A Decision Model for Investment Timing Using Real Options Approach

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ABSTRACT

Real options approach has recently received growing attentions in the research on business and technology management. Despite the applicability of the approach, it gained much momentum in practice. This paper addresses a problem of deciding the optimal investment time as the uncertainty resolves over time. The optimal timing for investment decisions was formulated, and the characteristics of the optimal decision were identified.

KEYWORDS

Decision-Making; Optimal investment timing; Real options approach; Telecommunications

1. Introduction

Real options approach is based on the idea of evaluating financial options. In the financial options evaluation, purchase or sales decisions would be based on the volatility of the future stock prices [1, 2]. In the real world, the uncertain future is considered as a risk to the business decisions. However, as the volatility in the financial option is considerable valuable, the uncertainty in the real decision problems can be a good source of opportunity rather than a risk if it is appropriately assessed and managed [3,4,5].

Rapid technological development and ever-changing business environment caused a lot of uncertainties to deal with. Because of the uncertainty, gathering information before major commitment and making contingent decisions based on the understanding is critical. In that sense, delaying commitment of resources or *delaying* option could be a good strategy to take [4,5,6,7].

In this paper, real options approach was used to estimate the value of delaying option in the invest decision-making. Using the valuation approach, a model for deciding optimal investment timing was developed. Understanding delaying option and making contingent decisions based on the understanding would help business managers to rationally evaluate the business situations and make intelligent investment decisions.

Empirical study is also done for a mobile telecommunications company in Korea. Since the Personal Communications Service (PCS) was introduced in 1997, five mobile telecommunications companies, 2 cellular and 3 PCS companies, have been competing seriously. With their heavy subsidy for mobile phones and marketing efforts, the number of subscribers have increased dramatically over to 20M and even surpassed the number of wired telephone users in 1999.

In 1998, the Korean government allowed to provide the mobile telecommunications service through the resale of the existing companies' channel capacities. A Korean company had to decide whether they want to enter the market or not. In this paper, the company's strategy for market entry was evaluated while market is maturing. Additionally, the value of waiting for the information, especially potential growth of the market was analyzed.

2. A Decision Model for Investment Timing

If an investor is absolutely certain how the outcome of an investment would turn out in the near future, then s/he could easily decide what to do. It would be the case of depositing money on the bank or investing on the risk-free CD. However, with lot of uncertainties involved in the investment decisions, it maybe worthwhile to consider delaying decision and see how the immediate uncertainties are resolved. With the information, decisions can be make more intelligently. However, it is likely to incur waiting cost. Therefore, the decision to delay the commitment would be based on the tradeoff between the value of waiting vs. cost of waiting.

Let *R(t)* be the value of waiting until time *t*. It is assumed to be a non-decreasing and twice differentiable function of *t*. Let *C(t)* be the cost of waiting until time *t*. It is also assumed to be a non-decreasing and twice differentiable function of *t*.

With the above assumptions, the problem of finding the optimal time *t** can be represented as follows

$$\text{Max}_t P(t) = Max_t \left[R(t) - C(t) \right]$$

, where P(t) is the net profit if decision is delayed until time *t*.

Let MVW and MCW be a marginal value of waiting and marginal cost of waiting, respectively. Marginal value of waiting (MVW) would come from the fact that decisions can be made depending on the uncertainty resolutions over time. Marginal cost of waiting (MCW) happens when cost is incurred when decision is delayed. For example, it could happen when potential customers are lost to the competitors. An investor would invest now if MVW were less than MCW. Otherwis e he would rather wait and see what happens.

The optimal decision time for profit maximization can be found when the first derivative of *P(t)* equals zero or the first derivatives of *R(t)* and *C(t)* are equal. That is,

$$\frac{\partial P(t)}{\partial t} = \frac{\partial R(t)}{\partial t} - \frac{\partial R(t)}{\partial t} = MVW_t - MCW_t = 0$$

Furthermore, for P(t) to have a single maximum point over $t \in [0, \quad T]$, the second-order condition would be required. Because it is a constrained optimization over $t \in [0, \quad T]$, the optimal time *t** can be can be characterized in the following two cases.

Case 1*:* *t** is interior point *of* [0, T], where $t^* = \underset{0 < t < T}{Arg} \left[\frac{\partial P(t)}{\partial t} = \frac{\partial U(t)}{\partial t} - \frac{\partial D(t)}{\partial t} = 0 \right]$.

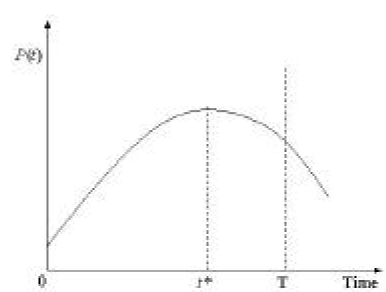

Figure 1: Interior Point Case

Case 2: t^* is a boundary point

$$\text{If} \quad \frac{\partial P(t)}{\partial t} > 0 \quad \text{then} \quad t^* = T.$$

$$\text{If} \quad \frac{\partial P(t)}{\partial t} < 0 \quad \text{then} \quad t^* = 0.$$

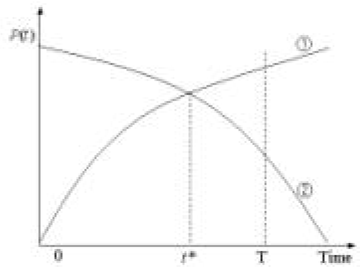

Figure 2: Boundary Point Case

When *MVW* and *MCW* are linear or constant, the optimal investment time can be easily be found by comparing the derivatives of them. For example, if the derivative of *MVW* is zero and that of *MCW* is positive, then t^* equals to 0. On the other hand, if the derivative of *MCW* is zero, then, the t^* equals to T, the expiration date of the investment opportunity.

However, more interesting case is when the derivative of *MVW* is negative and the derivative of *MCW* is positive. That represents the real market situation well and shown in Figure 3. The only time the optimal investment time can be an interior point is when *MVW* and *MCW* show the relationship shown in Figure 3-1. But, if the *MVW* and *MCW* increase as the Figure 3-2 and 3-3, then the optimal investment time becomes zero or T.

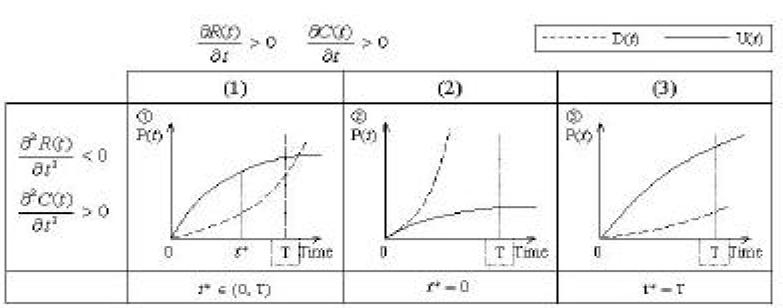

Figure 3: Characteristics of the optimal time

3. An Empirical Study

In the empirical study, a Korean mobile telecommunications company was chosen and their data were used to estimate the optimum market entry time. The company was trying to resale the channels of other Korean mobile telecommunications company. Their business plan was that they buy the channels from other company and bundle it with their existing telecommunications services. Their plan may make sense because customers want a bundled service with a single bill and single customer contact.

Forecast of Subscribers in Korean market

In order to estimate future subscribers for wireless telecommunications service, several assumptions are made. One of the assumptions was that total number of subscribers is to follow an *S*-shape curve which approaches to the ceiling value over time. Based on the assumptions, number of subscribers until the end of year 2002 was estimated using non-linear regression method.

The Result is as follows.

Table 1: Estimated number of subscribers by the regression model (Unit: Thousands)

	'99. Q3	'99. Q4	'00. Q1	'00. Q2	'00. Q3	'00. Q4	'01. Q1
Total subscriber	17,753	19,222	20,469	21,496	22,320	22,969	23,472
	'01. Q2	'01. Q3	'01. Q4	'02. Q1	'02. Q2	'02. Q3	'02. Q4
Total subscriber	23,856	24,147	24,366	25,511	25,638	25,732	25,802

Expected market size for the company

A survey was done for 1,500 people to understand their intentions to subscribe. The reference probability that would decide whether a non-subscriber will subscribe for a mobile telephone service was estimated to be $p^* = 0.3549$.

If $P_i \geq p^* = 0.3549$, then a subscription is expected, else no subscription is expected.

Based on the probability, the company is expected to acquire the following subscribers, if the company starts a wireless resale service in Oct. 1999.

Table 2: Estimation of the subscribers (Churn rate = 0.03/month) (Unit: actual)

	'99.Q3	'99.Q4	'00.Q1	'00.Q2	'00.Q3	'00.Q4	'01.Q1
Accumulated	134,557	181,533	236,768	297,191	358,291	415,353	464,820
New	134,557	46,976	55,235	60,423	61,100	57,062	49,467
Average	131,866	176,087	229,665	288,275	347,542	402,892	450,875
	'01.Q2	'01.Q3	'01.Q4	'02.Q1	'02.Q2	'02.Q3	'02.Q4
Accumulated	505,010	535,977	558,878	575,303	586,828	594,790	600,231
New	40,190	30,967	22,901	16,425	11,525	7,962	5,441
Average	489,860	519,898	542,112	558,044	569,223	576,946	582,224

Estimation of the number of subscribers caused by investment delay

The subscription probability for a non-user is assumed to be constant over time. That is,

$$\frac{f(t)}{1-F(t)} = q$$

, where *f(t)* is the probability of a customer would subscribe at time *t*, and F(t) is the probability of a customer to have subscribed by the time *t*.

Then, the company's potential subscriber will be

$$K_t = \frac{\text{The size of latent market at t}}{\text{Total latent market size}} \times K_0$$

$$= \frac{D_t - D_0}{MaximumPotential - D_0} \times K_0$$

Max Subscriber : Ceiling of the total number of subscriber

D(t) : Total number of subscriber at t

K_0 : Potential subscriber, if invested in Oct. 1999 (No delaying of decision)

Table 3: Change of subscribers caused by decision delay (Unit: thousands, Market Ceiling=30,467)

	’99.Q3	’99.Q4	’00.Q1	’00.Q2	’00.Q3	’00.Q4	’01.Q1
K_t	582	354	278	215	164	124	94
	’01.Q2	’01.Q3	’01.Q4	’02.Q1	’02.Q2	’02.Q3	’02.Q4
K_t	70	52	38	28	17	13	10

Estimation of Volatility

One way of estimating volatility or uncertainty of the investment project would be from the past stock prices. However, the company in study has no past history of providing any wireless business. Therefore, past 2 years stock price for the other telecommunications company data was used instead.

Let n : Number of data

S_i : Stock price at the end of i-*th* period ($i = 0, 1, \ldots, n$)

t : The number of period in a quarter

$\bar{u}$: The average value of u_i

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}\ , \qquad \ln\frac{S_T}{S} \approx \Phi\left[(\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\right]$$

According to this, the standard deviation of u_i is $\sigma\sqrt{T}$. Therefore, s is the estimate of $\sigma\sqrt{T}$ and s^* can be used as the estimate of s itself. s^* can be found as

$$s^* = \frac{s}{\sqrt{\tau}} \qquad (\text{S.E} = s^*/\sqrt{2n})$$

Average of u_i = 0.01419

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} = 0.0818$$

With that, τ and s* were estimated to be τ=1/13 and $s^* = s/\sqrt{t}$ =0.294864.

4. Evaluation of MVW & MCW

Black-Scholes model assumes the market conditions are constant until the option’s expiration date. Therefore, this study uses the cost of maintaining the subscriber level at t=0 (Oct. 1999), as the cost of delaying investment decision.

Here $e^{\sqrt{t}}$ was used as a customer retention weight to keep the latent subscriber from leaving to other service providers.

The value of the real option can be estimated by using the Black-Scholes model. The resulting value is shown in Figure 4. As shown in the figure, the cost increases far more rapidly than the value of the real option. This difference can be more easily recognized from Figure 5, the net profit. This figure shows that the company loses their value if investment decision is delayed.

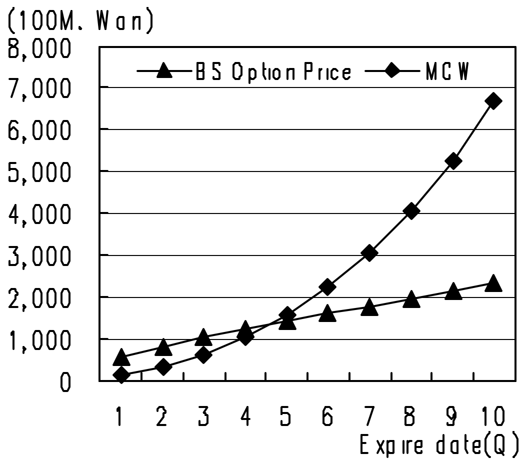

Figure 4: B-S Option value & Cost

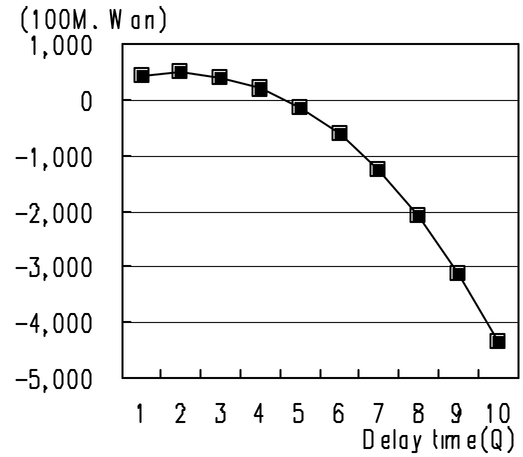

Figure 5: Net profit

Considering the fact that the Korean mobile telecommuncations market has already grown close to its saturation stage by the time the company was considering to start the wireless resale business, it might be useless to argue that it is better to invest now than wait. Too see the difference in market conditions, it is proposed to estimate investment timing assuming that the market is at its early stage. Figure 6 represents the market growth and Point B represents current situation (1998.8). Point A in Figure 6 is choosen as a suitable point for the earlier stage of the market growth, from where the value and the cost of waiting is calculated. The results are as follows.

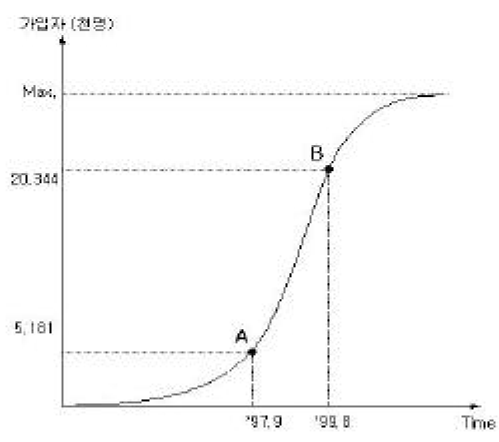

Figure 6: Market penetration

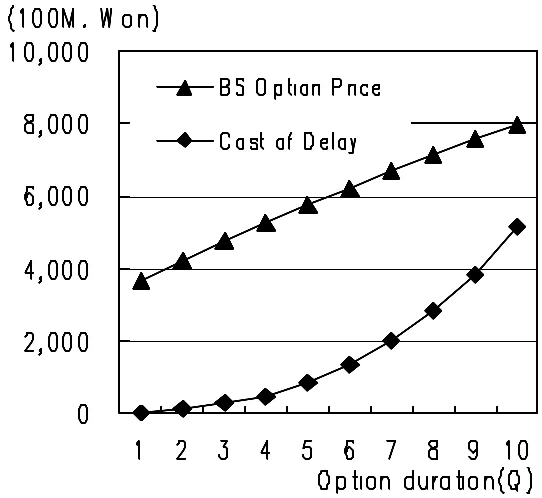

Figure 7: The *MVW* & the *MCW*(1997.10)

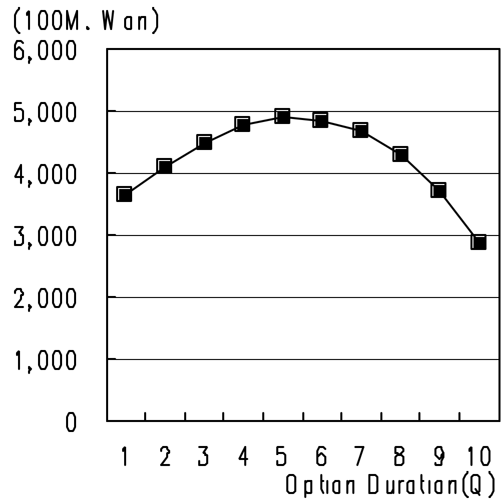

Figure 8: Net profit (1997.10)

From Figure 8, the optimal investment timing is to wait 5 quarters to observe the change in market conditions before making a decision to enter the market. In the early time, the cost of waiting is not substantial enough to make a hectic start for the service.

5. CONCLUSIONS

This paper addressed a problem of deciding the optimal investment time as the uncertainty resolves over time. The optimal timing for investment decisions was formulated, and the characteristics of the optimal investment time were identified. The investment time model was applied to a Korean company looking for the entry to the resale market for the personal mobile communications service. The analysis showed that it is better to wait about before committing resources. Because there is not much to lose for already saturating market, it seems to be better to wait and see how the great uncertainty regarding the potential market size of the market is later resolved. However, there may be some option value to enter the market early that was not considered in this paper. The customer base established by entering the market early may prove to be a good investment serving as a stepping-stone for the next generation service. That would be another research area to pursue.

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