Control and braiding of Majorana fermions bound to magnetic domain walls

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Control and braiding of Majorana fermions bound to magnetic domain walls

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Owing to the recent progress on endowing the electronic structure of magnetic nanowires with topological properties, the associated topological solitons in the magnetic texture—magnetic domain walls—appear as very natural hosts for exotic electronic excitations. Here, we propose to use the magnetic domain walls to engender Majorana fermions, which has several notable advantages compared to the existing approaches. First of all, the local tunneling density-of-states anomaly associated with the Majorana zero mode bound to a smooth magnetic soliton is immune to most of parasitic artifacts associated with the abrupt physical ends of a wire, which mar the existing experimental probes. Second, a viable route to move and braid Majorana fermions is offered by domain-wall motion. In particular, we envision the recently demonstrated heat-current induced motion of domain walls in insulating ferromagnets as a promising tool for nonintrusive displacement of Majorana modes. This leads us to propose a feasible scheme for braiding domain walls within a magnetic nanowire network, which manifests the nob-Abelian exchange statistics within the Majorana subspace.

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Introduction.—Majorana fermions (MFs) are of a great current interest in the field of topological quantum computation (TQC) for being non-Abelian anyons encoding quantum information[1]. A heterostructure based on conventional ingredients—a Rashba spin-orbit coupled (SOC) semiconducting wire subjected to a Zeeman field and proximity-induced *s*-wave superconductivity—supports MFs at the ends of the wire in topological phase[2,3]. Such wires can form a mesh, where the MFs can be braided by slowly adjusting gate voltages[4] or switching the supercurrents[5], and thus provides a promising platform for TQC.

Ongoing experimental efforts are devoted to observing MFs at the ends of a wire by measuring zero-bias peaks in the local tunneling density-of-states[6]. MFs, however, are obscured by the spurious effects caused by the abrupt physical ends of the wire. Realization of the aforementioned proposals for moving and braiding MFs is also stymied by their need for flawless control of electrostatic gates or supercurrents. A new idea that is able to render MFs more tangible and easily movable would, therefore, constitute a critical step towards TQC.

Spintronics aims at the active control and manipulation of spin degrees of freedom in condensed-matter systems[7]. Topologically stable magnetic textures, e.g., a domain wall (DW) in an easy-axis ferromagnetic wire or a vortex in an easy-plane ferromagnetic film, have been extensively studied out of fundamental interest as well as practical motivations exemplified by the racetrack memory[8]. Their dynamics can be driven by various means, e.g., an external magnetic field[9], an electric current (in metallic ferromagnets)[10], or a temperature gradient[11,12]. If we can identify a localized magnetic texture that supports MFs under suitable conditions, we would be able to manipulate them by controlling the magnetic host with standard spintronic techniques.

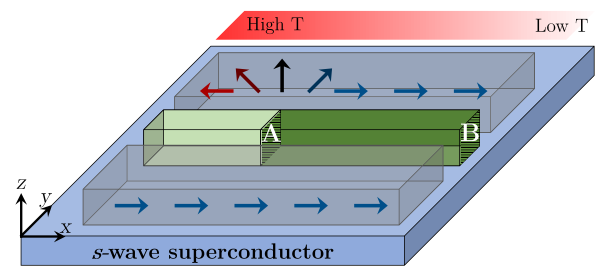

FIG. 1. (Color online) A schematic diagram of a device supporting two MFs in a SOC nanowire with proximity-induced exchange field and superconductivity. The wire is sandwiched between two easy-axis ferromagnets: one with a uniform magnetization and the other with a DW. In the right portion of the wire, where the magnetizations of the two ferromagnets are parallel, the net exchange field engenders the topological region, fomenting MFs at locations marked by A and B. The DW and its accompanying MF at A can be driven by applying a temperature gradient.

In this Rapid Communication, we propose to use the magnetic DWs to engender MFs, which has several notable advantages compared to the existing approaches. First of all, the local tunneling density-of-states anomaly associated with the Majorana zero mode bound to a smooth magnetic soliton is immune to most of parasitic artifacts associated with the abrupt physical ends of a wire, which mar the existing experimental probes (see Fig. 1). Second, a viable route to move and braid MFs is offered by DW motion. In particular, we envision the recently demonstrated heat-current induced motion of DWs in insulating ferromagnets as a promising tool for nonintrusive displacement of Majorana modes. This leads us to propose a feasible scheme for braiding MFs within a magnetic nanowire network, which manifests the nob-Abelian exchange statistics within the Majorana subspace (see Fig. 2).

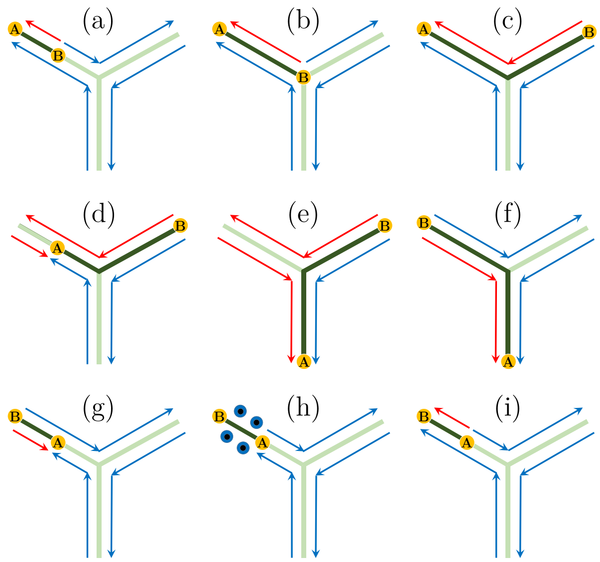

FIG. 2. (Color online) The exchange of two MFs bridged by a topological region (dark green). The arrows indicate the local directions of the magnetizations of ferromagnets. (a)-(g) MFs interchange positions via a series of thermally-driven DW motions. (g)-(i) Uniform rotation of the magnetization between MFs by 180° about the spin-orbit field direction transforms the state (g) to the state (i). This is essentially monodomain flipping, which can be done by standard techniques in spintronics.

MF at a magnetic DW.—Consider a semiconducting nanowire with Rashba SOC, which is deposited on an s-wave superconductor and is proximity-coupled to adjacent ferromagnets[2,3]. See Fig. 1 for a schematic design of a device. The superconducting quasiparticle spectrum is obtained by solving the BdG equation $\mathcal{H}^{\mathrm{BdG}}\Psi(x) = E\Psi(x)$, where

$$\mathcal{H}^{\mathrm{BdG}} = \left(-\frac{\hbar^2}{2m}\partial_x^2 - \mu + i\alpha\partial_x\sigma_2\right)\tau_3 + \mathbf{M}\cdot\boldsymbol{\sigma} + \Delta\tau_1 \quad (1)$$

acts on the spinor wavefunction $\Psi = (u_\uparrow, u_\downarrow, v_\downarrow, -v_\uparrow)^{\mathrm{T}}$. Here, m, μ, and α are the electron effective mass, the chemical potential, and the strength of SOC. The proximity-induced exchange field $\mathbf{M}$ is perpendicular to the spin-orbit field $\propto \hat{\mathbf{y}}$. The proximity-induced superconducting order parameter Δ is gauge shifted to be real and positive. Pauli-matrix vectors $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ act respectively on the spin and the electron-hole subspaces of the spinor Ψ. The corresponding quasiparticle creation operator is $\hat{\gamma}^\dagger = \int dx \sum_{\alpha=\uparrow,\downarrow}[u_\alpha(x)\hat{\psi}_\alpha^\dagger(x) + v_\alpha(x)\hat{\psi}_\alpha(x)]$. For the uniform exchange field, the "topological gap" at zero momentum is given by

$$E_g = |\mathbf{M}| - \sqrt{\Delta^2+\mu^2}. \quad (2)$$

When the gap E_g is positive, $|\mathbf{M}| > \sqrt{\Delta^2+\mu^2}$, the wire is in the topological phase, harboring a pair of MFs at its ends[13,14]. Otherwise, for the negative gap, the wire is in the normal phase, without MFs.

A spatially-varying exchange field induces the topological phase transition along the wire where $|\mathbf{M}|$ crosses $\sqrt{\Delta^2+\mu^2}$[14]. A DW in a ferromagnet adjacent to the wire is a natural object to bring about such a position-dependent field. We assume that the energy of the ferromagnet is given by

$$U[\mathbf{m}(x)] = \int dx \left[A|\partial_x\mathbf{m}|^2 - K_x m_x^2 + K_y m_y^2\right]/2, \quad (3)$$

where $\mathbf{m}(x)$ is the unit vector in the direction of local magnetization. Here, A, K_x, and K_y are the positive coefficients characterizing the stiffness of the magnetization, the easy-axis anisotropy along the wire, and the hard-axis anisotropy in the spin-orbit field direction, respectively[9]. Two ground states have uniform magnetization $\mathbf{m} \equiv \pm\hat{\mathbf{x}}$. A DW is a topological soliton solution minimizing the energy (3) with the boundary condition $\mathbf{m}(x\to\pm\infty) = \pm\hat{\mathbf{x}}$: its magnetization is $\mathbf{m}(x) = \tanh(x/\lambda)\hat{\mathbf{x}} + \mathrm{sech}(x/\lambda)\hat{\mathbf{z}}$ where $\lambda = \sqrt{A/K_x}$ is the DW width[9].

We sandwich the wire between two ferromagnets: one with a DW and the other with a uniform magnetization, as shown in Fig. 1. The proximity-induced exchange field is described by

$$\mathbf{M}(x) = M_1\left[\tanh(x/\lambda)\hat{\mathbf{x}} + \mathrm{sech}(x/\lambda)\hat{\mathbf{z}}\right] + M_2\hat{\mathbf{x}}, \quad (4)$$

which introduces a new length scale λ to the Hamiltonian. The presence of the DW causes spatial variance of the gap:

$$E_g(x) = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\tanh(x/\lambda)} - \sqrt{\Delta^2+\mu^2}. \quad (5)$$

For example, when $M_1 = M_2$ and $2M_1 > \sqrt{\Delta^2+\mu^2}$, there must be a pair of MFs in the wire: one at the right end, $x\to+\infty$, and the other one at the DW, $x_0 = \lambda\tanh^{-1}[(\Delta^2+\mu^2)/2M_1^2 - 1]$. The topological stability of the DW protects the hosted MF from disturbances of magnetization.

The technique to control a DW has been developed over decades in spintronics[7], which is directly translated into the ability to manipulate MFs[15]. DWs are conventionally driven by an external magnetic field[9] or a spin-polarized electrical current (in an itinerant ferromagnet)[10]. These methods, however, affect the electrons in the wire by altering the Hamiltonian. Instead, we propose to induce the motion of a DW by a temperature gradient[11], which is not intrusive to the electrons (see Fig. 1). The resultant velocity of the DW is proportional to temperature gradient. Multiple DWs on the single wire can be moved simultaneously, thereby providing the ability to control a series of MFs. DWs can be created by e.g., locally applying an external magnetic field opposite to the magnetization of a ferromagnet. They may be created away from a superconductor, and then be brought into a composite system by applying a temperature gradient.

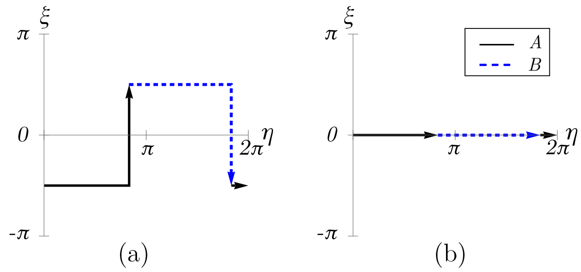

FIG. 3. (Color online) (a) The trajectories of the parameters (ξ,η) of MF A and B in the proposed exchange process shown in Fig. 2. The combined path of two trajectories forms a closed curve on the parameter space which is topologically a torus. (b) The trajectories of (ξ,η) in the process proposed by Alicea *et al.*[4]. Two combined paths of (a) and (b) are topologically equivalent.

Exchange of two MFs.—The exchange of two MFs is possible in the Y junction of three nanowires, each of which is sandwiched between two ferromagnets. Figure 2 shows the process to exchange MFs bridged by a topological region[16]. This proposal uses a well-controllable topological soliton (DW) as a host of MFs, and thus does not need intricate gate fabrication and control, unlike the exchange process proposed by Alicea *et al.*[4]. The non-Abelian exchange statistics of MFs is a universal property, meaning that it is invariant under the continuous deformation of the Hamiltonian as long as the gap $E_g(x)$ (2) remains finite, except at isolated locations of MFs[4,17]. Our proposal uses the spatially varying exchange field $\mathbf{M}(\mathbf{r})$ and the chemical potential fixed at zero, $\mu(\mathbf{r})\equiv 0$. These two functions, $\mathbf{M}(\mathbf{r})$ and $\mu(\mathbf{r})$, can be continuously deformed to a uniform exchange field in the z direction and a spatially-varying chemical potential of the form studied by Alicea *et al.*[4], without changing the gap. This is accomplished by first locally rotating the exchange field about the spin-orbit field direction to be oriented along the z axis: $\mathbf{M}'(\mathbf{r}) = M(\mathbf{r})\hat{\mathbf{z}}$, while keeping the gap $E_g(x)$ unchanged. The following transformation then connects our Hamiltonian ($s=0$) to that of Ref.[4] ($s=1$):

$$\begin{aligned} M(\mathbf{r};s) &= M(\mathbf{r}) + \left[-M(\mathbf{r}) + M_s\right]s, \\ \mu(\mathbf{r};s) &= \sqrt{\left\{\left[-M(\mathbf{r}) + M_s\right]s + \Delta\right\}^2 - \Delta^2}. \end{aligned} \tag{6}$$

where $M_s = \max_{\mathbf{r}} M(\mathbf{r})$. The gap-preserving transformation between two exchange processes, one performed by controlling the exchange field and the other by gating the chemical potential, ensures that the process shown in Fig. 2 exhibits the non-Abelian exchange statistics obtained in Ref.[4].

Alternatively, we may understand the topological equivalence of two processes following the general approach developed by Halperin *et al.*[17]. The state of each MF, denoted by A and B, can be described by two parameters. One is the wire orientation $\hat{\mathbf{w}}^a = (\cos\xi^a, \sin\xi^a)$ (a = A, B) in the xy-plane pointing from a nontopological region to a topological region, e.g., $\xi^A = 11\pi/6$ and $\xi^B = 5\pi/6$ in Fig. 2(a). The other parameter is the exchange-field direction $\hat{\mathbf{b}}^a = \cos\eta^a\,\hat{\mathbf{z}} + \sin\eta^a\,\hat{\mathbf{w}}^a$ acting on a MF, e.g., $\eta^A = -\pi/2$ and $\eta^B = \pi/2$ in Fig. 2(a). The state (ξ,η) of each MF evolves under the exchange process while making a trajectory on the parameter space which is topologically a torus. When the Hamiltonian goes back to the original form after the exchange, the combined path of trajectories of two MFs forms a closed curve on the torus. Figure 3(a) and (b) depict the combined curves of the parameters of our process and Alicea *et al.*[4]'s process, respectively. Both curves wind the torus once along the angle ξ: they are topologically equivalent. Thus the two processes should exhibit the same exchange statistics[17].

Analytical solutions for MFs.—In the strong SOC regime, $E_{\text{so}} = m\alpha^2/\hbar^2 \gg \max(|\mathbf{M}|,\Delta)$, we can use the BdG Hamiltonian density linearized at zero momentum:

$$\mathcal{H}^{\text{lin}} = -i\alpha\partial_x\sigma_2\tau_3 + M_x(x)\sigma_1 + M_z(x)\sigma_3 + \Delta\tau_1, \tag{7}$$

where the chemical potential is set to zero[14,18]. In the following discussions, we set $\alpha = 1$.

To obtain an intuitive idea of the existence of MFs bound to a DW, let us neglect the exchange field M_z for the moment. The Hamiltonian density $\mathcal{H}^{\text{lin}}$, then, can be block-diagonalized by employing Majorana operators instead of fermion operators[19]. Define four Hermitian Majorana operators $\hat{\gamma}^{A,B}_{\uparrow,\downarrow}$ in such a way that

$$\hat{\psi}_\downarrow = (\hat{\gamma}^A_\downarrow + i\hat{\gamma}^B_\downarrow)/\sqrt{2}, \quad \hat{\psi}_\uparrow = (\hat{\gamma}^B_\uparrow + i\hat{\gamma}^A_\uparrow)/\sqrt{2}. \tag{8}$$

Majorana operators are labeled by A and B for the convenience of the following discussion. The quasiparticle creation operator is $\hat{\gamma}^+ = \int dx \sum_{\substack{\alpha=\uparrow,\downarrow \\ \beta=A,B}} \left[u^\beta_\alpha(x)\hat{\gamma}^\beta_\alpha(x)\right]$, where the new 4-component spinor $\tilde{\Psi} = (u^A, u^B)^{\text{T}} \equiv (u^A_\uparrow, u^A_\downarrow, u^B_\uparrow, u^B_\downarrow)^{\text{T}}$ is related to the original spinor Ψ by the unitary transformation $\tilde{\Psi} = U\Psi$. In this new basis, the 4-component Schrödinger equation $i\partial_t\tilde{\Psi} = (U\mathcal{H}^{\text{lin}}U^\dagger)\tilde{\Psi}$ is cast as two Dirac equations

$$i\gamma^\mu\partial_\mu u^A + [-M_x(x) - \Delta]u^A = 0, \tag{9a}$$

$$i\gamma^\mu\partial_\mu u^B + [M_x(x) - \Delta]u^B = 0, \tag{9b}$$

where we have taken the representation of Dirac matrices to be $\gamma^0 = -\sigma_2$ and $\gamma^1 = \pm i\sigma_3$ ($-$ for u^A). The masses of A and B, $m^A(x) \equiv -M_x(x) - \Delta$ and $m^B(x) \equiv M_x(x) - \Delta$, are essentially the topological gap: $E_g(x) = m^A(x)$ when $M_x(x) > 0$ and $E_g(x) = m^B(x)$ when $M_x(x) < 0$. These Dirac equations coincide with that of solitons with fermion number 1/2 that Jackiw and Rebbi[20] studied. Each Dirac equation always has a unique static normalizable solution when its mass changes sign between two ends $x \to \pm\infty$. The exchange field $\mathbf{M} = [M_1\tanh(x/\lambda) + M_2]\hat{\mathbf{x}}$ yields two zero-energy

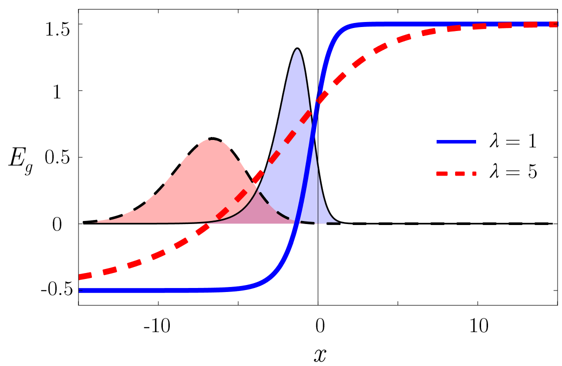

FIG. 4. (Color online) The solid and dashed lines show the topological gap $E_g(x)$ (2) corresponding to DW width $\lambda = 1$ and 5, respectively. Here, the superconducting order parameter is $\Delta = 0.5$, and the exchange field is $\mathbf{M} = [1 + \tanh(x/\lambda)]\hat{\mathbf{x}} + \mathrm{sech}(x/\lambda)\hat{\mathbf{z}}$. The amplitude squared $|\Psi(x)|^2 = \left|u_\downarrow^A(x)\right|^2 + \left|u_\uparrow^B(x)\right|^2$ of each MF in Eq. (11) is drawn by the shaded area. MFs are localized where the gap vanishes.

solutions[21]

$$\hat{\gamma}^A = \int dx\, e^{-(M_2+\Delta)x} \mathrm{sech}^{M_1\lambda}(x/\lambda) \hat{\gamma}_\uparrow^A(x), \tag{10a}$$

$$\hat{\gamma}^B = \int dx\, e^{-(M_2-\Delta)x} \mathrm{sech}^{M_1\lambda}(x/\lambda) \hat{\gamma}_\uparrow^B(x) \tag{10b}$$

up to the normalization factor. $\hat{\gamma}^A$ and $\hat{\gamma}^B$ are normalizable when $M_2 < M_1 - \Delta$ and $M_2 < M_1 + \Delta$, respectively.

When the uniform field M_2 is smaller than $M_1 - \Delta$, both masses $m^A(x)$ and $m^B(x)$ cross zero at $\lambda \tanh^{-1}[-(M_2 \pm \Delta)/M_1]$ (+ for A) and thus two MFs exist[22]. Perturbations split the degeneracy generally. For example, the kinetic energy $\left(\hbar^2\partial_x^2/2m\right)\tau_3$ omitted from the original BdG Hamiltonian density (1) hybridizes MFs with the energy $\lesssim \hbar^2 \max(M_1^2, \Delta^2)/2m\alpha^2$, which is finite, yet still much smaller than the bulk gap $\sim M_1, \Delta$.

When the uniform field exceeds $M_1 - \Delta$, while still smaller than $M_1 + \Delta$, the left part of the wire $x \ll -\lambda$ is driven into the nontopological phase, destroying MF A. The surviving MF B persists in the presence of finite M_z as long as the bulk gap remains finite. In particular, for the exchange field $\mathbf{M}(x)$ (4) with $M_1 = M_2 > \Delta/2$, one MF is located at $x_0 = \lambda \tanh^{-1}(\Delta^2/2M_1^2 - 1)$. The anticommutativity of the particle-hole transformation operator $\sigma_2\tau_2 K$ and the Hamiltonian density $\mathcal{H}^{\mathrm{lin}}$ (7) allows us to obtain a nondegenerate MF solution[13], given by

$$u_\uparrow^B(x) = e^{\Delta x}[1 - f^2(x)]^{1/4} P_{-1/2}^{-\nu+1/2}[f(x)], \tag{11a}$$

$$u_\downarrow^A(x) = -\nu e^{\Delta x}[1 - f^2(x)]^{1/4} P_{-1/2}^{-\nu-1/2}[f(x)] \tag{11b}$$

up to the common normalization factor, where $f(x) = (1 + e^{-2x/\lambda})^{-1/2}$ is introduced to simplify expressions. Here, $P_\alpha^\beta(x)$ is the associated Legendre function of the first kind with degree α and order β, and $\nu = 2M_1\lambda$ is the characteristic degree of our problem. Figure 4 shows the spatial profile of the topological gap and the amplitude squared of the MF solution. The relative magnitude of the two components can be obtained in two limiting cases: $|u_\downarrow^A(x)|/|u_\uparrow^B(x)| \simeq 2M_1\lambda$ when $M_1\lambda \ll 1$, and $\simeq 1$ when $M_1\lambda \gg 1$. For an abrupt wall $\lambda \to 0$, the solution (11) converges to the solution (10a) which was obtained in the absence of M_z.

Discussion.—We have proposed to bind MFs to magnetic DWs in the heterostructure of a SOC nanowire, an s-wave superconductor, and ferromagnet wires. We have also studied MFs analytically in the strong spin-orbit regime. Typical strong SOC semiconducting nanowires (e.g., InAs), magnetic insulators (e.g., EuO), and s-wave superconductors (e.g., Nb) provide the parameters $\alpha \sim 5\,\mathrm{meV\ nm}$, $|\mathbf{M}| \sim 1\,\mathrm{meV}$, and $\Delta \sim 0.5\,\mathrm{meV}$[13], which must be sufficient to engender MFs in DWs. MFs can be experimentally observed by measuring the local tunneling density-of-states anomaly[6], free from parasitic artifacts associated with the abrupt physical ends of a wire.

We are able to maneuver MFs with diverse means to control DW motion. In particular, we have proposed a process braiding MFs in the Y junction performed by thermally-driven DW motion, which exhibits non-Abelian exchange statistics. Thermally-driven motion of a DW has been observed in yttrium iron garnet films at room temperature[12]: the DW moves at the velocity $v \sim 100\,\mu\mathrm{m/s}$ for a temperature gradient $\nabla T \sim 2\,\mu\mathrm{eV}/\mu\mathrm{m}$[23]. According to a theory[11], the velocity of the DW is proportional to $T^{1/2}\nabla T$, which leads us to expect a sizable velocity at tens of μm/s even at $T \sim 3\,\mathrm{K}$. The resultant temperature drop over the DW width $\lambda \sim 60\,\mathrm{nm}$ is much smaller than the induced topological gap $\sim 200\,\mu\mathrm{eV}$[6].

We have focused on thermally-induced motion of a DW in this Rapid Communication. In principle, a temperature gradient in our proposal can be replaced by other spintronic techniques for moving a DW provided that they are not intrusive to MFs. For example, attaching a normal metal exhibiting strong spin-Hall effect such as a platinum to a ferromagnet allows us to drive a DW by an electric current through the metal via an interfacial spin accumulation[24].

We envision that realization of TQC can be brought forward by employing diverse objects and techniques of spintronics. It would be worth pursuing higher dimensional generalizations of our proposal based on a one-dimensinoal wire. For example, a thin film of $Fe_{0.5}Co_{0.5}Si$ subject to an external field (in the $\hat{\mathbf{z}}$ axis) perpendicular to the film supports a lattice of skyrmions which are two-dimensional topological solitons[25]. A skyrmion is a circular domain with $\mathbf{m} = \hat{\mathbf{z}}$ surrounded by a domain with $\mathbf{m} = -\hat{\mathbf{z}}$ (or vice versa), where $\mathbf{m}$ is the local direction of the magnetization. The edges of skyrmions form domain walls, and, therefore, may host MFs under suitable conditions, which can be braided by thermally-induced motions of skyrmions[26].

After the completion of this work, we became aware of the paper that speculates that a field-driven-control of a domain wall between two ground states of a chiral superconductor can give rise to a controllable transfer of edge excitations such as Majorana modes[27].

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[19] The Hamiltonian density $\mathcal{H}^{\mathrm{lin}}$ (7) can be classified as the BDI class with the particle-hole symmetry operator $\sigma_2\tau_2 K$, the BDI time reversal symmetry operator $\sigma_1\tau_1 K$ (different from the conventional one), and the BDI chiral symmetry operator $\sigma_3\tau_3$[28], where K is the complex conjugation operator. It can, thus, support multiple MFs at a domain wall. The BDI time reversal symmetry operator $\sigma_1\tau_1 K$ is transformed to a diagonal matrix $\tau_3 K$ by the unitary transformation U: MF solutions (10) respect the BDI time reversal symmetry.

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