligopolistic competition, as product R&D has properties similar to those of advertising; product R&D increases the demand by improving the quality of existing products or introducing a new product to supersede an existing one. Technological innovation is composed of product innovation and process innovation. In general, process innovation decreases the cost of production. Therefore, we propose that the effects of process R&D expenditure should be included in the model.

This paper extends the existing models in two directions to explain the relationship between R&D and market share. First, we derive a dynamic model in which oligopolistic firms perform not only product (quality-improving) innovation but also process (cost-reducing) innovation. Second, we analyze a case in which the perceived quality relative to the price of a product can determine its market share.

This paper is organized as follows. In Section 2, a basic model, in which firms compete with each other only through the perceived quality of product, is presented. In Section 3, an extended model analyzed the case in which both price and quality have an influence on the competition. In Section 4, propositions are developed for the relationship between market share and R&D, and the change of R&D composition in the life cycle, and the implications of propositions are discussed. In Section 5, concluding remarks are suggested.

## 2. The model of quality competition

In this section, we extend the Fershtman's (1984) model by regarding advertising as product R&D and including the effect of process R&D expenditure on the cost. The concept of 'goodwill' in the Fershtman's model is similar to that of 'product quality technology' (Nakao, 1982) and 'perceived quality' (Levin and Reiss, 1988) in the literature of R&D economics.

The profit function of the oligopolistic firm *i* at time *t* can be written as

$$\pi_{it} = (p - c_{it})Q_t s_{it} - a_{it} - x_{it},$$

where $i = 1, \ldots, N$ and $N$ is the equilibrium number of firms in the industry.

The market price $p$ and market demand $Q_t$ are assumed to be given, implying that all firms are price-takers. The market share is assumed to be determined by $G_{it}$, the product quality or goodwill stock, since all firms in the industry face the same price:

$$s_{it} = \frac{(G_{it}/p)^{\alpha}}{\sum_{j=1}^{N}(G_{jt}/p)^{\alpha}} = \frac{G_{it}{}^{\alpha}}{\sum_{j=1}^{N} G_{jt}{}^{\alpha}} \tag{1}$$

where $\alpha$ denotes a constant responsiveness of consumers to the perceived quality.[2]

An alternative interpretation of these assumptions is that firms compete with each other through product quality in the market because they might agree to price collusion or because consumers pay their attention only to the product quality in choosing a product.

We assumed that $G_{it}$ accumulates according to the following equation:

$$\frac{dG_{it}}{dt} = \dot{G}_{it} = a_{it} - \delta G_{it} \tag{2}$$

where $a_{it}$ is product R&D expenditure and $\delta$ is a constant depreciation rate of $G_{it}$.

We assumed that process R&D expenditure $x_{it}$ increases the stock of cost-reducing technology $A_{it}$ according to the following equation:

2 We assume, as Fershtman did, that $0 < \alpha < 1$ to indicate that $S_{it}$ is concave in $G_{it}$, which means that consumers respond positively to quality but returns to quality are diminishing returns.

$$\frac{dA_{it}}{dt} = \dot{A}_{it} = x_{it} - \rho A_{it} \qquad (3)$$

where $\rho$ is a constant depreciation rate of $A_{it}$.

Contrary to Nakao (1982) and Fershtman (1984), who assumed a constant average cost, we assume that an increase in the stock of cost-reducing technology decreases average cost $c_{it}$:[3]

$$c_{it} = c\,\mathrm{e}^{-bA_{it}} \qquad (4)$$

where $c$ denotes the initial average cost and $b$ denotes a parameter related to the marginal returns of cost-reducing technology to average cost.

Under these assumptions, each firm maximizes its net present value of discounted profit stream:

$$\max \int_0^{\infty} e^{-rt}\{(p - c_{it})Q_t s_{it} - a_{it} - x_{it}\}dt \quad \text{subject to}$$

$$\dot{G}_{it} = a_{it} - \delta G_{it}, \quad \dot{A}_{it} = x_{it} - \rho A_{it};$$

$G_{i0}$ and $A_{i0}$ are given; and

$G_{jt}$ and $A_{jt}$ are given for every $i \neq j$,

where $r$ denotes a constant discount rate.

We can define the current-value Hamiltonian as

[3] This assumption indicates that accumulated experience in process innovation (Rosenberg, 1983; Spence, 1984), rather than accumulated output in production, (Spence, 1981) reduces average cost.

$$H = \{(p - c_{it})Q_{it}s_{it} - a_{it} - x_{it}\} + \lambda_t(a_{it} - \delta G_{it}) + \mu_t(x_{it} - \rho A_{it}) .$$

By assuming that firms have Cournot-Nash conjectures regarding the R&D decisions of other firms (Nakao, 1982; Levin and Reiss, 1988), we can obtain first-order conditions and tranversality conditions:

$$\frac{\partial H}{\partial a_{it}} = -1 + \lambda_t = 0\,;\ \frac{\partial H}{\partial x_{it}} = -1 + \mu_t = 0 \tag{5}$$

$$\frac{\partial H}{\partial G_{it}} = (p_{it} - c_{it})Q_t \frac{\partial s_{it}}{\partial G_{it}} - \delta\lambda_t = -\dot{\lambda}_t + r\lambda_t \tag{6}$$

$$\frac{\partial H}{\partial A_{it}} = -\frac{\partial c_{it}}{\partial A_{it}} Q_t s_{it} - \rho\mu_t = -\dot{\mu}_t + r\mu_t \tag{7}$$

$$\lim_{t\to\infty} \lambda_t \mathrm{e}^{-r_t} G_{it} = 0\,;\ \lim_{t\to\infty} \mu_t \mathrm{e}^{-r_t} A_{it} = 0 . \tag{8}$$

We can derive an open-loop stationary Nash equilibrium point ($s_{it}{}^{*}$, $a_{it}{}^{*}$, $x_{it}{}^{*}$) from these conditions:[4]

$$a_{it}{}^{*} = \frac{\alpha\delta}{r+\delta}\left\{-pQ_t s_{it}^{*2} + (pQ_t + \frac{r+\rho}{b})s_{it}^{*} - \frac{r+\rho}{b}\right\} \tag{9}$$

$$x_{it}{}^{*} = \frac{\rho}{b}\left(\ln s_{it}^{*} + \ln Q_t + \ln\frac{bc}{r+\rho}\right). \tag{10}$$

Equation (9) shows that the relationship between product R&D and market share is inverted U-shaped. Product R&D expenditure has its maximum value at $s_{it} = \frac{1}{2} + \frac{r+\rho}{2bpQ_t}$ in this model, while advertising expenditure has its maximum value at $s_{it}$ = 1/2 in the Fershtman's (1984)

[4] Refer to Appendix A1 for details on the process for deriving Equations (9) and (10).

model, in which the effect of cost-reducing investment, such as process R&D, on cost is not considered. Equation (9) shows that the characteristics ($b$, $c$, and $\rho$) related to process R&D, as well as the demand characteristics ($\alpha$), have an influence on the optimal level of product R&D expenditure. Equation (10) indicates that process R&D expenditures of firms increase with their market share at a decreasing rate, since the optimal level of process R&D expenditures is concave in market share.

We define total R&D expenditure $r_{it}^*$ as the sum of product R&D expenditure and process R&D expenditure:

$$r_{it}^* = \frac{\alpha\delta}{r+\delta}\left\{-pQ_t s_{it}^{*2} + (pQ_t + \frac{r+\rho}{b})s_{it}^* - \frac{r+\rho}{b}\right\} + \frac{\rho}{b}\left(\ln s_{it}^* + \ln Q_t + \ln\frac{bc}{r+\rho}\right). \tag{11}$$

We differentiate total R&D expenditure with respect to market share to explain the revised Schumpeterian hypothesis:

$$\frac{dr_{it}^*}{ds_{it}^*} = \frac{\alpha\delta}{r+\delta}\left\{-2pQ_t s_{it}^* + (pQ_t + \frac{r+\rho}{b})\right\} + \frac{\rho}{bs_{it}^*} \tag{12}$$

$$\frac{d^2 r_{it}^*}{ds_{it}^{*2}} = -\frac{2\alpha\delta pQ_t}{r+\delta} - \frac{\rho}{bs_{it}^{*2}}. \tag{13}$$

Equations (12) and (13) show that the relationship between market share and total R&D expenditure under non-price competition is inverted U-shaped.

## 3. The model of competition through quality relative to price

In this section, we examine a case in which the perceived quality relative to price determines the product's market share. When the prices of products are different, consumers consider the price as well as the perceived quality in making their choices. For example, if there were several products in the market of the same perceived quality, the price would determine the market share of each product. Therefore, price, in addition to investment activities in oligopolistic competition, can be a strategic variable (Slade, 1995).

It is assumed that the market share of a product is more likely to be higher as the product's ratio of perceived quality to price increases. According to this assumption, the market share is described as:

$$s_{it} = \frac{(G_{it} / p_{it})^{\alpha}}{\sum_{j=1}^{N} (G_{jt} / p_{jt})^{\alpha}}. \tag{14}$$

Other assumptions of Equations (2), (3) and (4) remain valid in this model. Under these assumptions, each firm maximizes the net present value of its discounted profit stream such that:

$$\max \int_0^{\infty} e^{-rt} \{(p_{it} - c_{it}) Q_t s_{it} - a_{it} - x_{it}\} dt \quad \text{subject to}$$

$$\dot{G}_{it} = a_{it} - \delta G_{it}, \quad \dot{A}_{it} = x_{it} - \rho A_{it};$$

$G_{i0}$ and $A_{i0}$ are given; and

$G_{jt}$ and $A_{jt}$ are given for every $i \neq j$.

We can define the current-value Hamiltonian as

$$H = \{(p_{it} - c_{it})Q_{it}s_{it} - a_{it} - x_{it}\} + \lambda_t(a_{it} - \delta G_{it}) + \mu_t(x_{it} - \rho A_{it}) .$$

We can obtain first-order conditions and tranversality conditions in a way similar to that in Section 2. The first-order condition regarding price is included additionally:

$$\frac{\partial H}{\partial p_{it}} = Q_{it}s_{it} + (p_{it} - c_{it})\left(\frac{\partial Q_t}{\partial p_{it}} s_{it} + Q_t \frac{\partial s_{it}}{\partial p_{it}}\right) = 0 . \tag{15}$$

We manipulate one of the components in Equation (15) in the following form:

$$\frac{\partial Q_t}{\partial p_{it}} = -\varepsilon \frac{Q_t}{p_{it}} \tag{16}$$

where $\varepsilon = -\frac{P_t}{Q_t}\frac{\partial Q_t}{\partial p_t}$ denotes the a firm's price elasticity of market demand.

From those conditions, we can derive an open-loop stationary Nash equilibrium point ($s_{it}^*$, $a_{it}^*$, $x_{it}^*$):[5]

$$a_{it}^* = \frac{\alpha\delta}{1-\varepsilon}\left\{1 - \frac{r+\rho}{b(r+\delta)}\right\}(1 - s_{it}^*) \tag{17}$$

$$x_{it}^* = \frac{\rho}{b}\left(\ln s_{it}^* + \ln Q_t + \ln \frac{bc}{r+\rho}\right). \tag{18}$$

Equation (17) shows a negative relationship between product R&D expenditure and market share, which is completely different from the relationship presented in Section 2. Similar to

[5] Refer to Appendix A2 for details on the process for deriving Equations (17) and (18).

Equation (9), Equation (17) suggests that the characteristics regarding process R&D influence the optimal level of product R&D expenditure. Equation (18) is the same with Equation (10), which implies that process R&D expenditure increases with market share at a decreasing rate regardless of the effects of price on the competition. A combination of Equations (17) and (18) suggests that firms' product R&D expenditures tend to decrease with market share, while process R&D expenditures tend to increase with market share.

Summing Equations (17) and (18) yields total R&D expenditure:

$$r_{it}^{*} = \frac{\alpha\delta}{1-\varepsilon}\left\{1 - \frac{r+\rho}{b(r+\delta)}\right\}(1-s_{it}) + \frac{\rho}{b}\left(\ln s_{it}^{*} + \ln Q_{t} + \ln\frac{bc}{r+\rho}\right). \tag{19}$$

Differentiating Equation (19) with respect to market share yields Equations (20) and (21):

$$\frac{dr_{it}^{*}}{ds_{it}^{*}} = -\frac{\alpha\delta}{1-\varepsilon}\left\{1 - \frac{r+\rho}{b(r+\delta)}\right\} + \frac{\rho}{bs_{it}^{*}} \tag{20}$$

$$\frac{d^{2}r_{it}^{*}}{ds_{it}^{*2}} = -\frac{\rho}{bs_{it}^{*2}}. \tag{21}$$

Similar to Equations (12) and (13), Equations (20) and (21) show the inverted U-shaped relationship between market share and total R&D expenditure.

## 4. Implications

In this section, five propositions are developed to discuss the Schumpeterian hypothesis concerning R&D expenditure and firm size in terms of market share, and the relationship between R&D expenditure and industry characteristics is considered.

*Proposition 1. The relationship between R&D expenditure and market share is inverted U-shaped.*

If market share is regarded as firm size in the Schumpeterian hypothesis concerning firm size and R&D, Proposition 1 can be interpreted to mean that R&D expenditure increases with firm size below a certain level but decreases with firm size over that level. The level depends upon industry characteristics (e.g., $\alpha$, $\delta$, $\rho$, and $b$) and the allocation of market shares in industries, so the level varies across industries. Proposition 1 is consistent with one of the stylized facts found in the previous empirical studies (Cohen and Klepper, 1996a).

*Proposition 2. Low appropriability conditions of product innovation induce firms to increase product R&D expenditure, while low appropriability conditions of process innovation induce firms to decrease product R&D expenditure.*

Proof:

$\dfrac{da_{it}^{*}}{d\delta} = \dfrac{\alpha\delta}{(r+\delta)^2}\left\{-pQ_t s_{it}^{*2} + (pQ_t + \dfrac{r+\rho}{b})s_{it}^{*} - \dfrac{r+\rho}{b}\right\} > 0$ and $\dfrac{da_{it}^{*}}{d\rho} = -\dfrac{\alpha\delta(1-s_{it}^{*})}{(r+\delta)b} < 0$ are derived from Equation (9). $\dfrac{da_{it}^{*}}{d\delta} = \dfrac{\alpha(1-s_{it}^{*})}{1-\varepsilon}\left\{1-\dfrac{r+\rho}{b(r+\delta)}+\delta+\dfrac{r+\rho}{b(r+\delta)^2}\right\} > 0$ and $\dfrac{da_{it}^{*}}{d\rho} = -\dfrac{\alpha\delta(1-s_{it}^{*})}{b(1-\varepsilon)(r+\delta)} < 0$ are derived from Equation (17).

Proposition 2 can explain the effect of the appropriability condition[6] on product R&D expenditure. In accordance with Nakao's (1982) argument that the depreciation rate might be

[6] Appropriability conditions refer mechanisms in order to exploit the returns of technological innovation exclusively. They have been considered one of the main determinants of industrial R&D (Levin *et al.*, 1987; Cohen, 1995).

negatively associated with patent life, it is reasonable to argue the depreciation rates ($\delta$, $\rho$) of technology stock are negatively associated with appropriability conditions. Proposition 2 shows that a higher value of $\delta$ resulting from low appropriability of product innovation increases product R&D expenditure and that a higher value of $\rho$ resulting from low appropriability of process innovation decreases product R&D expenditure.

*Proposition 3. Low appropriability conditions of process innovation increase product R&D expenditure only for dominant firms with a higher market share, while appropriability conditions of product innovation have no influence on process R&D expenditure for any firms.*

Proof:

$\dfrac{dx_{it}^{\ *}}{d\rho} = \dfrac{1}{b}\left(\ln s_{it}^{*} + \ln Q_t + \ln \dfrac{bc}{r+\rho} - \dfrac{\rho}{r+\rho}\right) = \dfrac{1}{b}\left(\dfrac{b}{\rho} x_{it}^{*} - \dfrac{\rho}{r+\rho}\right)$ is derived from Equations (10) and (18).

Proposition 3 argues that a higher value of $\rho$ resulting from low appropriability increases process R&D expenditure only when a firm's market share or its expenditure of process R&D is higher than a certain level in Equations (10) and (18).

Propositions 2 and 3 propose that high appropriability conditions do not always induce firms to increase total R&D expenditure; rather, low appropriability conditions of product innovation might cause firms to increase their expenditures of product R&D to compensate for the negative effect of weak protecting mechanisms (e.g., a short patent life) on innovation. Further, high appropriability conditions of process innovation are more beneficial to firms with a higher expenditure of process R&D than to those with a lower expenditure.

We can also express the proportion of process R&D expenditure to total R&D expenditure on the basis of our model. Considering Equations (17) and (18) together, we can suggest the following Proposition 4.

*Proposition 4. The proportion of process R&D expenditure to total R&D expenditure for a firm,* $x_{jt} / r_{jt}$*, increases with its market share in the market in which consumers choose products based on the perceived quality of products relative to price.*

Proposition 4 directly follows a negative relationship between market share and product R&D in Equation (17) and a positive relationship between market share and process R&D in Equation (18). Larger firms with a higher market share devote a larger proportion of their R&D expenditures to process innovation. Proposition 4 is consistent with the empirical and theoretical findings of previous studies (Cohen and Klepper, 1996b; Saha, 2007).

*Proposition 5. The industrial proportion of process R&D expenditures to total R&D expenditures,* $\sum_j x_{jt} / \sum_j r_{jt}$*, increases as the number of firms decreases, in keeping with industry evolution over time.*

The pattern that Proposition 5 explains is one of the stylized patterns observed in the industry life cycle (Utterback and Abernathy, 1975; Klepper, 1996). Saha (2007) argued that this pattern occurs because the marginal buyer's willingness to pay decreases over time, causing a monopolistic firm increasingly to devote more of its R&D effort to making the product cheaper. Saha's model seems suitable as applied to explaining the process of the monopolist's market pioneering and the consequent pattern but implausible as applied to explaining relevant results occurring within the oligopoly market.

Our model of oligopolic firms provides a different view on this pattern. Another of the stylized patterns related to industry evolution is that the number of firms in an industry tends to decrease over time, in keeping with the industry life cycle. In particular, the number of firms tends to decreased drastically after the industry experiences a selection process such as an industry shakeout (Jovanovic and McDonald, 1994; Klepper, 2002; Klepper and Simons, 2005). We argue that the two patterns are interrelated. According to Equations (17) and (18), when an industry consists of many firms, each with a smaller market share, the firms devote a relatively lower expenditure to product R&D and a relatively higher expenditure to process R&D. The average market share of the remaining firms is higher after market selection than before, so industrial expenditures in product R&D decrease, but industrial expenditures in process R&D increase. Therefore, the industrial proportion of process R&D expenditures to total R&D expenditures will increase as the number of firms in the industry decreases. This tendency that Proposition 5 considers is similar to the empirical regularity that Klepper's model (1996) explains.

Contrary to conventional wisdom, the total number of surviving firms in some industries can sometimes increase or stagnate over time, as occurred in the printer and monitor markets in the Unites States (Filson, 2001). According to our model presented in Section 3, the industrial proportion of process R&D expenditures to total R&D expenditures in such markets should be low if there are many firms, each with a small market share. However, in fact several dominant firms operate in the printer market, so the industrial proportion of process R&D expenditures to total R&D expenditures might be not very low. Thus, results of the model in Section 3 seem inconsistent with the pattern observed in such markets. Alternatively, using the model presented in Section 2, we can understand a market where the number of firms increases over time. In this case, if consumers choose a product in the market based on the perceived quality of the product, the proportion of process R&D to total R&D would depend upon the market structure concerned with the allocation of market shares among firms, as well as other industry characteristics. Cohen and Klepper (1996b) found that the average share of process R&D in total R&D differs greatly across industries. Given that types of competition vary across industries, this finding might be expected in the light of the results of our model.

## 5. Concluding remarks

This paper contributes to the literature by providing a framework for the revised Schumpeterian hypothesis concerning the relationship between R&D and market share. Specifically, the paper discusses the relationship between R&D expenditure and market share and develops theoretical models that consider the effects of accumulated experience in R&D on the firm's market share and average cost. The models consider cases in which the perceived product quality relative to price determines the market share and in which only quality influences the market share. The paper also explains cross-sectional regularities regarding technological innovation, as well as general patterns of the industry life cycle.

Because of the difficulties inherent in dividing R&D into different types, such as process R&D and product R&D, and measuring those types, less research in the field of industrial economics has been done on the composition of R&D within industries than on other aspects of R&D (Cohen and Klepper, 1996b). Thus, we still lack sufficient knowledge about the composition of R&D (Fritsch and Meschede, 2001; Filson, 2002), and the composition of R&D deserves further study. By separating innovation into product innovation and process innovation in a dynamic model, the current research suggests not only how firms set their portfolio of the two types of innovation as their market share changes, but also the implications for innovation policies that can provide favorable environments to corporate R&D.

## Appendix

### A1. Derivation of Equations (9) and (10)

Equation (5) implies that $\lambda_t = 1$ and $\mu_t = 1$ and thus $\dot{\lambda}_t = 0$ and $\dot{\mu}_t = 0$. (A1)

Differentiating Equation (1) with respect to $G_{it}$ yields $\dfrac{\partial s_{it}}{\partial G_{it}} = \dfrac{\alpha s_{it}(1 - s_{it})}{G_{it}}$. (A2)

Therefore, Equation (6) can be transformed into $G_{it} = \dfrac{\alpha}{r+\delta}(p - c_{it})Q_t s_{it}(1 - s_{it})$. (A3)

Differentiating Equation (4) with respect to $A_{it}$ yields $\dfrac{\partial c_{it}}{\partial A_{it}} = -bc_{it}$. (A4)

Therefore, Equation (7) can be transformed into $c_{it} = \dfrac{r+\rho}{bQ_t s_{it}}$. (A5)

Incorporating Equation (A5) into Equation (A3) yields

$$G_{it} = \frac{\alpha}{r+\delta}\left\{-pQ_t {s_{it}}^2 + (pQ_t + \frac{r+\rho}{b})s_{it} - \frac{r+\rho}{b}\right\}. \quad \text{(A6)}$$

Incorporating Equation (A5) into Equation (4) yields

$$A_{it} = \frac{1}{b}\left(\ln s_{it} + \ln Q_t + \ln \frac{bc}{r+\rho}\right). \quad \text{(A7)}$$

Since $\dot{G}_{it} = 0$ and $\dot{A}_{it} = 0$ at the equilibrium, the optimal level of process R&D and product R&D satisfies $a_{it}^{*} = \delta G_{it}^{*}$ from Equation (2) and $x_{it}^{*} = \rho A_{it}^{*}$ from Equation (3).

Therefore, we can derive the optimal level of process R&D and product R&D in a function of market share $s_{it}^{*}$ at the equilibrium in Equations (9) and (10). (Q.E.D.)

**A2. Derivation of Equations (17) and (18)**

Equation (6) can be transformed into $G_{it} = \dfrac{\alpha}{r+\delta}(p_{it} - c_{it})Q_t s_{it}(1-s_{it})$. (A8)

Differentiating Equation (14) with respect to $p_{it}$ yields $\dfrac{\partial s_{it}}{\partial p_{it}} = -\dfrac{\alpha s_{it}(1-s_{it})}{p_{it} G_{it}}$. (A9)

Therefore, Equation (15) can be transformed into $\dfrac{p_{it} - c_{it}}{p_{it}} = \dfrac{1}{\varepsilon + \dfrac{\alpha(1-s_{it})}{G_{it}}}$. (A10)

Incorporating Equation (A5) into Equation (A10) yields

$$p_{it} = \frac{\varepsilon + \dfrac{\alpha(1-s_{it})}{G_{it}}}{\varepsilon + \dfrac{\alpha(1-s_{it})}{G_{it}} - 1} \cdot \frac{(r+\rho)}{bQ_t s_{it}}. \tag{A11}$$

Incorporating Equations (A11) and (A5) into Equation (A8) yields

$$G_{it} = \frac{\delta}{1-\varepsilon}\left\{1 - \frac{r+\rho}{b(r+\delta)}\right\}(1-s_{it}). \tag{A12}$$

$A_{it}$ can be described as in the same form of Equation (A7):

$$A_{it} = \frac{1}{b}\left( \ln s_{it} + \ln Q_t + \ln \frac{bc}{r+\rho} \right). \tag{A13}$$

Since $a_{it}^* = \delta G_{it}^*$ and $x_{it}^* = \rho A_{it}^*$ at the equilibrium, we can derive the optimal level of process R&D and product R&D in a function of market share $s_{it}^*$ at the equilibrium in Equations (17) and (18). (Q.E.D.)

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