Unbalance Response Control in Rotor-Bearing System using Semi-Active Fluid Bearing

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Abstract: This paper presents an unbalance response analysis for a rotor bearing system that utilizes a high-density fuel as its working fluid. The extensive simulations with a LP spool rotor indicate that the unbalance response can be significantly varied by manipulation of the fluid supply pressure. It suggests that use of semi-active fluid bearing can be very effective in attenuation of unbalance response of the spool rotor, when the fluid supply pressure is properly manipulated. For comparison, the characteristics of the conventional design using rolling element bearings are also analyzed.

Keywords: Unbalance Response, Semi-Active Fluid Bearing, High-Density Fuel

1 Introduction
In the past, working fluids, such as water and fuel, used in high-speed turbomachinery possessed far lower density and thus lower viscosity than lubricants so that lubricant contamination of working fluids was somehow inevitable. In recent years, there is a design concept of using high-density fuels not only as a working fluid but as a lubricant to increase the fuel efficiency as well as the power density, and thus lower viscosity than lubricants so that lubricant contamination of working fluids was somehow inevitable. In this work, in order to improve the unbalance response of the rotor-bearing system submerged in high-density fuel, a semi-active fluid bearing utilizing the fuel as its lubricant is proposed and its dynamic characteristics are investigated. In particular, the effects of the semi-active fluid bearing on the dynamic behavior of the rotor-bearing system are examined.

2 Modeling of rotor bearing system
For the rotor bearing system with \( N \) nodes, the equation of motion can be written as

\[
\begin{bmatrix}
M & 0 \\
0 & M
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} + \begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} + \Omega \begin{bmatrix}
0 & J_p \\
-J_p & 0
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} + \begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} = \begin{bmatrix}
f_x \\ f_y
\end{bmatrix}
\]  

Here, \( M \) and \( J_p \) denote the mass and polar mass moment of inertia matrices, respectively, related to the rotating disk and shaft; \( C_{xx}, C_{xy}, C_{yx}, \) and \( C_{yy} \) represent the non-symmetric damping matrices in \( x \) and \( y \) directions; \( K_{xx}, K_{xy}, K_{yx}, \) and \( K_{yy} \) represent the non-symmetric stiffness matrices in \( x \) and \( y \) directions; \( x \) and \( y \) are the \( 2N \times 1 \) coordinate vectors in \( x \) and \( y \) directions, respectively; \( f_x \) and \( f_y \) are the \( 2N \times 1 \) force vectors in \( x \) and \( y \) directions, respectively; \( \Omega \) is the rotational speed.

Rotor systems having complex components, i.e. turbine blades or flexible disks, can be modeled by mass, damping, and stiffness matrices with the help of finite element method. However, the dynamic behavior of the rotor in its lower modes can be approximated by a reduced degrees of freedom model. Such reduction techniques are commonly applied in general structural dynamics applications to reduce eigenvalue extraction time, to condense internal element degrees of freedom, or as part of a substructuring approach[13-18]. In this work, the modal truncation method is applied using the modes dominant at a specific rotational speed. Using the upper symmetric matrix in Eq.(1), the transformation matrix for modal truncation can be expressed as

\[
\begin{bmatrix}
M \\
0
\end{bmatrix} \Phi \Phi^T = \begin{bmatrix}
I_{m \times m} \\
0_{m \times m}
\end{bmatrix}
\]  

By using the Choleski decomposition and the well-known eigenroutines developed for symmetric positive definite matrices such as the vector inverse iteration method or the subspace iteration method, the \( m \) lowest real eigenvalues and the corresponding real eigenvectors are easily obtained, satisfying the orthonormality conditions

\[
\varphi^T \begin{bmatrix}
M \\
0
\end{bmatrix} \varphi = \lambda_{m \times m} \]  

\[
\varphi^T \begin{bmatrix}
K_{xx} \\
K_{xy}
\end{bmatrix} \varphi = \lambda_{m \times m} \]

where the superscript \( T \) denotes the matrix transpose, \( I_{m \times m} \) is the \( m \times m \) identity matrix, \( \lambda_{m \times m} \) is the \( m \times m \) diagonal matrix, and \( \varphi \) is the \( 2N \times m \) modal matrix. The \( 4N \times 2m \) transformation matrix \( \Phi \) then becomes

\[
\begin{bmatrix}
x \\ y
\end{bmatrix} = \Phi \begin{bmatrix}
q_x^* \\
q_y^*
\end{bmatrix} = \begin{bmatrix}
\varphi_{x} & 0 \\
0 & \varphi_{y}
\end{bmatrix} \begin{bmatrix}
q_x^* \\
q_y^*
\end{bmatrix}
\]

(4)

where \( q_x^* \) is the \( 2m \times 1 \) reduced coordinate vector, and, \( q_x^* \) and \( q_y^* \) are the \( m \times 1 \) reduced coordinate vectors defined in the \( x \) and \( y \) directions, respectively. Substitution of Eq.(4) into Eq.(1) and pre-multiplication by \( \varphi^T \) yield
From Eq.(5), the unbalance and the transient responses as well as the stability can be easily analyzed. The response at a given node can also be obtained by using the transform relation given in Eq.(4) and the shape function.

3 Development of bearing stiffness module

3.1 Rolling element bearings
Consider the low-pressure spool of jet engine supported by ball and roller bearings. The damping of rolling element bearings is normally so small that, its theoretical and experimental evaluation is very difficult[5]. For example, for the rolling element bearings at the speed of \( \omega = 3,140 \text{rpm} \), the damping ratio ranges from 0.0004 to 0.004. Therefore, a squeeze film damper is normally installed at the location of a ball bearing in order to provide additional damping.

3.2 Stiffness of roller bearings[6]
For a rigidly supported roller bearing subjected to radial load, the load-deformation relationship is given by

\[
Q = K_i \delta^a
\]

(6)

where, \( K_i = 7.86 \times 10^4 \cdot \beta^{1/9} \)

Here, \( l \) is the roller length and \( n \) equals to \( l \cdot \beta \). In Eq.(6), the centrifugal force of a roller is not considered. In high-speed operation of roller bearings, the rolling element centrifugal forces can be significantly large compared to the forces applied to the bearing. The centrifugal force per roller can be calculated from

\[
F_c = 3.39 \times 10^{-11} D^2 l d_m n_m^3
\]

(7)

Here, \( D \) is the roller diameter, \( d_m \) is the pitch diameter of bearing, and \( n_m \) indicates the orbital roller speed.

The load-deformation relationship can be obtained by a numerical method, considering the force equilibrium given by

\[
K_i \delta_{ij}^{o,11} - K_i \delta_{ij}^{i,11} - F_c = 0
\]

(8)

Here, \( \delta_{ij} \) and \( \delta_{ij} \) represent the deformations at the outer and inner raceways, respectively.

By using Eq.(8), the variation in the deformation with the radial load for various rotational speeds is shown in Figure 1. Note that, as the speed increases, the deformation increases for given radial load.

3.3 Squeeze film damper[5]
In general, a squeeze film damper is known to have the combined structure of ball and journal bearings, accommodating their advantages. Therefore, squeeze film dampers have been widely used at a damping device for aircraft turbine engines. For the circular center orbit motion of a journal, the damping and stiffness coefficients can be expressed by

\[
K_{SFD} = \frac{R L_3 \mu \omega}{c^3} \frac{2 \varepsilon}{(1 - \varepsilon^2)^2}
\]

(9a)

\[
C_{SFD} = \frac{R L_3 \mu \pi}{c^3} \frac{3}{2(1 - \varepsilon^2)^{3/2}}
\]

(9b)

Here, \( c \) is the clearance of the squeeze film damper; \( R \) is the radius of the journal; \( L \) is the bearing length; \( \mu \) and \( \varepsilon \) are the viscosity of the lubricant and the eccentricity, respectively. Figure 2 shows the effect of parameter change to the dynamic coefficients of damper.

3.4 Semi-active fluid bearings
3.4.1 Characteristics of semi-active fluid bearing
Semi-active hydrostatic bearing has recently drawn much attention among engineers because machine parts supported on hydrostatically lubricated shafts move with incomparable smoothness. This mechanical advantage can be acquired since the bearing and journal surfaces are separated by a film of liquid forced between them under pressure.

In a semi-active fluid bearing, pressurized fluid, with the level of pressure varied upon the operational condition, is delivered to the bearing through recessed inlet ports. As

![Figure 1 Load-deformation charts for roller bearings](image1)

![Figure 2 Stiffness and damping coefficients of SFD](image2)
the rotor attempts to move in the bearing, pocket pressure increases in the direction of motion and decreases in the direction opposite to motion. This effect creates a pressure differential in opposing bearing pockets, and a corresponding restoring force to the rotor. Benefits of active fluid bearings can be summarized as:

- Enhanced stability (keeping the journal near the center of the bearing clearance, with an attitude angle near zero degrees)
- Adjustable stiffness and damping (by the manipulation of a supply pressure)
- Machine size reduction (due to the improved load capacity)
- Safety backup (if high-pressure supply fluid is lost, the bearing cavitates. The basic geometry is lost when the bearing cavitates. This equation is a statement of continuity of mass flow but the continuity is resultant force exerted by that pocket. This equation is a statement of continuity of mass flow but the continuity is lost when the bearing cavitates. The basic geometry is illustrated in Figure 4.

![Figure 3 The schematic of semi-active fluid bearing](image)

![Figure 4 Coordinate system for load and displacement](image)

### 3.4.2 Dynamic coefficients of a semi-active fluid bearing

In a semi-active fluid bearing, the continuity equation for one pocket defines the relationship between the various effects which determine the pocket pressure and hence the resultant force exerted by that pocket. This equation is a statement of continuity of mass flow but the continuity is lost when the bearing cavitates. The basic geometry is illustrated in Figure 4.

\[
q_i = p_i \frac{Bh_i^3}{\mu} + \frac{\gamma_c}{2} \left( p_i - p_{i+1} \right) \frac{Bc_i^3}{\mu} + \frac{\gamma_e}{2} \left( p_i - p_{i-1} \right) \frac{Bc_{i-1}}{\mu} + \frac{\pi DN}{2} \left( L - a \right) \left( c_i - c_{i-1} \right) + D \left( L - a \right) \sin \left( \frac{\pi}{n} \right) dh_i \left( \frac{dt}{n} \right) + \kappa V_r \frac{\partial p_i}{\partial t}
\]

where, \( B = \pi D / 6an \)

\[
\gamma_c = na \left( L - a \right) / \pi Dh
\]

Here, \( a \) and \( b \) represent the axial flow and inter-pocket land width, respectively; \( D \) is the shaft diameter and \( L \) is the bearing length; \( N \) is the rotational speed of shaft in rev/sec and \( p_i \) represents the pressure at \( i \)-th pocket; \( V_r \) and \( \kappa \) represent the oil volume associated with one pocket, and the compressibility of oil, respectively. The nature of term \( q_i \) depends on the control device and, for the capillary, can be given by

\[
q_i = \left( p_i - p_{i-1} \right) \frac{\pi d_e^2}{128 \mu l_c}
\]

Here, \( d_e \) and \( l_c \) represent the diameter and length of a capillary, respectively.

(a) At zero speed \((N=0)\) and zero velocity \((de/dt=0)\), the hydrostatic force \(W_{hs}\) acts in the direction of eccentricity and gives rise to a definition of hydrostatic stiffness, i.e.,

\[
\lambda_{hs} = \frac{dW_{hs}}{de}
\]

By using Eq.(12) and the dimensionless form of Eq.(10), the hydrostatic stiffness can be expressed by

\[
\lambda_{hs} = \frac{P_l LD \frac{3n^2}{h_o} \sin^2 \left( \frac{\pi}{n} \right) \left( 1 - \frac{a}{L} \right) \beta}{z + 1 + 2\gamma_c \sin^2 \left( \frac{\pi}{n} \right)}
\]

Here \( \beta \) means the concentric pressure ratio, \( p_l/P_s \) and the control device parameter \( z \) can be expressed by \( \beta/(1-\beta) \). The hydrostatic stiffness increases with the number of pockets \( n \) and reduces with the circumferential flow factor \( \gamma_c \).

(b) The effect of rotation is to introduce an additional force \(W_{hd}\) at 90 deg lagging \(W_{hs}\). Thus, the hydrodynamic stiffness is

\[
\lambda_{hd} = \frac{dW_{hd}}{de}
\]

and can be expressed by

\[
\lambda_{hd} = \frac{LD \frac{12n^2}{h_o} \sin^2 \left( \frac{\pi}{n} \right) \left( 1 - \frac{a}{L} \right)^2 \left( \frac{L}{D} \right)^2 N \mu \left( \frac{D}{C_d} \right)^2}{z + 1 + 2\gamma_c \sin^2 \left( \frac{\pi}{n} \right)}
\]
Note that the hydrodynamic stiffness is directly proportional to the rotational speed \( N \).

(c) The effect of squeeze velocity \( {de}/{dt} \) is to introduce an additional force \( W_{sq} \) acting in opposition to \( {de}/{dt} \). This is a damping force that is defined as

\[
C_{sq} = \frac{dW_{sq}}{de}
\]  

(16)

By using Eq. (12), the damping coefficient \( C_{sq} \) can be given by

\[
C_{sq} = \frac{LD}{h_o} \frac{12n^2}{\pi} \sin^2 \left( \frac{\pi}{n} \right) \left( \frac{a}{L} \right) \left( 1 - \frac{a}{L} \right)^2 \left( \frac{L}{D} \right)^2 \frac{z + 1 + 2\gamma c \sin^2 \left( \frac{\pi}{n} \right) \mu}{C_d}
\]

(17)

4 Unbalance response of rotor
4.1 Definition of the rotor bearing system

In this work, the low-pressure spool of a turbine engine is studied. The model for LP spool is shown in Figure 5.

Since the shape of the rotor is relatively long but thin, the shaft can be considered to be flexible. The operational speed range of LP spool is from 0 to 35,600rpm. Table 1 lists the assumed mass unbalance according to API standards.

<table>
<thead>
<tr>
<th>Table 1 Unbalance of LP spool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of unbalance</td>
</tr>
<tr>
<td>Magnitude of unbalance</td>
</tr>
</tbody>
</table>

4.2 Unbalance responses for the conventional design

Ball and roller bearings are installed in conventional design and their specifications are as following:
- No.1 brg.: Split inner ring ball bearing, \( D=9.525mm \), No. of balls=15, Inner diameter=45mm, Outer diameter=75mm, Angle of contact=33–38 deg, Damping method–squeeze film damper
- No.2 brg.: Double-row cylindrical roller bearing, \( D=5.0mm \), No. of rollers=18, Roller length=5mm, Inner diameter=35mm, Outer diameter=55mm, Damping method–none
- No.5 brg.: same as No.2 brg., Damping method–metal ring damper

Figure 6 shows the unbalance response at No.1 bearing. Note that excessive vibration occurs near the rotational speed of about 22,000rpm. This resonance may be avoided by fast acceleration of the rotor but, if the operation speed is set near this speed, the conventional design cannot stabilize the system.

4.3 Unbalance response for semi-active fluid bearings

From the experimental results, the density and viscosity of the high-density fuel proposed for a working fluid can be modeled by

\[
\rho = 805.0 - \frac{9 T}{15}
\]

(18a)

\[
\log \mu = 0.3031 - 0.0113T + 7.527 \times 10^{-5} T^2 - 2.823 \times 10^{-7} T^3
\]

(18b)

Here \( T \) and \( \rho \) represent the temperature and density of the fuel. In Table 2, the specifications for the designed semi-active fluid bearing are listed.

<table>
<thead>
<tr>
<th>Table 2 Design specifications for semi-active fluid bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design parameters</td>
</tr>
<tr>
<td>Bearing length ( L )</td>
</tr>
<tr>
<td>Shaft diameter ( D )</td>
</tr>
<tr>
<td>Radial clearance ( h_o )</td>
</tr>
<tr>
<td>No. of pockets ( n )</td>
</tr>
<tr>
<td>Axial flow land width ( a )</td>
</tr>
<tr>
<td>Concentric pressure ratio ( \beta )</td>
</tr>
<tr>
<td>Control device parameter ( z )</td>
</tr>
<tr>
<td>Inter-pocket land width ( b )</td>
</tr>
<tr>
<td>Circumferential flow factor ( \gamma_c )</td>
</tr>
<tr>
<td>Diametric clearance ( C_d )</td>
</tr>
</tbody>
</table>

A semi-active fluid bearing replaces the ball bearing and squeeze film damper at the bearing location No.1. The two bearings, Nos.2 and 5, are roller bearings and the stiffness model derived in section 3.2 is used.

For the LP spool with the semi-active fluid bearing, the whirl chart is obtained as shown in Figure 7, when the supply pressure equals to 1MPa. The temperature at the inlet/outlet of the bearing is assumed to be 200°C. Evaluated critical speeds are listed at Table 3.

<table>
<thead>
<tr>
<th>Table 3 Critical speeds for LP spool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical speeds (rpm)</td>
</tr>
<tr>
<td>CASE</td>
</tr>
<tr>
<td>Conventional</td>
</tr>
<tr>
<td>Proposed</td>
</tr>
</tbody>
</table>

Figure 8 shows the mode shapes at the rotational speed of 35,600rpm. Unbalance responses for the supply pressure of 1MPa, 3MPa, and 6MPa are shown in Figure...
9. The supply pressure levels are chosen within the allowable range for the conventional hydrostatic bearing. Application of the supply pressure level higher than 6MPa may be required to keep the vibration level below threshold value.

From these results, it is found that the maximum amplitude of unbalance responses can be improved by the application of semi-active fluid bearings and controlled not to exceed the predetermined threshold value.

5 Conclusions
Semi-active fluid bearing system is proposed and its dynamic characteristics are analyzed in order to effectively attenuate the unbalance response of a rotor-bearing system that utilizes a high-density fuel as its working fluid. Stiffness model for rolling element bearings is also derived considering the centrifugal force at high speed. The extensive simulations with a LP spool rotor prove that the unbalance response can be significantly reduced within a threshold limit by properly manipulating the fluid supply pressure.

Acknowledgments
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