Testing for Chaos in Stock Return Volatility: *An Application of R/S Analysis to the Korean Stock Market*

Cheolsoo Park and Ingoo Han

Graduate School of Management
Korea Advanced Institute of Science and Technology
Tel : 958-3673, E-Mail : cspark@msd.kaist.ac.kr

ABSTRACT

The stock market break of Oct. 1987 has drawn widespread attention and attempts at explanation. Chaos has captured the fancy of many financial economists. The attractiveness of chaos is its ability to generate large movements which appear to be random, with gerater frequency than linear models. This paper examines the sort of stock return volatility that could be generated by a chaotic market and explores a simple set of sufficient conditions for chaos to occur in the stock market. And we investigate whether there is long memory, or chaos, in the Korean stock market.

Key Words : Volatility, R/S Analysis, Chaos, Neural Network

I. INTRODUCTION

Recently, several studies have raised questions concerning the efficiency of stock markets. The stock market break of Oct. 1987 has drawn widespread attention and attempts at explanation. Chaos is a nonlinear determinstic process which 'looks' random. There is a very good description of chaos and its origins in the popular book by James Gleick (1987), entitled Chaos. Also Baumol and Benhabib (1989) gives a good survey of economic models which produce chaotic behavior.

And Summers(1986) argued that fact, although they have nothing to do with fundamentals, in fact play an important role in financial asset pricing, and Brock, Laknishok, and LeBaron(1992) found that abnormal returns could arise due to simple technical trading rules such as the moving average. In addition, Fama and French(1988) discovered that stock returns are positively correlated over long horizons and negatively correlated over short horizons.

Since then, many studies have attempted to investigate the long-tern behavior of stock prices, using artificial intelligence techniques for long memory analysis, chaotic dynamics, or neural networks which are not compatible with the efficient market hypothesis.

Among these, Peters (1994) employed R/S analysis in studying the long-term behavior of stock prices. R/S analysis was actually first introduced by a British hydrologist, H. E. Hurst, in 1950s to investigate rainfall along the Nile River, and it was later refined by Mandelbrot (1972) to study chaotic dynamics.

Peters argued that there is strong evidence of the existence of chaos in the stock returns from the Dow Jones Industrial Average and that efficient market theory cannot explain stock price movements as actually observed. However, Lo (1991), using so-called modified R/S analysis, found to the contrary that there is no evidence of chaos in the U.S. stock market, Cheung and Lai (1993, 1995), furthermore, found no evidence of chaos in either the gold markets or major stock markets around the world.

In this study, we seek to determine whether there is chaos in the Korean stock market, using so-called classical R/S analysis as developed by Hurst(1951), Mandelbrot (1972), and Peters (1994). We perform R/S analysis on the AR residuals of daily stock returns as well as on the original daily stock returns themselves in order to circumvent the bias problems which arise due to short-term dependence in daily returns. Unlike Peters, however, we find not strong evidence of the existence of chaos in the Korean stock market.

We also study chaos in the volatility of daily stock returns. Since we cannot actually observe volatility itself directly, we employ an GARCH(Generalized Autoregressive Conditional Heteroskedasticity) model and neural network model to estimate the daily stock returns. According to our R/S analysis on the volatility, there seems to be chaos in the volatility of daily stock returns.

This paper is organized as follows. In section II, an analysis for the descriptive statistics of stock returns from the Korean stock market is provided. R/S analysis is introduced in section III, and its application to the stock markets is presented. The results of the R/S analysis on daily stock returns are then provided in section IV and those on volatility in section V. Concluding remarks are in section VI.

II. DESCRIPTIVE STATISTICS OF STOCK RETURNS

In this section, we study the distributional properties of the nominal daily stock returns obtained from the KOSPI(Korea Stock Price Index). That is, The kurtosis of stock returns calculated based on the Dow Jones Industrial Index is much greater than that of a normal distribution, implying that stock returns give rise to a fat-tailed distribution. The existence of fat-tailed distributions is often cited as evidence of a long memory system generated by a nonlinear stochastic process.

Table 1 shows the mean, variance, skewness, and kurtosis of Korean daily stock returns from January 1980 to December 1996. It seems that the kurtosis of daily stock returns was much greater than that of a normal distribution as in the U.S. stock market. The skewness of Korean daily stock returns is 0.14, so we cannot reject the null hypothesis for normal distribution. The kurtosis, however, is 5.86, which is a sufficiently large figure to allow us to reject the null hypothesis. We can therefore say that Korean daily stock returns do not follow a normal distribution.

The daily limit on price movements in the Korea stock market is one of the reasons for the high kurtosis. Because stock prices can only move within a certain range each trading day, the daily stock returns naturally tend to be concentrated around the mean. Thus, the variance is small and the kurtosis is very high. What is more, the limit also causes the daily stock returns to be strongly autocorrelated. In cases where the price of a particular stock hits the upper bound of the limit during any given trading day, the price will probably continue to rise the next day. This kind of phenomenon applies to price declines of the same magnitude as well. Table 2 shows the Ljung-Box Q-statistics, and these permit us to see the degree of autocorrelation in daily stock returns. It is in fact found to be very strong, thus requiring us to apply the AR model to exclude the effects of short-term dependence. We need to perform R/S analysis not only on daily stock returns but also on the residuals from the AR model as well.

<Table 1> Stock Return Statistics

Mean	0.00043
Variance	0.00014
Skewness	0.14101
Kurtosis	5.86038

* The size of the sample population is 4,980.

<Table 2> Autocorrelation of Daily Stock Returns

	Ljng-Box Q-statistics	Significance level
Q(1)	92.134	0.000000
Q(50)	109.512	0.000000
Q(200)	326.186	0.000000
Q(600)	349.629	0.000001
Q(1500)	1435.127	0.000513

III. R/S ANALYSIS AND ITS APPLICATION

1. Basic Concepts of R/S Analysis

Mandelbort (1972) proposed the R/S statistic from rescaled range (or R/S) analysis, originally developed by Hurst(1951), as a tool for testing for chaos. The R/S statistic is simply the ratio of the range and standard deviation. In other words, the R/S statistic is the rescaled range of the partial sum of deviations normalized by the standard deviation.

For a simple example of R/S analysis, Let's begin with a time series, X_1, X_2, ...X_n, of n consecutive values. The mean value of the time series is defined as :

$$X_n = \frac{1}{n} \sum_{j=1}^{n} X_j \qquad (1)$$

The range is defined as:

$$R_n \equiv \underset{1 \le k \le n}{Max} \sum_{j=1}^{k} (X_j - \overline{X_n}) - \underset{1 \le k \le n}{Min} \sum_{j=1}^{k} (X_j - \overline{X_n}) \qquad (2)$$

The range is the difference between the maximum value and the minimum value out of the n cumulative sums of (X_j - $\overline{X_n}$). In this case, the maximum value will always be greater than zero, and the minimum value will always be less than zero. Hence, the range R_n, is always non-negative.

The rescaling factor, S_n is the sample standard deviation of X_j

$$S_n = \sqrt{\frac{1}{n} \sum_{j} (X_j - \overline{X_n})^2} \qquad (3)$$

The R/S statistic, $Q(=(R/S)_n)$ can be obtained by normalizing the range, R_n with the sample standard deviation, S_n

$$Q_n \equiv \frac{R_n}{S_n} \qquad (4)$$

In most stochastic models, including the random walk and ARMA models, the ratio of $\log(Q_n)$ to $\log(n)$ converges to 1/2. In other words, in stochastic models, the Hurst exponent, $\hat{H}$ is 1/2 in equation (5). In equation (5), c is constant, and $\hat{H}$ is the Hurst exponent. The Hurst exponent can be used to test the existence of chaos. Mandelbrot proved that there exists chaos in a time series when $\hat{H} \neq 1/2$.

$$\hat{H} = c \bullet \frac{\log(Q_n)}{\log(n)} \qquad (5)$$

Classical R/S Analysis can be used for any time series without and restriction, but the results it produces may be biased if there is short-term dependence.

2. Application to Stock Returns.

In this section, we apply R/S analysis to daily stock returns. We begin with a time series of stock prices of length L. Convert this into a time series of length $n = L - 1$ with daily stock returns r_i

$$r_i = \ln(P_{i+1} / P_i),\ i=1,2,3,.....,(L-1) \qquad (6)$$

where P_i is the stock price index in period i, Now, we divide the entire sample period into M contiguous subperiods of length K, such that $K=n/M$. In this case, the number of elements in the mth subperiod, S_m, is K. Then, $r_{k,m}$ is the kth element of the mth subperiod.

The sample means of each subperiod, S_m, μ_m are defined as:

$$\mu_m = \frac{1}{K} \sum_{k=1}^{K} r_{k,m} \qquad (7)$$

We need to calculate the range in order to obtain the rescaled range. In each subperiod, S_m, we calculate the cumulated sum of deviations Z as follows:

$$Z_{A,m} = \sum_{k=1}^{A} (r_{k,m} - m_m), \quad 1 \le A \le K. \qquad (8)$$

$Z_{A,m}$ is a cumulated sum up to the Ath stock return is subperiod S_m. In each subperiod, the number of cumulated sums is therefore K.

The range of daily stock returns in subperiod S_m can be obtained by finding the difference between the maximum value and the minimum value of $Z_{A,m}$.

$$R_{S_m} = max(Z_{a,m}) - min(Z_{a,m}), \; 1 \le A \le K. \qquad (9)$$

The rescaling factor, the standard deviation of subperiod S_m, is defined as:

$$s_{S_m} = \frac{\sqrt{\sum_{k=1}^{K} (r_{k,m} - \mu_m)^2}}{K} \qquad (10)$$

And the (R/S) value of subperiod S_m is defined as follows.

$$(R/S)_{S_m} = \frac{R_{S_m}}{s_{S_m}} \qquad (11)$$

We repeat this process for each subperiod and obtain the corresponding (R/S) values. Then, the mean of the (R/S) values can be calculated, and the rescaled range (R/S), which is of K magnitude in each subperiod, can be obtained.

$$Q_K \equiv (R/S)_K = \frac{1}{M} \sum_{m=1}^{M} (R_{S_m} / s_{S_m}) \qquad (12)$$

We can obtain a series of (R/S) statistics from the daily stock returns as we repeat the above process by changing the sample size. With this series of R/S statistics, we can then estimate the Hurst exponent, by which we can verify the existence of chaos, Consider of the following equation.

$$Q_n = c \bullet n^H \qquad (13)$$

In equation (13), c is a constant, and H is the Hurst exponent. H equals 0.5 in the case of Brownian motion. The R/S value, Q_n fluctuates as much as H times with the progress of time.

The Hurst exponent can be approximated by plotting the log(R/S)$_n$ versus the log(n) and solving for the slope through an ordinary least squares regression. Taking the log in equation (13), we have

$$log(Q_n) = log(c) + H \bullet log(n) \qquad (14)$$

If a time series had normal distribution, then H=0.5. For a time series which is chaotic, however, H is greater than 0.5. We can test whether H is 0.5 by a traditional hypothesis test on regression coefficients. It should be noted, though, that this is not a formal test to verify the existence of chaos: even though the null hypothesis. H=0.5, is rejected by tests on regression coefficients, this cannot be used as direct evidence that chaos does not exist.

IV. R/S TESTS ON KOREAN STOCK MARKET DATA

In order to obtain statistically significant results from R/S analysis, we need a large number of observations from a sufficiently long period of time. Because there is no formal criteria for determining just how many observations or how long a period is required, however, we use all of the available stock market data in Korea from January 1980 to December 1996, a period which includes more than 4,980 trading days.

1. R/S Test on Stock Returns

As shown in Table 8, The Hurst exponent estimated from the daily stock returns, r_p is 0.59732, and the standard deviation is 0.0042. According to the hypothesis test on regression coefficients, the Hurst exponent for r_t is statistically different from 0.5: it is significantly smaller than 0.7. The Hurst exponent estimated is based on data from the Dow Jones Industrial Average of the United States.

As noted in the previous section, since there is no formal test statistic for the Hurst exponent, the above result is not sufficient evidence on chaos. The fact that the Hurst exponent in the Korean stock market is much lower than that in the U.S. implies that chaos may not exist in Korean daily stock returns. In addition, considering the autocorrelation in Korean daily stock returns, it is possible that the Hurst exponent would be biased. which also implies low possibility of chaos in Korean stock prices.

Lo (1991) argued that short-term dependence can produce biased results in R/S analysis and therefore proposed modified R/S analysis. which is robust with regard to the short-term dependence. We apply Lo's method to the analysis of Korean stock returns.

Let V denote the limiting distribution of $Qn/\sqrt{n}$ when there is no short-term dependence. Then, the limiting distribution of $Qn/\sqrt{n}$ with short-term dependence can be expressed as follows.

$$\frac{1}{\sqrt{n}}\xi_n \Rightarrow \xi V \qquad (15)$$

where Q_n is an R/S value, n is the sample size, and ξ is factor which takes into account the effects of short-term dependence.

ξ can be obtained as follows. Let's first assume that the daily stock return, r_p follow stationary AR(1). That is,

$$r_t = \rho r_{t-1} + \eta_t \quad \eta_t \sim WN(0,\sigma_\eta^2), |\rho| \in (0,1) \qquad (16)$$

Then ξ is defined as follows.

$$\xi \equiv \sqrt{\frac{1+\rho}{1-\rho}} \qquad (17)$$

In this case, the limiting distribution of $Qn/\sqrt{n}$ is ξV.

Table 3 shows R/S values, $Qn/\sqrt{n}$. When the sample size n is 4,980, the R/S value, Q_n is estimated as 137.5421, and $Qn/\sqrt{n}$ is 1.9376. This itself may not be sufficient evidence to determine whether or not there is chaos in daily stock returns, however. If daily stock returns do not follow the AR(1) process, we cannot use ξ to correct problems caused by short-term dependence, and estimation of ξ will not be as simple as the above. Thus, without evidence that daily stock returns follow stationary AR(1), the above test does not confirm the existence of chaos.

<Table 3> $Qn/\sqrt{n}$ Statistics

N	R/S	$\frac{1}{\sqrt{N}} Q_n$
60	8.9624	1.3411
100	13.2928	1.3899
160	18.7485	1.4248
240	21.6006	1.4472
320	25.5278	1.4935
480	33.3421	1.4399
1200	61.3451	1.5837
2400	101.1231	1.8975
4980	137.5421	1.9376

2. R/S Test on Randomly Scrambled Stock Returns

Because of the short-term dependence in daily stock returns, the previous test do not offer any clear conclusion with regard to the existence of chaos in Korean daily stock returns. A random scrambling method, however, can be used as an indirect solution for problems arising due to short-term dependence. This method is based on the correlation dimension method developed by Schinkman and LeBaron(1980).

If a time series follows a chaotic process, the order in the time series would be important. Accordingly, if the time series was scrambled without order, the Hurst exponent would be lower than the original Hurst exponent, H. As shown in Table 8, the Hurst exponent, H_S, from randomly scrambled daily stock returns is 0.5476, which seems significantly lower than the original Hurst exponent(H=0.59732). Since the standard error of the regression coefficient is 0.00611, the null hypothesis of the two Hurst exponents using equal, $H=H_S$, is rejected at any significance level.

3. R/S Tests on AR(n) Residuals

As noted in the previous section, the daily returns observed on the KOSPI are significantly autocorrelated, so the Hurst exponent and R/S statistic estimated based on them might be biased. What we do in this section to get rid of this bias is, first, estimate the AR models for daily stock returns, and then, perform R/S tests on AR residuals.

$$r_t = a + \sum_{i=1}^{j} b_i r_{t-i} + e_t \qquad j = 1,2,3. \qquad (18)$$

The bias problem caused by autocorrelation in estimating the Hurst exponent can be resolved partially by using the residuals from the AR(n) model. There should be a much lower degree of autocorrelation in the residuals from AR(1) than in the original time series. Comparing the Hurst exponents from daily stock returns and from the AR(1) residuals, we can study the effects of short-term dependence on estimation of the Hurst exponents from daily stock returns and from the AR(1) residuals is 0.5872, which is slightly smaller than that for original daily stock returns.

<Table 4> **Performance of Stochastic Models**

Model	MSE	r^2
AR (1)	0.6521	0.5821
AR (2)	0.6368	0.5929
AR (3)	0.5586	0.6131
ARMA(3, 3)	0.6115	0.6068

Since the AR(1) residuals could be serially correlated, we perform the R/S test on the AR(3) residuals. We expect that we can get rid of any serial correlation in residuals with AR(3) and that test results will be different from the previous test results. As it turns out, however, we get the similar results as in the AR(1) case. The Hurst exponent is estimated as 0.6104, not greatly different compared to H, or $H_{AR(1)}$. This means that short-term dependence does not give rise to serious problems in estimating the Hurst exponents. Considering that the Hurst exponent for randomly scrambled stock returns is about 0.5476 and that there is no serial correlation in AR(3), this suggests that there might be chaos in daily stock returns. However, it seems that there are no nonperiodic cycles, which implies no chaos. Table 4 presents results for the best stochastic models

V. R/S TESTS ON THE VOLATILITY

Volatility - the variation in prices for a given time interval - is the most revealing indicator of market behavior. Not only does it give the size of current and recent price movements, it can also provide an indication of variables about which there little real information, such as who is present in the market and the volumes being traded. In addition, it can provide evidence of the persistence of trends in price movements.

Many studies have shown which periods of high and low volatility trend to persist in the markets longer than can be accounted for by the efficient market theory. The markets appear to have 'memory' - at least of volatility if not of the prices themselves.

Meanwhile, during periods of low volatility prices tend to follow trends for longer than expected, while trends persist for less time than expected during periods of high volatility. So volatility is an indicator of the persistence of price trends.

In this test we modeled the change in volatility between t and t+1 based on previous changes in volatility and based on the volatility at time t. We used to test the neural network, as well as to test conditional variances model. Figure 1 presents overall architecture of volatility estimation models.

1. Neural network Model

Artificial neural network (ANN) is a model that simulates a biological neural network. ANN have become mature tools for capital market analysis in finance. ANN have successfully been applied to areas such as stock price prediction, option trading, forecasting business distress, bond rating and security trading system.

The data set used for both ANN modeling consisted of 4980 frames of daily stock return observed on the KOSPI, of which the first 2500 were used for model identification and ANN training and the last 2480 for model validation and ANN testing.

The residual analysis indicated that AR(3) model was the most appropriate, suggesting the use of a feedforward ANN with 3 external inputs. The results are presented in Table 5, where the notation *x-y-z* stands for an ANN with *x* external inputs, *y* hidden layers and *z* output.

2. Conditional Variances Model

We employ the AR(3)-GARCH(1,1) model for the estimation of conditional variances of daily stock returns.

$$\begin{aligned} r_t &= \alpha + \sum_{j=1}^{3} \beta_j r_{t-j} + \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t) \\ h_t &= a + b\varepsilon_{t-1}^2 + ch_{t-1} \end{aligned} \qquad (19)$$

The results of estimation are presented in Table 6. We find that there is a significant ARCH effect in daily stock returns, so there is time variation in conditional variances.

To be sure, we must also determine the validity of the model, and we can do so by performing several specification tests and residual diagnostic tests. the first of these is the likelihood ration test on the null hypothesis. In this test, b=c=0 and the x^2 statistic under the null hypothesis is 915.55, hence the null hypothesis is rejected. As reported in Table 7, the Ljung-Box Q-statistics indicate no serial correlation in residuals normalized by conditional variances. Finally, the heteroskedasticity test by Domowitz and Hakkio (1985) and Bollerslev (1990) is performed, and the null hypothesis that the model is correctly specified cannot be rejected. Thus, the results from the various tests provide no evidence for incorrect model specification.

3. R/S Tests on Conditional Variances model and Neural Network Model

We need to generate the series of conditional variances and neural network model for the R/S tests on volatility. A series of conditional variances is generated according to equation (19).

The Hurst exponent estimated with this series is 0.7547, as shown in Table 8. This can be interpreted as an indication of the existence of chaos in the volatility of daily stock returns. Even considering the bias in the Hurst exponent caused by short-term dependence in conditional variances, it seems that we should therefore not reject the hypothesis that there exists chaos in the volatility of daily stock returns. This being the case, we can conclude that there indeed exists chaos in the volatility of daily stock returns.

VI. CONCLUSIONS

In this study, using so-called classical R/S analysis by Husrt(1951) and Mandelbrot(1972). We investigate whether there is long memory, or chaos, in the Korean stock market. This of course requires not only a large amount of data but a set of data which is observed over a fairly long period time. We therefore employed all of the available data in the Korean stock market (4,980) data points from January 1980 to December 1996.

Since the classical R/S analysis on the data with short-term dependence may produce significantly biased results, we also perform tests on the AR residuals as well as on the original stock returns. The Hurst exponents for daily stock returns and for AR residuals are approximately 0.6. The statistical tests on the regression coefficients reveal that the coefficients are significantly different from 0.5, which indicates that there is chaos in stock returns. However, we need to use other additional statistical test to verify the existence of this chaos because these are not formal statistical test, and no formal test in R/S analysis has yet been devised. We nevertheless find strong evidence of the existence of chaos in the volatility stock returns. Volatility cannot of course be observed directly, so we use the GARCH model to estimate it. the Hurst exponent for volatility is larger than 0.75, which strongly suggests the existence of chaos.

In summary, we cannot offer clear evidence of chaos in daily stock prices in Korea, but there does seem to be chaos in volatility. As mentioned earlier, however, we do not have any formal statistical tests in R/S analysis at our disposal, and the Hurst exponent might be biased due to autocorrelation. We, therefore, cannot state definitively whether there is chaos or not in daily stock returns and volatility. We need other statistical methods such as modified R/S analysis by Lo (1991) for more conclusive investigation and artificial intelligence techniques such as recurrent neural networks.

<Table 5> Perfomance of Feedforward ANN's

Architecture	MSE	r^2
3-3-1	0.5026	0.6875
3-6-1	0.5241	0.6531
4-4-1	0.6289	0.6394
8-8-1	0.5268	0.6687

<Table 6> Estimation of AR(3)-GARCH(1,1) for Stock Returns

	Coefficients	Standard errors	t-statistics	Significance level
α	2.4132e-004	1.3321e-004	2.18472	0.02978
β_1	0.0899	0.0123	6.40344	0.00000
β_2	-0.0425	0.0137	-2.96798	0.00211
β_3	0.0344	0.0121	2.77825	0.00325
A	1.01392e-005	1.160e-005	9.32522	0.00000
B	0.2216	0.0756	12.64622	0.00001
C	0.72653	0.0187	36.9345	0.00000

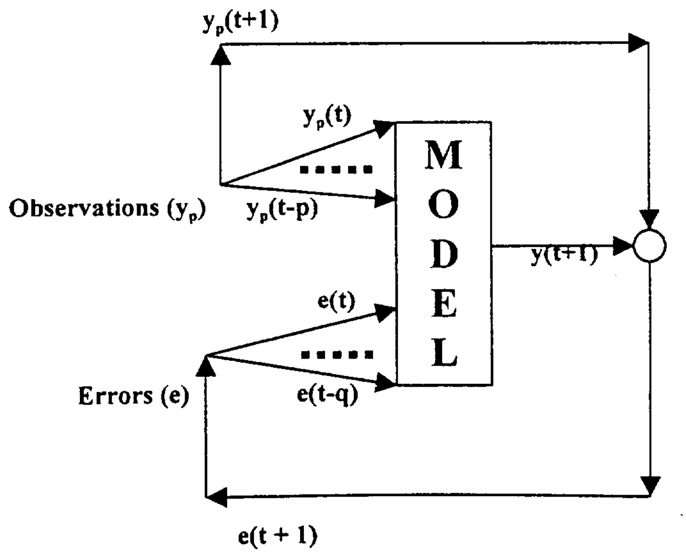
<Table 7> **Diagnostic chicking**

	Ljung-Box Q-statistic $\varepsilon_t / \sqrt{h_t}$	Ljung-Box Q-statistic ε_t^2 / h_t
Q(10)	15.8524 (0.10394)	4.0250 (0.94621)
Q(50)	38.1543 (0.00848*)	7.5803 (0.99430)
Q(100)	119.0813 (0.09370)	38.6362 (1.00000)
Q(200)	191.4497 (0.65553)	121.3761 (0.99999)
Q(1500)	1157.4514 (0.80658)	796.2582 (1.00000)

<Table 8> **Hurst Exponents**

Models	**Hurst Exponents**	**Standard Deviation**
Daily stock returns	0.59732	0.00425
Randomly scrambled daily stock returns (scrambled r_t)	0.5476	0.00611
AR(1) residuals	0.5872	0.00417
AR(3) residuals	0.6104	0.00398
ANN(3-3-1)	0.7138	0.00562
Conditional variances	0.7547	0.00936

<Fig. 1> The overall architecture



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