BLIND CHANNEL TRACKING FOR LONG-CODE WCDMA WITH LINEAR INTERPOLATION MODEL

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ABSTRACT

A new technique for tracking of fast fading channels in long code CDMA is proposed exploiting multipath diversity of mobile channels. Based on a linear interpolation model, the proposed method blindly estimates the channel coefficients at the selected positions within a slot up to a scale factor and tracks the time-varying channel using an interpolation. The unknown scale factor can be resolved using only one pilot symbol which increases the bandwidth efficiency. The proposed method can be implemented using several front-ends including the conventional matched filter, decorrelator, and regularized decorrelator. A new identifiability condition is established and the performance of the proposed method is assessed through the mean square error and bit error rate.

1. INTRODUCTION

Third generation code division multiple access (CDMA) systems such as WCDMA adopted coherent detection with pilot symbols in the reverse link to increase the system capacity. Channel estimation plays a important role in coherent schemes. To track a time-varying channel in a fading environment, usually a pilot channel is superimposed on the data streams in a code-division multiplexing (CDM) or pilot symbols are inserted periodically in a time-division multiplexing (TDM) [1]. Although the CDM pilot channel is more attractive for channel tracking, it usually increases the peak-to-average power ratio of the transmitting signal, which reduces the power efficiency of the mobile station.

Several channel tracking methods have been proposed based on time multiplexed pilot symbols and interpolation techniques, e.g., [2] [3]. These methods utilize only pilot symbols over multiple slots, which requires pilot symbols with high SNR and a frequent insertion of pilot symbols. Others considered blind or semi-blind approaches to address this problem [4] [5]. However, they are mostly based on the block fading assumption with blocks of a large size which is not suitable for fast fading where channel changes rapidly within a block (usually the time interval between pilot symbols).

In this paper, we present a new channel tracking technique which utilizes multipath diversity and tracks fast fading channels effectively with one pilot symbol within a block which can be significantly larger than the channel coherence time. Based on a linear interpolation model, the proposed method blindly estimates the channel coefficients at selected estimating points within a block up to a unknown scale factor and obtained the channel for the whole slot with the estimate at the selected positions. The proposed method can be implemented with the conventional matched filter where the spreading gain is high and the multiaccess interference is not severe or with a decorrelating front-end which can be efficiently implemented with a state-space inversion technique with a comparable amount of complexity with short spreading code systems [6].

The paper is organized as follows. The data model of a CDMA system is described in Section 2. A blind channel tracking method based on multipath diversity and a linear interpolation channel model is proposed in Section 3. In Section 4, the performance of the proposed method is assessed by Monte Carlo simulations and compared with the estimation with block fading model.

2. DATA MODEL

We consider an asynchronous CDMA system with K users with long spreading sequences of spreading gain G. We assume that transmissions are slotted with size of M symbols.

As illustrated in Fig.1, user i data stream $s_i(t)$ is spread with the long scrambling code $c_i(t)$ and scaled by A_i and transmitted though a multipath time-varying channel. We assume that the channel of a particular user i consists of L independent multipaths each of which is a bandlimited deterministic waveform with bandwidth f_D (the maximum Doppler frequency). To deal with fast fading, we let the multipath coefficients vary from symbol to symbol while remaining constant over one symbol period T_s. The delays of

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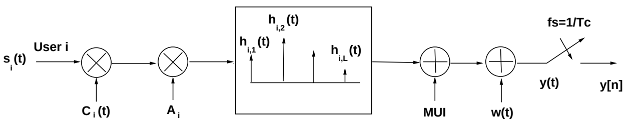

Fig. 1. System model

dominant multipaths are assumed to be invariant within one slot[1]. We assume that all users are chiprate synchronized[2], that is, the delays between paths are multiples of chip interval T_c. Specifically, the continuous-time time-varying channel impulse response of user i is given as

$$h_i(t,\tau) = \sum_{l=1}^{L} h_{il}(t)\delta(\tau - lT_c - d_iT_c), \qquad (1)$$

where $h_{il}(t)$ is the channel waveform for the lth path, d_i the delay of user i relative to the slot reference.

Consider the received signal corresponding to user i first. The signal $y_i(t)$ is passed through a chip waveform matched filter and sampled synchronously at the chiprate. Since the channel is linear and has a finite impulse response, the noiseless chiprate sample $y_i[n]$ at the nth chip interval is expressed as an output of a time-varying response

$$y_i[n] = \sum_{l=1}^{L} p_i[n-l]h_i[n,l], \qquad (2)$$

$$p_i[n] \triangleq c_i[n]s_i[\lfloor \frac{n}{G} \rfloor], \qquad (3)$$

where $h_i[n,l]$ is the channel coefficient of the lth path at the nth chip interval, $\{c_i[n]\}_{n\geq 1}$ the chiprate spreading sequence, and $\{s_i[m]\}_{m\geq 1}$ is the symbol sequence for user i. The received noiseless signal vector $\mathbf{y}_{im}$ corresponding to the mth symbol $s_{im} (\triangleq s_i[m])$ is given in a matrix form as

$$\mathbf{y}_{im} = \mathbf{T}_{im}\mathbf{h}_{im}s_{im}, \qquad (4)$$

where $\mathbf{T}_{im}$ is the Toeplitz matrix whose first column is made of $(m-1)G + d_i$ zeros followed by vector $\mathbf{c}_{im} = [c_i[mG+1], \ldots, c_i[(m+1)G]]^T$ and additional zeros to make the size of $\mathbf{y}_{im}$ the total number of chips of the entire M-symbol slot. (See Fig. 2.) The multipath channel vector at the m the symbol interval $\mathbf{h}_{im}$ is defined as

$$\mathbf{h}_{im} \triangleq \begin{bmatrix} h_{im}^{(1)} \\ h_{im}^{(2)} \\ \vdots \\ h_{im}^{(L)} \end{bmatrix}, \quad h_{im}^{(l)} \triangleq h_{il}(mT_s), \; l = 1, \ldots, L. \qquad (5)$$

[1] Fast fading results mainly from the phase variation of carrier not from the delay changes

[2] With Nyquist sampling, the chiprate synchronism is not required. In that case, the derivation here will corresponds to the even (or odd) subsequence with a $T_c/2$ sampling period.

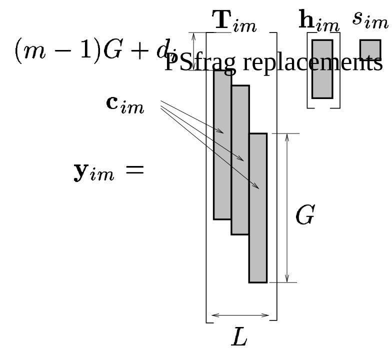

Fig. 2. Code matrix for one symbol

For user i, the total received noiseless signal is given by

$$\begin{aligned} \mathbf{y}_i &= \sum_{m=1}^{M} \mathbf{T}_{im}\mathbf{h}_{im}s_{im}, \\ &= \mathbf{T}_i \text{diag}(\mathbf{h}_{i1}, \ldots, \mathbf{h}_{iM})\mathbf{s}_i, \end{aligned} \qquad (6)$$

where $\mathbf{s}_i = [s_{i1} \ldots s_{iM}]^T$ and $\mathbf{T}_i \triangleq [\mathbf{T}_{i1}, \cdots, \mathbf{T}_{iM}]$ which has a special structure of sparse block Toeplitz form which is exploited for efficient implementation of matrix inversion [6]. Including $K'(\leq K)$ dominant users, we have the complete data model as

$$\begin{aligned} \mathbf{y} &= \sum_{i=1}^{K'} \mathbf{T}_i \text{diag}(\mathbf{h}_{i1}, \ldots, \mathbf{h}_{iM})\mathbf{s}_i + \mathbf{w}, \\ &= \mathbf{THs} + \mathbf{w}, \end{aligned} \qquad (7)$$

where the overall code matrix $\mathbf{T} = [\mathbf{T}_1, \ldots, \mathbf{T}_{K'})]$, $\mathbf{H} = \text{diag}(\mathbf{h}_{11}, \ldots, \mathbf{h}_{1M}, \mathbf{h}_{21}, \ldots, \mathbf{h}_{2M}, \ldots \mathbf{h}_{K'M})$, and $\mathbf{w}$ is additive Gaussian noise which includes the signals of users not modeled in $\mathbf{T}$. We assume the following

A1: The code matrix $\mathbf{T}$ is known and has full column rank.

A2: The noise vector is circularly symmetric complex Gaussian $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ with possibly unknown σ^2.

Assumption (A1) implies that the receiver knows the codes and delays of users of interest. This assumption is usually valid for the uplink. Assumption of full column rank is easily satisfied with proper choice of K'. In the case of long spreading gain, we can model a single user only regarding all other user signals as additive noise.

3. FAST FADING CHANNEL ESTIMATION

3.1. Linear interpolation model

The channel over a time slot can be modeled as a sum of weighted basis or an interpolation of samples at several positions. We consider a linear interpolation channel model under the deterministic parameter assumption. Since the number of samples within a slot is limited, we focus on the N-sample time domain approach which includes a broad range of interpolation techniques such as piecewise linear, polynomial, ideal low pass interpolations. A similar modeling of a time-varying channel with a truncated basis was proposed in [8].

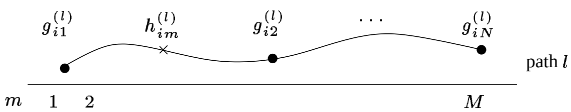

Fig. 3. N sample interpolation model

We assume that the channel at an arbitrary symbol interval within a slot is a linear combination of channels at selected estimating points which are not necessarily the pilot positions. Consider the lth multipath of user i's channel. The channel $h_{im}^{(l)}$ at the mth symbol interval is modeled as

$$h_{im}^{(l)} = \alpha_{m1} g_{i1}^{(l)} + \cdots + \alpha_{mN} g_{iN}^{(l)},\ m = 1, \ldots, M, \quad (8)$$

where α_{mn} is the interpolation coefficient for symbol m and sample n, and $g_{in}^{(l)}$ is the channel coefficient at the nth estimation position. Stacking all the multipaths corresponding to the same symbol and user gives

$$\begin{aligned} \mathbf{h}_{im} &= \begin{bmatrix} g_{i1}^{(1)} & g_{i2}^{(1)} & \cdots & g_{iN}^{(1)} \\ g_{i1}^{(2)} & g_{i2}^{(2)} & \cdots & g_{iN}^{(2)} \\ \vdots & & & \vdots \\ g_{i1}^{(L)} & g_{i2}^{(L)} & \cdots & g_{iN}^{(L)} \end{bmatrix} \begin{bmatrix} \alpha_{m1} \\ \alpha_{m2} \\ \vdots \\ \alpha_{mN} \end{bmatrix} \\ &\triangleq \mathbf{G}_i \mathbf{a}_m, \end{aligned} \quad (9)$$

where $\mathbf{G}_i$ and $\mathbf{a}_m$ are defined correspondingly. The interpolation coefficient vector $\mathbf{a}_m$ is determined by the selected interpolation method and $\mathbf{G}_i$ is the unknown parameter matrix which contains the channels at the selected estimating points. Notice that $\mathbf{G}_i$ is invariant within a slot. The use of interpolation model converts the problem of a time-varying channel estimation to the estimation of invariant parameters within a slot. We further assume the following

A3: The matrix $\mathbf{G}$ has full column rank.

Assumption (A3) implies that the number of multipaths is larger than or equal to that of the estimating points. Due to the abundance of the multipaths in mobile channels, this assumption can be satisfied with the proper selection of the number of estimating points N. If we select two end points of slot as the estimating points with pilot symbol placement at each end, the model can be considered as the common interpolation based on the pilot symbols. When N is one, the interpolation model reduces to the general block fading model. The assumption also requires that N is designed considering the fading rate so that the channel vectors at different sampling points are linearly independent almost surely.

3.2. Blind multiuser channel estimation algorithm

We propose a blind channel estimation based on a linear interpolation model exploiting multipath diversity of channel. We assume that the channel and symbols are deterministic parameters.

3.2.1. Front-End Processing

The multiuser signals are separated by various front-ends such as the conventional matched filter, decorrelator, or regularized decorrelator. Although the conventional matched filter cannot separate multiuser signals perfectly, it works satisfactorily with large spreading factors. For the systems with small spreading gains, we can use more advanced front-end processing like decorrelator. For long code systems, the computational complexity of the decorrelator is prohibitive. However, the required matrix inversion can be efficiently implemented by an algorithm using the state-space technique[6]. The complexity of the proposed inversion technique is proportional to the product of the slot length and the square of the number of users($\sim GM(K')^2$).

The output of the front end is given by

$$\mathbf{z} = \mathbf{T}^H \mathbf{y},\ \ \mathbf{z} = \mathbf{T}^\dagger \mathbf{y},\ \ \mathbf{z} = (\mathbf{T}^H\mathbf{T} + \sigma^2 \mathbf{I})^{-1}\mathbf{y}, \quad (10)$$

where $(\cdot)^H$ and$(\cdot)^\dagger$ represent the conjugate transpose and pseudo inverse, respectively. Segment the front-end output $\mathbf{z}$ according to the symbol and user. Let $\mathbf{z}_{im}$ be the subvector of length L corresponding to mth symbol of user i. Due to the diagonal structure of $\mathbf{H}$ in (7), the vector $\mathbf{z}_{im}$ is given by

$$\begin{aligned} \mathbf{z}_{im} &= \mathbf{h}_{im} s_{im} + \mathbf{n}_{im}, \\ &= \mathbf{G}_i \mathbf{a}_m s_{im} + \mathbf{n}_{im}, \quad m = 1, \ldots, M, \end{aligned} \quad (11)$$

where the noise $\mathbf{n}_{im}$ contains the additive noise. It also contains the other user interference in the case of the conventional matched filter.

3.2.2. Subspace Identification with Cross Referencing

Consider the noise free case first. With the deterministic assumption on $\mathbf{G}_i$, the column space of $\mathbf{G}_i$ is obtained by singular value decomposition (SVD)

$$\mathbf{Z}_i \triangleq [\mathbf{z}_{i1}, \mathbf{z}_{i2}, \ldots, \mathbf{z}_{iM}] = \mathbf{U}_i \Sigma_i \mathbf{V}_i^H. \quad (12)$$

The parameter matrix $\mathbf{G}_i$ is given by

$$\mathbf{G}_i = \mathbf{U}_i \mathbf{S}_i, \quad (13)$$

where $\mathbf{S}_i$ is a invertible $N \times N$ unknown square matrix from the assumption A(3). Projecting $\mathbf{z}_{im}$ to the column space of $\mathbf{U}_i$, we have the following system of equations

$$\begin{aligned} \mathbf{x}_{im} &\triangleq \mathbf{U}_i^H \mathbf{z}_{im}, \\ &= \mathbf{S}_i \mathbf{a}_m s_{im}, \ \ m = 1, \dots, M. \end{aligned} \tag{14}$$

Define $\mathbf{W}_i$ as the inverse of $\mathbf{S}_i$ and let $\mathbf{W}_i$ have the following row partition

$$\mathbf{W}_i = \begin{bmatrix} \mathbf{w}_{i1}^H \\ \mathbf{w}_{i2}^H \\ \vdots \\ \mathbf{w}_{iN}^H \end{bmatrix}. \tag{15}$$

Multiplying $\mathbf{W}_i$ from the left in (14), we have

$$\begin{bmatrix} \mathbf{w}_{i1}^H \\ \mathbf{w}_{i2}^H \\ \vdots \\ \mathbf{w}_{iN}^H \end{bmatrix} \mathbf{x}_{im} = \begin{bmatrix} \alpha_{m1} \\ \alpha_{m2} \\ \vdots \\ \alpha_{mN} \end{bmatrix} s_{im}. \tag{16}$$

Assume that $s_{im} \neq 0$ for given m which is valid for most modulation schemes such as BPSK, QPSK, and QAM. Notice that each row of $\mathbf{W}_i \mathbf{x}_{im}$ is s_{im} scaled by a known interpolation coefficient α_{mn}, which makes it possible to eliminate the unknown s_{im} by cross referencing [9] . Specifically, multiplying row j,k by α_{mk}, α_{mj} respectively gives the same value $\alpha_{mj}\alpha_{mk}s_{im}$. Taking difference between two rows related to mth symbol data, we obtain the following equation

$$\tilde{\mathbf{X}}_{im} \mathbf{w}_i = \mathbf{0}, \tag{17}$$

where $\mathbf{w}_i \triangleq [\mathbf{w}_{i1}^H, \cdots, \mathbf{w}_{iN}^H]^H$ and

$$\tilde{\mathbf{X}}_{im} \triangleq \begin{bmatrix} \tilde{\mathbf{x}}_{im1}^H & -\tilde{\mathbf{x}}_{im2}^H & 0 & \cdots & \cdots & 0 \\ \tilde{\mathbf{x}}_{im1}^H & 0 & -\tilde{\mathbf{x}}_{im3}^H & & & \vdots \\ \vdots & 0 & 0 & \ddots & & \vdots \\ \vdots & & & & \ddots & 0 \\ \tilde{\mathbf{x}}_{im1}^H & 0 & \cdots & \cdots & 0 & -\tilde{\mathbf{x}}_{imN}^H \\ 0 & \tilde{\mathbf{x}}_{im2}^H & -\tilde{\mathbf{x}}_{im3}^H & 0 & \cdots & 0 \\ 0 & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & \tilde{\mathbf{x}}_{im2}^H & 0 & \cdots & 0 & -\tilde{\mathbf{x}}_{imN}^H \\ 0 & 0 & \tilde{\mathbf{x}}_{im3}^H & -\tilde{\mathbf{x}}_{im4}^H & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \tilde{\mathbf{x}}_{im,N-1}^H & -\tilde{\mathbf{x}}_{imN}^H \end{bmatrix}$$

and $\tilde{\mathbf{x}}_{imj} \triangleq \alpha_{mk}\mathbf{x}_{im}$ and $\tilde{\mathbf{x}}_{imk} \triangleq \alpha_{mj}\mathbf{x}_{im}$ for the pair (j, k). Now, combining all the symbol data gives

$$\mathbf{X}_i \mathbf{w}_i = \mathbf{0}, \tag{18}$$

where $\mathbf{X}_i \triangleq [\tilde{\mathbf{X}}_{i1}^H, \cdots, \tilde{\mathbf{X}}_{iM}^H]^H$. The identifiability of the proposed method is given by the following proposition.

Proposition 1 (Identifiability) *The $MN(N-1)/2 \times N^2$ matrix $\mathbf{X}_i$ is a matrix with rank $N^2 - 1$, i.e. the column rank is deficient by one. Hence, $\mathbf{w}_i$ is the unique null space of $\mathbf{X}_i$ and is blindly identifiable up to a scale factor.*

Proof. See [7].

Hence, the parameter matrix $\mathbf{G}_i$ is given by (13). The channel at an arbitrary position within a slot is obtained by interpolation.

For noisy observation, we can construct a least squares estimator based on (18) which is given by

$$\hat{\mathbf{w}}_i = \arg\min_{||\mathbf{w}_i||^2} ||\mathbf{X}_i \mathbf{w}_i||^2. \tag{19}$$

4. NUMERICAL RESULTS

In this section, we present some numerical results on the performance of the proposed method. The error performance of the proposed estimation results from two factors. One is the noise added in the signal and the other is the modeling error using a finite sample interpolation. We evaluated the mean square error (MSE) of the method due to noise only using a channel generated according to a interpolation model. We also assessed the modeling error using a lowpass channel with Jakes's spectrum[10].

Fig. 4 shows the MSE performance of the proposed method for a single user system with the interpolation channel with sync coefficients and three estimating points including both ends and the middle of a slot. We used the decorrelating front-end. The scrambling code was generated randomly with spreading factor $G = 32$ and slot length $M = 80$ symbols and fixed throughout the Monte Caro runs. The number of multipaths L is 4 with equal average magnitude. The signal-to-noise ratio is defined as $\frac{Avg(||\mathbf{h}_{im}||^2)G}{\sigma^2}$. The scale ambiguity is resolved using one pilot symbol placed at the left end of slot. As shown in the figure, the method showed a good MSE performance and almost reaches the the Cramér-Rao bound (CRB) at medium and high SNR.

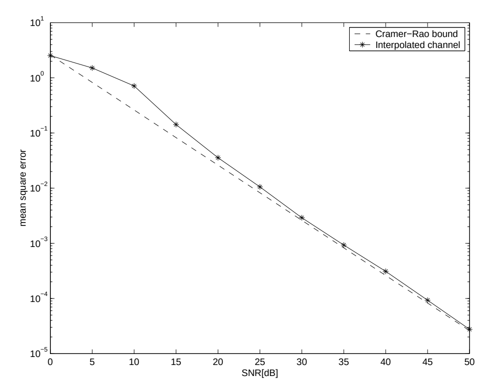

Fig. 4. Mean square error: interpolation channel

The MSE performance of the proposed estimator to a lowpass channel is shown in Fig. 5. The channel waveform was generated with Jakes's model with fading rate $f_D T_{slot} = 0.75$ and truncated for one slot length. Other parameters were the same as in the interpolation channel case. The proposed algorithm improved MSE performance much over the estimation using the pilot symbol at the left end of slot and the block fading assumption. The new estimator performed better than the training based estimation even at the pilot symbol position. This is because the proposed method utilized the whole slot observation to estimate the sample points while the training based method didn't. However, the proposed method also showed a performance floor at high SNR due to the imperfect modeling of the actual channel. Fig. 6 shows the bit error rate (BER) of a RAKE receiver with the estimated channel. The increase of BER with respect to SNR for the estimation with block fading model shows that the additive noise works beneficially for detection since the estimator works adversely for detection due to lack of tracking capability in a very fast fading environment.

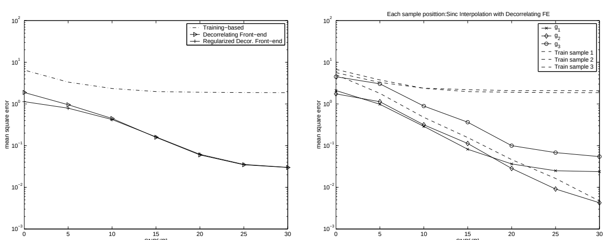

Fig. 5. Mean square error: lowpass channel (left: whole slot, right: each estimating point)

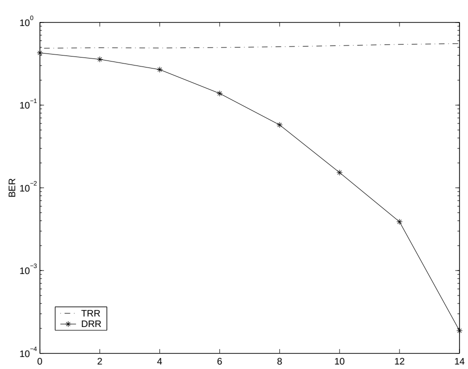

Fig. 6. Bit error rate: lowpass channel

5. CONCLUSION

We proposed a new blind channel estimation technique which effectively tracks fast fading channels in long code CDMA systems. Exploiting the multipath diversity and interpolation model, the proposed method shows a significant improvement over the channel estimation with block fading model without insertion of additional pilot symbols. The proposed method is useful in a scattering rich environment and requires the rough knowledge of fading rate.

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