Asymptotic Locally Optimal Detector for Large-Scale Sensor Networks under the Poisson Regime

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***Abstract*— We consider the distributed detection problem with a large number of identical sensors deployed over a region where the phenomenon of interest (POI) has different signal strength depending on the location. Each sensor makes a decision based on its own measurement of the spatially varying signal and the local decision of each sensor is sent to a fusion center through a multiple access channel. The fusion center decides whether the POI has occurred in the region, under a global size constraint in the Neyman-Pearson formulation. Assuming that the initial distribution of sensors is a homogeneous spatial Poisson process, we show that the Poisson process of 'alarmed' sensors satisfies the locally asymptotic normality (LAN) condition as the number of sensor goes to infinity and derive a new asymptotically locally most powerful detector for the spatially varying signal. We show that (1) an optimal test statistic is a weighted sum of local decisions, (2) the optimal weight function is the shape of the spatial signal, and (3) the exact value of the spatial signal is not required. For the case of independent, identical distributed (i.i.d.) sensor observation, we show that the counting-based detector is also asymptotic locally optimal.**

I. Introduction

Recently large-scale sensor networks have been proposed in many applications such as environmental monitoring, scientific research, and surveillance. A large number of sensors are expected to be deployed over a region, to measure the phenomenon of interest (POI) and to transmit their local data via wireless channels to a central site which performs global processing. For these large-scale sensor networks, the number of sensors is envisioned to be 10,000 to 100,000 and the sensors cover a geographically wide area. Hence, sensors must be cheap, and be manufactured in a massive volume, so that data from any one sensor may be unreliable. In this paper, we consider a detection problem based on local sensor data in such a large-scale sensor network.

Distributed detection using multiple sensors and optimal fusion rules has been widely investigated, see, e.g., [10]. Many authors have derived optimal fusion rules based on different sets of assumptions [11], [12], [13]. However, most fusion rules are obtained under the assumption that the hypotheses of the underlying phenomenon are simple, i.e., discrete and finite; further, these approaches require the knowledge of the false alarm and detection probability of each sensor decision under each of the M possible hypotheses. However, in applications such as the detection of biological/chemical agents or radioactivity in a certain area, it is difficult to determine the local detection probability beforehand at each sensor, and to formulate the detection problem in a simple hypotheses testing framework since the signal strength of POI is continuous and unknown beforehand. In [14], the authors considered the detection of an unknown signal via multilevel quantization and simple fusion rules. In addition to the unknown signal strength, for a large-scale sensor network, it is reasonable to assume that POI has different signal strength over the area, and that the observation of each sensor depends on its location and is not identically distributed since sensors are deployed over a geographically large area.

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We consider the optimal fusion problem in such large-scale sensor networks in the Neyman-Pearson context as the number of sensors goes to infinity within a fixed geographical area; each sensor observes an unknown spatially-varying signal. Asymptotic performance of distributed (as the number of sensors goes to infinity) detection was analyzed by several authors [15], [16]. Their analyses are based on the error exponent or convergence rate of the error probability to zero. However, when the number of observation samples is sufficiently large so that the error probability of reasonable detectors is already very close to zero, the convergence rate may not be a proper measure in the asymptotic regime. We consider a different criterion - asymptotic local optimality for such a large-sensor network. Assuming that the initial spatial distribution of sensors is a homogeneous Poisson process and that only the spatial variation of the signal is known, we derive an asymptotically local optimal detector; we also derive an optimal way to utilize the spatial information with the theory of locally asymptotic normality (LAN).

The paper is organized as follows. The data model of the sensor system that we consider is described and a brief summary of Poisson processes and LAN theory is introduced in section II. In section III, the asymptotically locally optimal detector is proposed under the Poisson assumption for the sensor distribution, and an estimation of spatial information is discussed.

II. System Model

We consider a large-scale sensor network with identical binary sensors deployed over a wide area; we want to decide whether POI has occurred in the area. Each sensor makes a decision based on its own observation and the local decisions are collected through a multiple access channel (MAC) at a central station or fusion center where a global decision is made under a size (PFA) constraint.

Since very many sensors are distributed over a wide area, we assume that POI has spatially-varying strength over the area. We assume that the exact value of the signal is unknown but the information of the relative signal strength with respect to location is available, if the phenomenon were to occur. For the example of detecting hazardous chemicals or radioactivity in a region, the strength of the phenomenon is highest at the center of the phenomenon and decays as the location becomes far from the center. We assume that the spatial signal is deterministic and denote the strength of the signal by

$$\gamma(\mathbf{x}) = \theta s(\mathbf{x}), \tag{1}$$

where $\mathbf{x}$ denotes the location, $\theta \in \Theta \triangleq [0, \infty)$ is an unknown amplitude, and $s(\mathbf{x})$ is a known function which incorporates the information about the spatial variation of the underlying phenomenon.

A. Single Sensor

We assume that sensors make their local decisions independently without collaborating with other sensors. Since the exact value of the signal strength is unknown, we design each sensor to detect the following hypotheses

$$\begin{array}{ll} H_0: & \gamma(\mathbf{x}) = 0, \\ H_1: & \gamma(\mathbf{x}) > 0, \end{array} \tag{2}$$

with local size constraint of α_0. The hypotheses (2) is equivalently expressed by

$$\begin{array}{ll} H_0: & \theta = 0, \\ H_1: & \theta > 0. \end{array} \tag{3}$$

The local decision of each sensor S_i, located at $\mathbf{x}$, is denoted by

$$u_i = \begin{cases} 0 & \text{if } H_0 \text{ selected}, \\ 1 & \text{otherwise}. \end{cases} \tag{4}$$

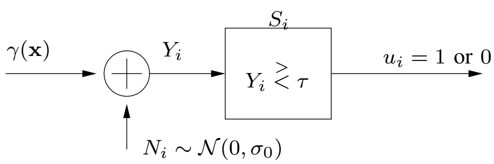

Fig. 1. Sensor located at $\mathbf{x}$

One possible sensor observation model is the additive Gaussian noise model shown in Fig. 1, where the sensor input Y_i is given by

$$Y_i = \gamma(\mathbf{x}) + N_i, \quad N_i \sim \mathcal{N}(0, \sigma_0), \tag{5}$$

where N_i is the independent sensor input noise. In this case, the local decision rule for (3) at each sensor is the UMP detector given by

$$Y_i \overset{>_{H_1}}{<_{H_0}} \tau_0, \tag{6}$$

where $\tau_0 = \sigma_0 Q^{-1}(\alpha_0)^1$. We define the following probability

$$p(\mathbf{x}) \triangleq \Pr\{u_i = 1\}. \tag{7}$$

Then, $p(\mathbf{x})$ is a function of the signal strength at $\mathbf{x}$ and is given by

$$p(\mathbf{x}) = g(\gamma(x)). \tag{8}$$

For the additive Gaussian observation model, $p(\mathbf{x})$ is expressed as $Q\left(\frac{\tau_0 - \gamma(\mathbf{x})}{\sigma_0}\right)$.

B. Parametric Poisson Model

Consider that a large number of the sensors described in II-A are deployed uniformly and randomly over a space A; each sensor makes its binary decision u_i depending on the signal strength at its location and then transmits the local decision to a fusion center through a MAC where some sensor data can be lost.

We assume that the initial distribution of sensors over the space is a homogeneous Poisson process with intensity λ_h. Since each sensor decision is independent and based on the signal strength at its location, the local decision making of each sensor can be viewed as a location-dependent thinning procedure of the original sensor distribution with probability $p(\mathbf{x})$ and the distribution of the *alarmed* sensors forms a nonhomogeneous spatial Poisson process. We also model the data collection through the MAC as another thinning that is uniform over the space with probability p_m which reflects the data lost during the transmission period through multiple access channel to the collector.

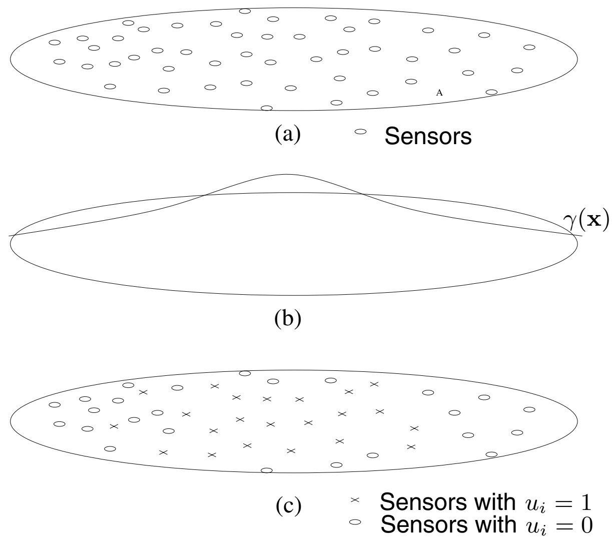

Fig. 2. (a) Initial sensor deployment over area (b) Signal strength of underlying phenomenon (c)Local decisions of sensors

Hence, the distribution of *alarmed* sensors, i.e., sensors with $u_i = 1$, at the final data collector or fusion center, is a nonhomogeneous Poisson process whose local intensity is given by

$$\lambda(\mathbf{x}) = \lambda_h p_m p(\mathbf{x}) = \lambda_h p_m g(\theta s(\mathbf{x})). \tag{9}$$

When the function $g(\cdot)$ is linear or θ is in a small neighborhood of $\theta = 0$, the Poisson distribution of alarmed sensors is described by a nonhomogeneous intensity model parameterized by amplitude θ and is given by

$$\lambda(\theta, \mathbf{x}) = \theta f(\mathbf{x}) + \lambda_0, \tag{10}$$

where

$$f(\mathbf{x}) = \lambda_h p_m g'(0) s(\mathbf{x}), \text{ and } \lambda_0 = \lambda_h p_m g(0). \tag{11}$$

The Poisson assumption on the initial sensor distribution effectively changes the global detection problem to that of of deciding from which intensity model the spatial distribution of alarmed sensors has occurred. Notice that the intensity variation $f(\mathbf{x})$ of alarmed sensors is a scaled version of the spatial signal shape $s(\mathbf{x})$.

C. Review of Poisson Process

The Poisson distribution X_A in space A is expressed in a simple manner by the counting measure notation which is given by [8]

$$X_A(B) = \sum_{i:\ \mathbf{x}_i \in A} \epsilon_{\mathbf{x}_i}(B), \quad \forall B \subset A, \tag{12}$$

where $\mathbf{x}_i, i = 1, \ldots, N_A$ are random points in A, N_A is a Poisson distributed random variable with mean $\Lambda(A)$, and

$$\epsilon_{\mathbf{x}_i}(B) \triangleq \begin{cases} 1, & \mathbf{x}_i \in B \\ 0, & \mathbf{x}_i \notin B \end{cases} \quad \forall B \subset A. \tag{13}$$

[1] $Q(x)$ denotes the tail probability $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2} dt$.

We define the stochastic integral for a given function f as

$$I(f) \triangleq \int_A f(\mathbf{x}) X_A(dx) = \sum_{i:\ \mathbf{x}_i \in A} f(\mathbf{x}_i). \tag{14}$$

We denote the probability density of a realization X_A of Poisson process with intensity $\lambda(\theta, \mathbf{x})$ by $d\mathbf{P}_\theta(X_A)$ which is given by [7]

$$d\mathbf{P}_\theta(X_A) = \exp\left(\int_A \log \lambda(\theta, \mathbf{x}) X_A(d\mathbf{x}) - \int_A \lambda(\theta, \mathbf{x}) d\mathbf{x}\right). \tag{15}$$

D. Sequence of Statistical Experiments and Locally Asymptotic Normality (LAN)

The theory of locally asymptotic normality (LAN) was first introduced by Le Cam [1]. In this section, we briefly introduce the LAN theory and its application in asymptotic detection. The LAN theory provides conceptual simplifications of asymptotic statistics via the existence of a randomized statistic in a limit Gaussian experiment using convergence in distribution of loglikelihood ratios. It makes it possible to establish a minimax bound for the risk of arbitrary estimators and to construct asymptotically locally most powerful criteria for composite hypothesis tests [1][2].

We define a statistical experiment as a statistical model $(\Omega, \mathcal{X}, \mathcal{P})$ where $\mathcal{P} = \{\mathbf{P}_\theta, \theta \in \Theta\}$. That is, the probability distribution of an event $X \in \mathcal{X}$ belongs to the family of distributions $\mathcal{P} = \{\mathbf{P}_\theta, \theta \in \Theta\}$ where the true parameter θ is unknown. The statistical experiment is simply denoted by $\{P_\theta, \theta \in \Theta\}$. We consider the sequence of statistical experiments $(\Omega^{(n)}, \mathcal{X}^{(n)}, \mathcal{P}^{(n)})$ where $\mathcal{P}^{(n)} = \{P_\theta^{(n)}, \theta \in \Theta\}$ with the same parameter space Θ for all n.

Definition 1: The sequence of statistical experiments $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta\}$ is locally asymptotically normal (LAN) at θ_0 if there exist matrices $\mathbf{r}_n(\theta_0)$ and $\mathbf{I}_{\theta_0}$ and random vectors Δ_{n,θ_0} such that $\mathcal{L}(\Delta_{n,\theta_0}|\theta_0) \Rightarrow \mathcal{N}(0, \mathbf{I}_{\theta_0})$ and for every $\mathbf{h}$

$$\log \frac{d\mathbf{P}_{\theta_0 + \mathbf{r}_n(\theta_0)\mathbf{h}}^{(n)}}{d\mathbf{P}_{\theta_0}^{(n)}}(X^{(n)}) = \mathbf{h}^T \Delta_{n,\theta_0}(X^{(n)}) - \frac{1}{2}\mathbf{h}^T \mathbf{I}_{\theta_0}\mathbf{h} + o_{P_{\theta_0}^{(n)}}(1),$$

where $\mathcal{L}(\Delta_{n,\theta_0}|\theta_0) \Rightarrow N(0, \mathbf{I}_{\theta_0})$ denotes that Δ_{n,θ_0} converges in distribution to $\mathcal{N}(0, \mathbf{I}_{\theta_0})$ under $P_{\theta_0}^{(n)}$ and $o_{P_{\theta_0}^{(n)}}(1)$ represents a term that converges to zero in $P_{\theta_0}^{(n)}$ probability.

Here, Δ_{n,θ_0} is called the central sequence and $\mathbf{I}_{\theta_0}$ is called the Fisher information matrix (FIM) which actually coincides with the conventional definition of the FIM for smooth parametric families. The i.i.d. drawings of random variables $X_1, \ldots, X_n$ where $X_1 \sim \mathbf{P}_\theta \triangleq \mathcal{N}(\theta, 1)$ provides a good example of a LAN family where $\mathbf{P}_\theta^{(n)} = \mathbf{P}_\theta^n$, $r_n(\theta) = \frac{1}{\sqrt{n}}$, $\Delta_{n,\theta} = \frac{1}{\sqrt{n}} \sum_{i=1}^n (X_i - \theta)$, and $\Theta = \mathbb{R}$.

When the sequence of experiments satisfies the LAN conditions, we can construct an asymptotic local upper bound on the power (detection probability), *and* a sequence of tests that achieves the bound [1][2].

Theorem 1: Let $\{\phi_n\}$ be any sequence of asymptotic α-tests for hypothesis (3). That is,

$$\limsup_{n \to \infty} \mathbb{E}_{n,0}\phi_n \le \alpha.$$

Suppose that $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta = [0, \infty)\}$ is LAN at $\theta = 0$ with normalizing sequence $r_n(0) \to 0$, central sequence $\Delta_{n,0}$, and FIM I_0. Then, for any $M > 0$,

$$\limsup_{n \to \infty} \sup_{0 < r_n(0)^{-1}\theta \le M} \left[\mathbb{E}_{n,\theta}\phi_n - Q(z_\alpha - r_n(0)^{-1}\theta I_0^{1/2})\right] \le 0. \tag{16}$$

where $\mathbb{E}_{n,\theta}$ denotes the expectation under $P_\theta^{(n)}$ probability. Furthermore, the following procedure is an asymptotic α-test for the hypotheses (3) that achieves the bound

$$\text{Take } H_0 \quad \text{if} \quad I_0^{-1/2}\Delta_{n,0} \le z_\alpha \tag{17}$$

$$\text{Take } H_1 \quad \text{if} \quad I_0^{-1/2}\Delta_{n,0} > z_\alpha, \tag{18}$$

where $\Delta_{n,0}$ is the central sequence and $z_\alpha = Q^{-1}(\alpha)$.

A detector that achieves the asymptotic local upper bound with the asymptotic size α is called the asymptotically locally most powerful (ALMP) detector with size α. The meaning of ALMP can be described as as follows. Suppose that the alternative hypothesis is bounded away from zero, $\theta \ge \theta_1 > 0$. As the sample size of the observations goes to infinity, the two sequences of distributions $\{\mathbf{P}_0^{(n)}\}$ and $\{\mathbf{P}_{\theta_1}^{(n)}\}$ become asymptotically entirely separated for most interesting cases and the power of any reasonable detector approaches unity as the sample size becomes large (with possibly different convergence rate). Suppose that we have two such detectors and a sufficiently large number of samples. Then, the powers of two detectors are already very close to unity, and the convergence rate is no longer a proper measure for assessing the performance of detector in the asymptotic regime. (This is the case for the large-scale sensor network we consider this paper.) Hence, a different asymptotic criterion is used to find an optimal detector in an asymptotic situation. The detection is focused on the alternative which is very close to the null hypothesis $\theta = 0$ where the distribution of null and alternative hypotheses are still nonseparable. Here, the interesting range for the alternative hypothesis is $\theta \in (0, r_n(0)M)$. Since θ is the amplitude parameter, the asymptotic criterion is the very low signal-to-noise ratio (SNR) range in our case. For the one-sided detection problem, the ALMP detector is most powerful not only in the local neighborhood of the null parameter but also in the entire parameter space $\Theta = [0, \infty)$ [3].

III. Detection of Spatially-varying Signal

In Section II-B, we assumed that the initial sensor distribution is Poisson, and showed that the original detection problem using identical binary sensors, deployed over a space with a spatially-varying signal, is converted to the problem of detecting a Poisson process with different intensities. In this section, using the LAN theory, we derive an asymptotically locally most powerful detector for the problem (3) as the number of sensors goes to infinity, in a fixed space and under the Poisson assumption.

We construct a sequence of statistical experiments of Poisson processes of alarmed sensors under the Poisson assumption on the initial sensor locations. An asymptotic scenario of infinite number of sensors is easily described by increasing the initial intensity λ_h of sensor deployment.

Model 1 (Fixed area and infinite sensor model): The intensity of Poisson process of initial sensor distribution over the space A with finite area is given by

$$\lambda_h = n\lambda_{h0}. \tag{19}$$

Then, for each $n \ge 1$, the local intensity of Poisson process of the alarmed sensors is given using (10, 11) by

$$\lambda^{(n)}(\theta, \mathbf{x}) = \theta n f(\mathbf{x}) + n\lambda_0, \tag{20}$$

and the sequence of experiments $\{\mathbf{P}_\theta^{(n)}, \theta \in [0, \infty)\}$ is given by (15). $X_A^{(n)}$ is the realization of Poisson processes of alarmed sensors on area A with probability $\mathbf{P}_\theta^{(n)}$.

Theorem 2: For Model 1, suppose that $f(\mathbf{x})$ satisfies the following conditions

(C.1) $f(\mathbf{x}) \geq 0, \quad x \in A$,
(C.2) $\sup_{\mathbf{x}\in A} f(\mathbf{x}) < \infty$,
(C.3) $\int_A f(\mathbf{x})d\mathbf{x} > 0$.

Then, the statistical model $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta\}$ is LAN at $\theta = 0$:

$$\log \frac{d\mathbf{P}^{(n)}_{r_n(0)h}}{d\mathbf{P}^{(n)}_0}(X_A^{(n)}) = h\Delta_{n,0} - \frac{1}{2}h^2 + o_{P_0^{(n)}}(1), \tag{21}$$

where the central sequence and normalizing sequence are given by

$$\Delta_{n,0} = \int_{A_n} r_n(0)^{-1/2}\left(\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})}\right)\left[X_A^{(n)}(d\mathbf{x}) - \Lambda_0^{(n)}(d\mathbf{x})\right], \tag{22}$$

$$r_n(0) = J_n(0)^{-1/2}, \quad J_n(0) = \int_A \left(\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})}\right)^2 \Lambda_0^{(n)}(d\mathbf{x}),$$

$$\dot{\lambda}^{(n)}(\theta,\mathbf{x}) = \frac{\partial}{\partial v}\lambda^{(n)}(v,\mathbf{x})|_{v=\theta}, \quad \Lambda_0^{(n)}(d\mathbf{x}) = \lambda^{(n)}(0,\mathbf{x})d\mathbf{x}.$$

Proof may be found in [17], and is omitted here due to space limitations.

The conditions (C.1)-(C.3) are general enough to incorporate various spatial variations including a step function, a linear decay, or an exponential decay

$$s(x, y) = e^{-\eta r}, \ r = \sqrt{x^2 + y^2}. \tag{23}$$

Theorem 3: For model 1, let the conditions (C.1)-(C.3) be satisfied. Then, the spatial function $s(\mathbf{x})$ provides an optimal weight under the Poisson assumption on the initial sensor distribution in the sense that it achieves the asymptotic local upper bound of the power under given size constraint as the number of sensors goes to infinity.

Proof:

$$\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})} = \frac{nf(\mathbf{x})}{\theta n f(\mathbf{x}) + n\lambda_0}|_{\theta=0} = \lambda_0^{-1} f(\mathbf{x}),$$

$$r_n^{-1/2}(0) = n^{-1/2}\lambda_0^{1/2}\left(\int_A f^2(\mathbf{x})d\mathbf{x}\right)^{-1/2}.$$

$$\Delta_{n,0} = r_n^{-1/2}(0)\int_A f(\mathbf{x})[X^{(n)}(d\mathbf{x}) - \Lambda_0^{(n)}(d\mathbf{x})], \tag{24}$$

$$= n^{-1/2}\lambda_0^{-1/2}\left(\int_A s^2(\mathbf{x})d\mathbf{x}\right)^{-1/2} \left(\sum_{i:\ \mathbf{x}_i\in A} s(\mathbf{x}_i) - n\lambda_0\int_A s(\mathbf{x})d\mathbf{x}\right). \tag{25}$$

■

Notice that the asymptotic optimal test statistic is the weighted sum of alarmed sensors. The weight $s(\mathbf{x})$ is related to the shape of underlying spatial signal $\gamma(\mathbf{x})$. Since $\Delta_{n,0}$ is normalized to have a limit distribution of $\mathcal{N}(0,1)$, any scaling of $s(\mathbf{x})$ is irrelevant in forming the test statistic. Hence, the exact value of the spatial signal is not necessary. The relative strength of the signal over the location is enough for the global detection with a given size.

The weighting of a local decision with the relative signal strength at the sensor location in Theorem 3 can be considered as a matched filtering in the spatial domain even though it is different from the conventional matched filtering since the received signal is random points with an intensity function rather than the distorted version of the transmitted signal. The use of intensity function in the detection of Poisson processes has been investigated. In [9], the author considered a binary on-off detection problem in optical transmissions. The author assumed that the photon generation epochs were Poisson distributed and showed that the optimal weight is related to the intensity of input light under Bayesian formulation between two simple hypotheses. However, the exact knowledge of the intensity of light is required rather than the shape of intensity.

Corollary 1: For i.i.d. sensor observations over A, the counting-based detector is asymptotically locally most powerful α-test under the Poisson assumption on sensor distribution.

Proof: In this case, the spatial signal shape $s(\mathbf{x})$ is given by

$$s(\mathbf{x}) \equiv 1,$$

and the central sequence is given by

$$\Delta_{n,0} = (n\lambda_0|A|)^{-1/2}(N^{(n)}(A) - n\lambda_0|A|), \tag{26}$$

where $N^{(n)}(A)$ is the number of alarmed sensors in space A and $|A|$ is the area of A. ■

IV. Numerical Results

We considered a two dimensional space A which is circular with radius one. The spatial signal shape we considered is the symmetric exponential in (23) with different decaying rates. The average number of sensors in A was chosen to be 1,000. For the local sensor function, we first used the additive Gaussian noise model (5) and the UMP detector with local size 0.1 described in Section II-A. We also considered the linear model for the power function $g(\cdot)$ which approximated the additive Gaussian input noise model.

For the simulation of power and false alarm probability, 10,000 Monte Carlo runs were executed. For each run, the following procedures were performed. The locations of sensors were randomly generated according to a homogeneous Poisson process with the given mean intensity [6]. For the additive Gaussian sensor model, the local threshold was calculated from the local size constraint and set to be the same for all sensors. A zero-mean Gaussian noise with variance one was generated independently for each sensor and added to the signal strength calculated from the location of sensor and the amplitude parameter to form a sensor observation. The threshold detection was made based on the sum of the signal and noise for the local decision. The global decision was made based on the test statistic $\Delta_{n,0}$ and the number of alarmed sensors for the ALMP detector and the counting-based detector, respectively. The global thresholds for both detectors were determined via the Gaussian limit distribution. Throughout the simulations, the probability of successful data collection from a sensor was set to one. The initial homogeneous density λ_h, the local false alarm probability, and the signal shape were assumed to be known, and the true values were used for the simulation.

Fig. 3 shows the analytic asymptotic upper bound in (16) and simulated powers with respect to the false alarm probability. The decay rate η for the exponential signal was 3. As shown in the figure, the power of ALMP detector almost achieves the upper bound with an average of 1000 sensors in the area.

Fig. 4 show the ROC of the proposed ALMP and the counting-based detector for the additive Gaussian sensor model with local size 0.1. The spatial variation was chosen to be fast within the region with $\eta = 9$. The ALMP detector utilizing the spatial information drastically improves the detector performance over the counting-based approach. The ROC for the linear sensor power model is also plotted. It is shown that the power of the additive Gaussian sensor model is actually larger than its linear approximation. This

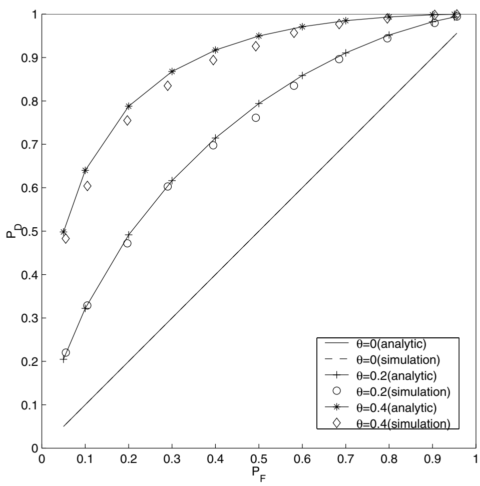

Fig. 3. ROC - analytic versus simulation.

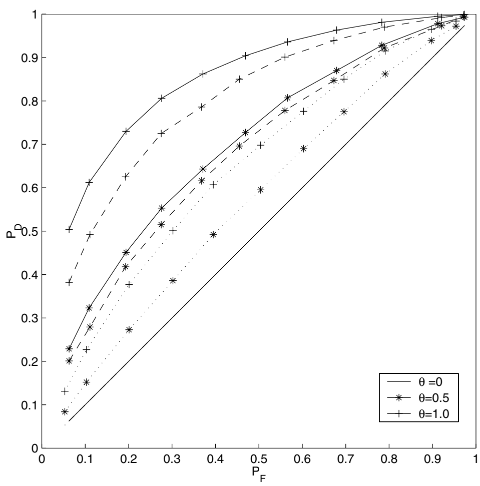

Fig. 4. ROC - additive Gaussian sensor model and $\eta = 9$ (solid line: ALMP detector with the additive Gaussian noise model, dashed line 1(- - -): ALMP detector with the linear sensor power model, dashed line 2($\cdots$): counting-based detector).

is because the cumulative distribution function of Gaussian $\mathcal{N}(0,1)$ is convex when its value is less than $1/2$, which is the case for the selected local size. Hence, the power of each sensor for the Gaussian model becomes larger than its linear approximation as the amplitude θ increases. The global power increases correspondingly.

V. Conclusion

Assuming Poisson distribution of sensor locations and the availability of location information, we proposed an efficient way of utilizing the spatial variation of the underlying phenomenon to optimize the global decision under Neyman-Pearson context. We obtained the asymptotically locally optimal test for a large sensor network with spatially varying sensor observation under the Poisson assumption.[2]

[2]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

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