Two-Stage Forecasting Using Change-Point Detection and Artificial Neural Networks for Stock Price Index*

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Abstract

The prediction of stock price index is a very difficult problem because of the complexity of the stock market data. It has been studied by a number of researchers since they strongly affect other economic and financial parameters. The movement of stock price index has a series of change points due to the strategies of institutional investors. This study presents a two-stage forecasting model of stock price index using change-point detection and artificial neural networks. The basic concept of this proposed model is to obtain intervals divided by change points, to identify them as change-point groups, and to use them in stock price index forecasting. First, the proposed model tries to detect successive change points in stock price index. Then, the model forecasts the change-point group with the backpropagation neural network (BPN). Finally, the model forecasts the output with BPN. This study then examines the predictability of the integrated neural network model for stock price index forecasting using change-point detection.

Keywords: Change-point detection; Backpropagation neural network; Stock Price Index; Pettitt test

1. Introduction

Stock market prediction is a matter of common interest among investors, speculators, and industries. Prior studies on stock market prediction using artificial neural networks (ANN) have been executed during the past decades. These studies used various types of ANN to predict the stock price index and the direction of its change. Previous work in stock market prediction has tended to use statistical techniques and AI techniques in isolation. However, an integrated approach, which makes full use of statistical approaches and AI techniques, offers the promise of better performance than each method alone [Chatfield, 1993].

The early days of these studies focused on estimating the level of return on stock price index. Kimoto et al. [1990], one of the earliest studies for stock market prediction using AI, employed several learning algorithms and prediction methods for the Tokyo stock exchange prices index (TOPIX) prediction system. Their system used modular neural networks to learn the relationships among various factors. Kamijo and Tanigawa [1990] used recurrent neural networks for analyzing candlestick charts. Ahmadi [1990] used backpropagation neural networks with the generalized delta rule to predict the stock market. They intended to test the *Arbitrage Pricing Theory* (APT) using ANN. Yoon and Swales [1991] also performed predictions using qualitative and quantitative data. Some researchers investigated the

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issue of predicting the stock index futures market. Trippi and DeSieno [1992] and Choi et al. [1995] predicted the daily direction of change in the S&P 500 index futures using ANN. Duke and Long [1993] executed the daily predictions of the German government bond futures using feedforward backpropagation neural networks.

Recent research tends to include novel factors and to hybridize several AI techniques. Hiemstra [1995] proposed fuzzy expert systems to predict stock market returns. He suggested that ANN and fuzzy logic could capture the complexities of functional mapping because they do not require the specification of the function to approximate. A more recent study of Kohara et al. [1997] incorporated prior knowledge to improve the performance of stock market prediction. Tsaih et al. [1998] integrated the rule-based technique and ANN to predict the direction of the S&P 500 stock index futures on a daily basis. In this study, we suggest the integrated neural network model based on the statistical change-point detection.

In general, macroeconomic time series data is known to have a series of change points since they are controlled by government's monetary policy [Mishkin, 1995; Oh and Han, 2000]. However, previous studies did not consider the structural break or the change-point in stock price index forecasting. The government takes intentional action to control the currency flow that has direct influence upon fundamental economic indices. For the stock price index, institutional investors play a very important role in determining its ups and downs since they are major investors in terms of marking and volume for trading stocks. They respond sensitively to such economic indices like stock price indexs, the consumer price index, anticipated inflation, etc. Therefore, we can conjecture that the movement of the stock price index also has a series of change points.

Based on these inherent characteristics in stock price index, this study suggests the change-point detection for stock price index forecasting. The proposed model consists of two stages. The first stage is to detect successive change points in the stock price index dataset, and to forecast the change-point group with BPN. The next stage is to forecast the output with BPN. This study then examines the predictability of the integrated neural network models for stock price index forecasting using change-point detection. To explore the predictability, we divided the stock price index data into the training data over one period and the testing data over the next period. The predictability of stock price index is examined using the metrics of the root mean squared error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE).

In section 2, we outline the development of change-point detection and its application to the financial economics. Section 3 describes the proposed integrated neural network model details. Section 4 reports the processes and the results of the case study. Finally, the concluding remarks are presented in Section 5.

2. Review of Change-Point Detection

2.1. Application of Change-Point Detection in the Financial Economics

Financial analysts and econometricians have frequently used piecewise-linear models which also include change-point models. They are known as models with structural breaks in economic literature. In these models, the parameters are assumed to shift — typically once — during a given sample period and the goal is to estimate the two sets of parameters as well as the change point or structural break.

This technique has been applied to macroeconomic time series. The first study in this field is conducted by Rappoport and Reichlin [1989] and Perron [1989, 1990]. From then on several statistics have been developed which work well in a change-point framework, all of which are considered in the context of breaking the trend variables [Banerjee et al., 1992; Christiano, 1992; Zivot and Andrews, 1992; Perron, 1995; Vogelsang and Perron, 1995]. In those cases where only a shift in the mean is present, the statistics proposed in the papers of Perron [1990] or Perron and Vogelsang [1992] stand out. However, some variables do not show just one change point. Rather, it is common for them to exhibit the presence of multiple change points. Thus, it may be necessary to introduce multiple change points in the specifications of the models. For example, Lumsdaine and Papell [1997] considered the presence of two or more change points in the trend variables. In this study, it is assumed that the Stock price indexs can have two or more change points as well as just one change point.

There are few artificial intelligence models to consider the change-point detection problems. Most of the previous research has a focus on the finding of unknown change points for the past, not the forecast for the future [Wolkenhauer and Edmunds, 1997; Li and Yu, 1999]. However, piecewise nonlinear model using structural change is known to significantly improve the performance for time series forecasting [Wasserman, 1989; Gorr, 1994; White, 1994; Oh and Han 2001]. Our model obtains intervals divided by change points in the training phase, identifies them as change-point groups in the training phase, and forecasts to which group each sample is assigned in the testing phase. It will be tested whether the introduction of change points to our model may improve the predictability of stock price index.

In this study, a series of change points will be detected by the Pettitt test, a nonparametric change-point detection method, since nonparametric statistical property is a suitable match for a neural network model that is a kind of nonparametric method (White, 1992). In addition, the Pettitt test is a kind of Mann-Whitney type statistic, which has remarkably stable distribution and provides a robust test of the change point resistant to outliers [Pettitt, 1980b]. In this point, the introduction of the Pettitt test is fairly appropriate to the analysis of chaotic stock price index data.

2.2. The Pettitt test

In this study, a series of change points will be detected by the Pettitt test [Pettitt, 1979; Pettitt, 1980a], a nonparametric change-point detection method, since nonparametric statistical property is a suitable match for a neural network model that is a kind of nonparametric method [Vostrikova, 1981]. In this point, the introduction of the Pettitt test is fairly appropriate for the analysis of chaotic time series data. The Pettitt test is explained as follows.

Consider a sequence of random variables $X_1, X_2, \ldots, X_T$, then the sequence is said to have a change-point at τ if X_t for $t = 1, 2, \ldots, \tau$ have a common distribution function $F_1(x)$ and X_t for $t = \tau+1, \tau+2, \ldots, T$ have a common distribution $F_2(x)$, and $F_1(x) \neq F_2(x)$. We consider the problem of testing the null hypothesis of *no-change*, $H_0: \tau = T$, against the alternative hypothesis of *change*, $H_A: 1 \le \tau < T$, using a non-parametric statistic.

An appealing non-parametric test to detect a change would be to use a version of the Mann-Whitney two-sample test. Let

$$D_{ij} = \mathrm{sgn}(X_i - X_j) \qquad (1)$$

where $\mathrm{sgn}(x) = 1$ if $x > 0$, 0 if $x = 0$, -1 if $x < 0$, then consider

$$U_{t,T} = \sum_{i=1}^{t} \sum_{j=t+1}^{T} D_{ij}\,. \qquad (2)$$

The statistic $U_{t,T}$ is equivalent to a Mann-Whitney statistic for testing that the two samples $X_1, \ldots, X_t$ and $X_{t+1}, \ldots, X_T$ come from the same population. The statistic $U_{t,T}$ is then considered for values of t with $1 \le t < T$. For the test of H_0: *no change* against H_A: *change*, we propose the use of the statistic

$$K_T = \max_{1 \le t < T} \left| U_{t,T} \right|. \qquad (3)$$

The limiting distribution of K_T is $\Pr \cong 2\exp\left\{-6k^2/(T^2 + T^3)\right\}$ for $T \to \infty$.

The Pettitt test detects a possible change point in the time sequence dataset. Once the change point is detected through the test, the dataset is divided into two intervals. The intervals before and after the change point form homogeneous groups which take heterogeneous characteristics from each other. This process becomes a fundamental part of the binary segmentation method explained in Section 3.

3. Description of the Proposed Model

Statistical techniques and neural network learning methods have been integrated to forecast the Stock price indices. The advantages of combining multiple techniques to yield synergism for discovery and prediction have been widely recognized [Gottman, 1981; Kaufman et al., 1991]. BPN is applied to our model since BPN has been used successfully in many applications such as classification, forecasting and pattern recognition [Patterson, 1996].

In this section, we discuss the architecture and the characteristics of our model to integrate the change-point detection and the BPN. Based on the Pettitt test, the proposed model consists of two stages: (1) the change-point-assisted group prediction (CPG) stage and (2) the output forecasting neural network (OFN) stage. The BPN is used as a classification tool in CPG and as a forecasting tool in OFN.

3.1. The CPG Stage: Construction and analysis on homogeneous groups

In this stage, we make the change-point-assisted neural network model for the intervals based on the Pettitt test, which is composed of two steps.

Step 1: Change-Point Detection

In the first step, we apply the Pettitt test to the stock price index at time t in the training phase. The Pettitt test mentioned in Section 2 is method for finding just one change point in time series data. Based on this method, multiple change points can be obtained under the binary segmentation method [Vostrikova, 1981]. With H_0 as in Section 2, under the alternative hypothesis we now assume that there are R changes in the parameters, where R is a known integer. The alternative can be formulated as

$$H_A^{(R)}: \text{there are integers } 1 < k_1 < k_2 < \ldots < k_R < n$$
$$\text{such that } \theta_1 = \cdots = \theta_{k_1} \neq \theta_{k_1+1} = \cdots$$
$$= \theta_{k_2} \neq \theta_{k_2+1} = \cdots = \theta_{k_R} \neq \theta_{k_R+1} = \cdots = \theta_n$$
$$\text{for the parameter } \theta\text{'s.}$$

We note that the test statistics under the null hypothesis will remain consistent against $H_A^{(R)}$ as well, despite the fact that they were derived under the assumption that $R = 1$. Without the loss of generality, we can deduce that the tests mentioned in Section 2 are extended to the form for "no change" against the "R changes" alternative $H_A^{(R)}$.

Vostrikova [1981] suggested a binary segmentation method as follows. First, use the change-point detection test. If H_0 is rejected, the find $\hat{k}_1$ that is the time where Equation (3) is satisfied. Next divide the random sample into two subsamples $\{X_i : 1 \le i \le \hat{k}_1\}$ and $\{X_i : \hat{k}_1 < i \le n\}$, and test both subsamples for further changes. One continues this segmentation procedure until no subsamples contain further change points. If exactly R changes are found, then one rejects H_0 in favor of H_A.

We, first of all, have to decide the number of change points. If just one change point is assumed to occur in a given dataset, only the first step will be performed. Otherwise, all of the three steps will be performed successively. This process plays a role of clustering which constructs groups as well as maintains the time sequence. In this point, the Step 1 is distinguished from other clustering methods such as the k-means nearest neighbor method and the hierarchical clustering method which classify data samples by the Euclidean distance between cases without considering the time sequence.

Step 2: Change-Point Group Detection with BPN

The significant intervals in the Step 1 are grouped to detect the regularities hidden in stock price index. Such groups represent a set of meaningful trends encompassing stock price index. Since those trends help to find regularity among the related output values more clearly, the neural network model can have a better ability of generalization for the unknown data. This is indeed a very useful point for sample design. In general, the error for forecasting may be reduced by making the subsampling units within groups homogeneous and the variation between groups heterogeneous [Cochran, 1977]. After the appropriate groups hidden in stock price index are detected by the Step 1, BPN is applied to the input data samples at time t with group outputs for $t+1$. In this sense, CPG is a model that is trained to find an appropriate group for each given sample.

3.2. The OFN stage: Forecast the output with BPN

OFN is built by applying the BPN model to each group. OFN is a mapping function between the input sample and the corresponding desired output (i.e. stock price index). Once OFN is built, then the sample can be used to forecast the stock price index.

4. Empirical Results

Research data used in this study comes from the weekly Korea Stock Price Index (KOSPI) from January 1987 to August 1996. The total number of samples includes 502 trading weeks. Input variables are 13 technical indicators. These variables are selected based on the review of domain experts. The description of input variables is presented in Table 1 [Achelis, 1995; Chang et al., 1996; Choi, 1995; Edwards and Magee, 1997; Gifford, 1995].

Table 1. Description of input variables (C_t : Closing price at time t)

Variable Name	Formula
MA6 (6-day moving average)	$\frac{1}{6}\times\sum_{i=1}^{6} C_{t-i+1}$
RSI (Relative strength index)	$100-\dfrac{100}{1+\dfrac{\sum_{i=1}^{6} Up_{t-i+1}/6}{\sum_{i=1}^{6} Dw_{t-i+1}/6}}$ where Up: Upward price change, Dw: Downward price change
OBV (On balance volume)	$OBV_{t-1}+kV_t$ $k=1$ if $C_t>C_{t-1}$, -1 if $C_t<C_{t-1}$, 0 if $C_t=C_{t-1}$. where V_t means the volume at time t
%K (Stochastic %K)	$\frac{C_t-LL_{t-6}}{HH_{t-6}-LL_{t-6}}\times 100$ where LL_n and HH_n mean lowest low and highest high in the last n days respectively.
%D (Stochastic %D)	$\dfrac{\sum_{i=1}^{6} \%K_{t-i+1}}{6}$
Momentum	C_t-C_{t-5}
Psychology	$\frac{1}{6}\times\sum_{i=1}^{6} UD_{t-i+1}$ ($UD_t=1$ if $C_t>C_{t-1}$, 0 if $C_t\le C_{t-1}$.)

Variable Name	Formula
Disparity6 (6-day disparity)	$\frac{C_t}{MA_6} \times 100$
Disparity25 (25-day disparity)	$\frac{C_t}{MA_{25}} \times 100$
ROC6 (6-day price rate-of-change)	$\frac{C_t}{C_{t-6}} \times 100$
VR (Volume ratio)	$\left[\left(\sum_{i=1}^{6} VU_{t-i+1} + \frac{1}{2}\sum_{i=1}^{6} VS_{t-i+1}\right) \Big/ \left(\sum_{i=1}^{6} VD_{t-i+1} + \sum_{i=1}^{6} VS_{t-i+1}\right)\right] \times 100$ where VU, VD and VS mean the volume of an up, down and steady day of the stock price index respectively.
MA25 (25-day moving avearage)	$\frac{1}{25} \times \sum_{i=1}^{25} C_{t-i+1}$
ROC25 (25-day price rate of change)	$\frac{C_t}{C_{t-25}} \times 100$

The data used in this study is weekly index on Korea Composite Stock Price Index (KOSPI) from January 1987 to August 1996. The training phase involves observations from January 1987 to December 1992 while the testing phase runs from January 1993 to August 1996. The stock price index data is presented in Figure 1. Figure 1 shows that the movement of stock price index highly fluctuates.

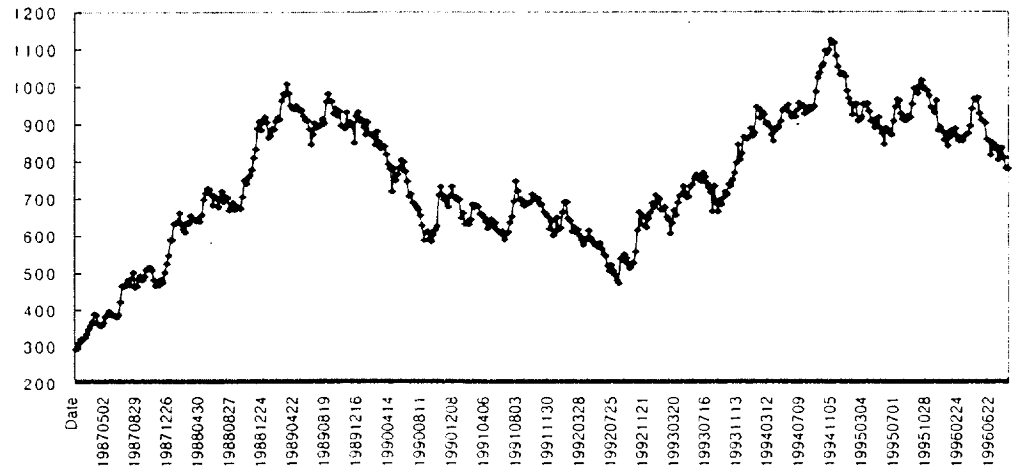

Figure 1. Weekly KOSPI data from January 1987 to August 1996

The Pettitt test is applied to the stock price index data. In this study, KOSPI data is varied from just one change-point to three change-points. For more change-points, the proposed model can be applied by the binary segmentation method. The study employs two neural network models. One model, labeled Pure_NN, involves four input variables at time t to generate a forecast for $t+1$. The second types, labeled Prop_NN(2) and Prop_NN(4), are the two-staged BPN model for 2 groups and 4 groups, respectively. The first step is the CPG stage that forecasts the change-point group while the next step is the OFN stage that forecasts the output. For validation, three learning models are also compared

To highlight the performance of the models, the actual values of stock price index and their predicted values are shown in Figure 2. The predicted values of the pure BPN model (i.e. Pure_NN) moves apart from the actual values in some intervals.

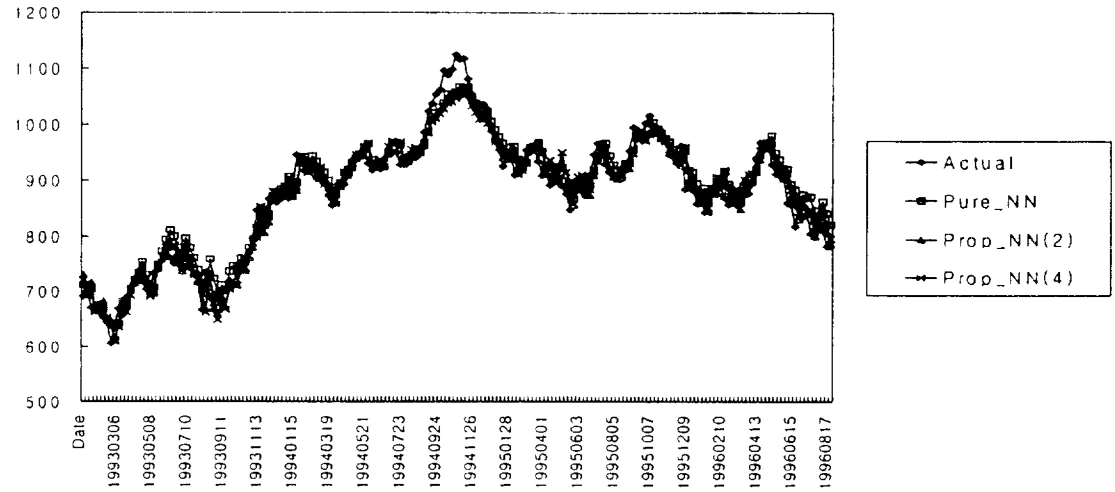

Figure 2. Actual vs predicted values due to the models for KOSPI

Numerical values for the performance metrics by the predictive model are given in Table 2. Figure 3 presents histograms of RMSE, MAE and MAPE for the forecast of each learning model. According to RMSE, MAE and MAPE, the outcomes indicate that the proposed neural network models are superior to the pure BPN model. In particular, the proposed model with 2 groups turns out to be the best model. This means that the number of change-point group can be an important factor to improve the performance.

Table 2.
Performance results of KOSPI forecasting based on RMSE, MAE and MAPE

Model	RMSE	MAE	MAPE (%)
Pure_NN	28.18	22.66	2.650
Prop_NN(2)	25.08	19.90	2.298
Prop_NN(4)	26.96	21.18	2.448

We use the pairwise t-test to examine whether the differences exist in the predicted values of models according to the absolute percentage error (APE). This metric is chosen since it is commonly used [Carbone and Armstrong, 1982] and is highly robust [Armstrong and Collopy, 1992; Makridakis, 1993]. Since the forecasts are not statistically independent and not always normally distributed, we compare the APEs of forecast using the pairwise t-test. Where sample sizes are reasonably large, this test is robust to the distribution of the data, to nonhomogeneity of variances, and to statistical dependence [Iman and Conover, 1983]. Table 3 shows t-values and p-values. The neural network models using change-point detection perform significantly better than the pure BPN model at a 1% or 10% significant level. Therefore, the proposed models are demonstrated to obtain improved performance using the change-point detection approach.

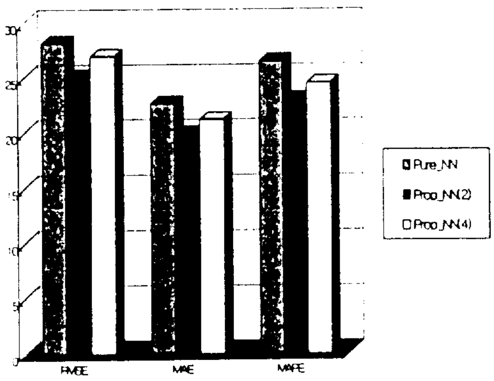

Figure 3. Histogram of RMSE, MAE and MAPE resulting from forecasts of KOSPI

In summary, the neural network models using the change-point detection turns out to have a high potential in stock price index forecasting. This is attributable to the fact that it categorizes the stock price index data into homogeneous groups and extracts regularities from each homogeneous group. Therefore, the neural network models using change-point detection can cope with the noise or irregularities more efficiently than the pure BPN model.

Table 3. Pairwise t-tests for the difference in residuals between the pure BPN model and the proposed neural network models for KOSPI based on the APE with the significance level in parentheses.

	Prop_NN(4)	Pure_NN
Prop_NN(2)	2.075 (0.039)**	2.969 (0.003)***
Prop_NN(4)		1.718 (0.087)*

***Significant at 1%; **Significant at 5%; *Significant at 10%

5. Concluding Remarks

This study has suggested change-point detection to support neural network models in stock price index forecasting. The basic concept of this proposed model is to obtain significant intervals divided by the change points, to identify them as change-point groups, and to use them in stock price index forecasting. We propose the integrated neural network model which consists of three stages. In the first stage, we conduct the nonparametric statistical test to construct the homogeneous groups. In the second stage, we apply BPN to forecast the change-point group. In the final stage, we also apply BPN to forecast the output.

The neural network models using change-point detection perform significantly better than the pure BPN model at a 1% or 10% significant level. These experimental results imply the change-point detection has a high potential to improve the performance. Our integrated neural network model is demonstrated to be a useful intelligent data analysis method with the concept of change-point detection. In conclusion, we have shown that the proposed model improves the predictability of stock price index significantly.

The proposed model has the promising possibility of improving the performance if further studies are to focus on the optimal decision of the number of change point and the various approaches in the construction of change-point groups. In the OFN stage, other intelligent techniques besides BPN can be used to forecast the output. In addition, the proposed model may be applied to other chaotic time series data such as stock market prediction and exchange rate prediction.

References

Achelis, S.B., *Technical analysis from A to Z*, Chicago: Probus Publishing, 1995.

Ahmadi, H., "Testability of the arbitrage pricing theory by neural networks," *Proceedings of the International Conference on Neural Networks* (pp.385-393), San Diego, California, 1990.

Armstrong, J.S. and F. Collopy, "Error measures for generalizing about forecasting methods: Empirical comparisons," *International Journal of Forecasting*, 8 (1992), 69-80.

Banergee, A., R. Lumsdaine and J. Stock, "Recursive and sequential tests of the unit root and trend break hypothesis: Theory and international evidence," *Journal of Business and Economic Statistics*, 10 (1992), 271-287.

Carbone, R., and J.S. Armstrong, "Evaluation of extrapolative forecasting methods: Results of academicians and practitioners," *Journal of Forecasting*, 1 (1982), 215-217.

Chang, J., Y. Jung,, K. Yeon, J. Jun, D. Shin and H. Kim, *Technical indicators and analysis methods*, Seoul: Jinritamgu Publishing, 1996.

Chatfield, C., "Neural networks: Forecasting breakthrough or passing fad?" *International*

Journal of Forecasting, 9 (1993), 1-3.
Choi, J., *Technical Indicators*. Seoul: Jinritamgu Publishing, 1995.
Christiano, L.J., "Searching for a break in GNP," *Journal of Business and Economic Statistics*, 10, 3 (1992). 237-250.
Cochran, W.G., *Sampling techniques*. New York: John Wiley & Sons, 1977.
Duke, L.S. and J.A. Long, "Neural network futures trading - A feasibility study," Society for Worldwide Interbank Financial Telecomm., *Adaptive intelligent systems* (pp.121-132), Amsterdam: Elsevier Science Publishers, 1993.
Edwards, R.D. and J. Magee, *Technical analysis of stock trends*, Chicago, Illinois: John Magee, 1997.
Gifford, E., *Investor's Guide to Technical Analysis: Predicting Price Action in the Markets*, London: Pitman Publishing, 1995.
Gorr, W., "Research respective on neural networks," *International Journal of Forecasting*, 10, 1 (1994), 1-4.
Gottman, J.M. *Time Series Analysis*. New York: Cambridge University Press, 1981.
Hiemstra, Y., "Modeling structured nonlinear knowledge to predict stock market returns," In R. R. Trippi, *Chaos & Nonlinear Dynamics in the Financial Markets: Theory, Evidence and Applications* (pp.163-175), Chicago, Illinois: Irwin, 1995.
Iman, R. and W.J. Conover, *Modern business statistics*, New York: Wiley, 1983.
Kamijo, K. and T. Tanigawa, T. "Stock price pattern recognition: A recurrent neural network approach," *Proceedings of the I. Joint Conference on Neural Networks* (pp.215-221), San Diego, California, 1990.
Kaufman, K.A., R.S. Michalski and L. Kerschberg, "Mining for knowledge in databases: Goals and general description of the INLEN system," In G. Piatetsky-Shapiro and W.J. Frawley (Eds.), *Knowledge discovery in databases* (pp. 449-462). Cambridge, MA: AAAI / MIT Press, 1991.
Kimoto, T., K. Asakawa, M. Yoda and M. Takeoka, "Stock market prediction system with modular neural network," *Proceedings of the I. Joint Conference on Neural Networks* (pp.1-6), San Diego, California, 1990.
Kohara, K., T. Ishikawa, Y. Fukuhara and Y. Nakamura, "Stock price prediction using prior knowledge and neural networks," *International Journal of Intelligent Systems in Accounting, Finance and Management*, 6, 1 (1997), 11-22.
Li, H.L. and J.R. Yu, "A piecewise regression analysis with automatic change-point detection," *Intelligent Data Analysis*, 3 (1999), 75-85.
Lumsdaine, R.L. and D.H. Papell, "Multiple trends and the unit root hypothesis," *The Review of Economics and Statistics*, 79 (1997), 212-218.
Makridakis, S, "Accuracy measures: Theoretical and practical concerns," *I. Journal of Forecasting*, 9 (1993), 527-529.
Oh, K.J. and I. Han, "An Intelligent Clustering Forecasting System based on Change-Point Detection and Artificial Neural Networks: Application to Financial Economics," Forthcoming in *Proceedings of the Thirty-Fourth Hawaii International Conference on System Sciences (HICSS)*, Hawaii, U.S.A, 2001.
Oh, K.J. and I. Han, "Using change-point detection to support artificial neural networks for stock price indexs forecasting," *Expert Systems with Applications*, 19, 2 (2000), 105-115.
Patterson, D.W., *Artificial neural networks*. New York: Prentice Hall, 1996.
Perron, P., "The great crash, the oil price shock, and the unit root hypothesis," *Econometrica*, 57 (1989), 1361-1402.
Perron, P., "Testing for a unit root in time series with a changing mean," *Journal of Besiness and Economic Statistics*, 8 (1990), 153-162.
Perron, P., Further evidence on breaking trend functions in macroeconomic variables, Manuscript, Université de Montreal, Canada, 1995.
Perron, P. and T. Vogelsang, "Nonstationarity and level shifts with an application to purchasing power parity," *Journal of Business and Economic Statistics*, 10 (1992), 301-320.
Pettitt, A.N., "A non-parametric approach to the change-point problem," *Applied Statistics*, 28, 2 (1979), 126-135.
Pettitt, A.N., "A simple cumulative sum type statistic for the change-point problem with zero-one observations," *Biometrika*, 67 (1980a), 79-84.
Pettitt, A.N., "Some results on estimating a change-point using nonparametric type statistics," *Journal of Statistical Computation and Simulation*, 11 (1980b), 261-272.
Rapport, P. and L. Reichlin, "Segmented trends and non-stationary time series," *The Economic Journal*, 99 (1989), 168-177.
Trippi, R.R. and D. DeSieno, "Trading equity index futures with a neural network," *The Journal of Portfolio Management*, 19 (1992), 27-33.
Tsaih, R., Y. Hsu and C.C. Lai, "Forecasting S&P 500 stock index futures with a hybrid AI system," *Decision Support Systems*, *23*, 2 (1998), 161-174.
Vogelsang, T. and P. Perron, Additional tests for a unit root allowing for a break in the trend function at an unknown time. Manuscript, Department of Economics, Ithaca, New York, 1995.
Vostrikova, L.J., "Detecting *disorder* in multi-dimensional random process, *Sov. Math. Dokl.*, 24 (1981), 55-59.
Wasserman, P.D. *Neural Computing: Theory and Practice*, New York: Van Nostrand Reinhold, 1989.
White, H., "Connectionist nonparametric regression: Multilayer feedforward networks can learn arbitrary mappings," In H. White (Ed.), *Artificial Neural Networks: Approximations and Learning Theory*. Oxford, UK: Blackwell, 1992.
White, H., "Can neural networks forecast in the big leagues? Comparing network forecasts to the pros?" *Proceedings of International Symposium of Forecasting*, Stockholm, Sweden, June, p.24,

1994.

Wolkenhauer, O. and J.M.Edmunds, "Possibilistic testing of distribution functions for change detection. *Intelligent Data Analysis*, 1 (1997), 119-127.

Yoon, Y. and G. Swales "Predicting stock price performance: A neural network approach," *Proceedings of the 24th Annual Hawaii International Conference on System Sciences* (pp.156-162), Hawaii, 1991.

Zivot, E. and D.W.K. Andrews, "Further evidence on the great crash, the oil-price shocks, and the unit-root hypothesis," *Journal of Business and Economic Statistics*, 10 (1992), 251-270.