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Magnetic Domain Wall Floating on a Spin Superfluid

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We theoretically investigate the transfer of angular momentum between a spin superfluid and a domain wall in an exchange coupled easy-axis and easy-plane magnetic insulator system. A domain wall in the easy-axis magnet absorbs spin angular momentum via disrupting the flow of a superfluid spin current in the easy-plane magnet. Focusing on an open geometry, where the spin current is injected electrically via a nonequilibrium spin accumulation, we derive analytical expressions for the resultant superfluid-mediated motion of the domain wall. The analytical results are supported by micromagnetic simulations. The proposed phenomenon extends the regime of magnon-driven domain-wall motion to the case when the magnons are condensed and exhibit superfluidity. Furthermore, by controlling the pinning of the domain wall, we propose a realization of a reconfigurable spin transistor. The long-distance dissipationless character of spin superfluids can thus be exploited for manipulating soliton-based memory and logic devices.

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Introduction.—Spin currents carried by collective excitation of magnets, in lieu of charge currents, have recently attracted vibrant experimental and theoretical activities opening a subfield of spintronics dubbed *magnonics* [1]. This is motivated in part by the prospects of constructing low-dissipation spintronic devices. Apart from allowing for the Joule heating-free transfer of spin signals, magnons also offer the possibility of imparting their spin angular momentum to topological solitons [2]. These solitons [3], such as domain walls and skyrmions, are robust against fluctuations and are thus considered as ideal candidates for encoding nonvolatile information [4]. Recent experimental demonstrations of thermal magnon-induced domain-wall [5] and skyrmion motion [6] could thus provide a basis for all-magnonic nonvolatile memory (such as the race-track register [4]) and logic devices [7].

On another front, these magnons offer a unique possibility of forming coherent condensates at room temperature, as demonstrated experimentally by parametric (microwave) pumping in a magnetic insulator [8]. Such condensates present an exciting opportunity for magnonics by supporting a long-distance coherent superfluid-like transport of the spin current [9], as opposed to the exponentially decaying spin currents carried by the incoherent thermal magnons. In addition to the pumped systems, such spin superfluidity is also supported by easy-plane magnets having a $U(1)$ order parameter [10]. More recently, these spin superfluids are gaining increased attention with proposals of realizing them in various easy-plane systems [11] [12]. The superfluid nature of spin currents results in: an algebraically decaying transport of spin [12], magnetic analogues of the Josephson effect [11, 13], dissipation via phase slips [14], and macroscopic qubit functionality [15]. While these proposals establish the feasibility of an efficient transport of the spin information, the possibility of transferring angular momentum by these superfluid-like spin currents to topological solitons remains unexplored. In this Letter, we fill this gap by

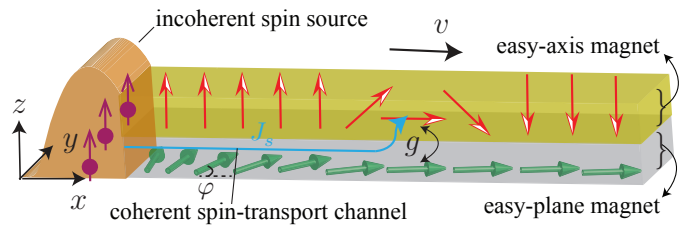


FIG. 1. A bilayer of an easy- z -axis magnet exchange coupled (with the coupling strength g) to an easy- xy -plane magnet. A z -polarized spin current is injected from an incoherent spin source and propagates as a superfluid spin current through the easy-plane magnet. This spin current is $\propto \nabla\varphi$, where φ is the azimuthal angle of the spin order parameter within the xy plane. This spin current is interrupted and absorbed by a domain wall in the easy-axis magnet, where it is converted into its sliding motion at speed v .

proposing a scheme for coupling spin currents carried by superfluids to magnetic solitons.

The main idea is to form an exchange coupled bilayer of an easy-plane and an easy-axis magnetic insulator. The bilayer is invariant under global spin rotations about an axis of symmetry, which coincides with the easy axis and the normal to the easy plane. See Fig. 1 for a schematic (where z is the symmetry axis). The easy-plane magnet plays the role of a spin superfluid and the easy-axis magnet harbors a domain wall. When a spin current polarized along the symmetry axis is injected into the bilayer, it is transported coherently by the gradient of the azimuthal angle (φ) of the spin density in the easy-plane magnet [10]. A static domain wall blocks the flow of this spin current by pinning φ underneath the domain wall. The pinning occurs due to the finite exchange coupling between the spin order parameters in the easy-axis and the easy-plane magnets. However, the $U(1)$ symmetry of the combined system demands conservation of the total angular momentum along the symmetry axis.

Consequently, the coherently transported spin current in the easy-plane magnet is absorbed by the domain wall and converted into its motion. The problem of deriving analytical expressions for this spin transfer-induced domain-wall motion and using it to propose a spin transistor are the main focuses of this Letter. Our proposal extends the concept of magnon-induced torques (due to the exponentially decaying incoherent magnon currents [16]) to the more efficient case, where the magnons are condensed and exhibit superfluidity.

Model.—We focus on a quasi one-dimensional model with a bilayer strip extended along the x axis and discuss two possible routes for forming the proposed system. That is, when an easy-axis ferromagnet is exchange coupled to a spin superfluid formed by (1) an easy-plane ferromagnet (referred to as FM/FM), or (2) a Heisenberg antiferromagnet (referred to as AFM/FM). For clarity, in the remainder of the main text we specialize to the FM/FM case. Similar results apply *mutatis mutandis* to the AFM/FM case [17]. In the FM/FM case, the free-energy density (per unit area in the xy plane) of the system can be written as $\mathcal{F} = \mathcal{F}_{\text{is}} + \mathcal{F}_{\text{sf}} + U$, with

$$\begin{aligned}\mathcal{F}_{\text{is}} &= \bar{A}\bar{t}\partial_x\mathbf{m}^2/2 - \bar{K}\bar{t}m_z^2/2, \\ \mathcal{F}_{\text{sf}} &= At\partial_x\mathbf{n}^2/2 + Ktn_z^2/2,\end{aligned}\quad (1)$$

and $U = -g\mathbf{m} \cdot \mathbf{n}$. Here, \bar{A} (A), \bar{K} (K), \bar{t} (t) and \mathbf{m} (\mathbf{n}) represent the magnetic stiffness, the anisotropy, the thickness, and the unit vector oriented along the order parameter in the easy-axis (easy-plane) magnet, respectively, while g parametrizes the strength of the exchange coupling between the easy-axis and the easy-plane magnets. The easy-axis and the easy-plane characters are enforced by having $\bar{K} > 0$ and $K > 0$. Within the easy-axis magnet, the equilibrium configuration of interest is that of a single domain wall (referred to as the region II) connecting magnetic domains (referred to as regions I and III for \mathbf{m} along $+\mathbf{z}$ and $-\mathbf{z}$, respectively). See Fig. 2 for a schematic. Furthermore, we focus on the small exchange coupling regime, where $g \ll Kt$ and $g \ll \bar{K}\bar{t}$. Within this regime, the equilibrium out-of-plane canting of \mathbf{n} and the deviation of \mathbf{m} away from the z axis (within regions I and III) are small. The proposed bilayer can be realized by using perpendicular racetrack material (such as cobalt iron boron [18] or cobalt/nickel multilayers [19]) for the easy-axis magnet, while magnetic insulators (such as yttrium iron garnet) for the easy-plane magnet. The exchange coupling between the easy-axis and the easy-plane system can be controlled via insertion of a non magnetic layer, such as copper [20].

Coupled spin hydrodynamics.—We begin by outlining a hydrodynamic theory for describing the proposed spin superfluid mediated domain-wall motion. The central idea is to write down the continuity equation for the flow of the z component of the spin current in the bilayer. In regions I and III this spin current is transported within

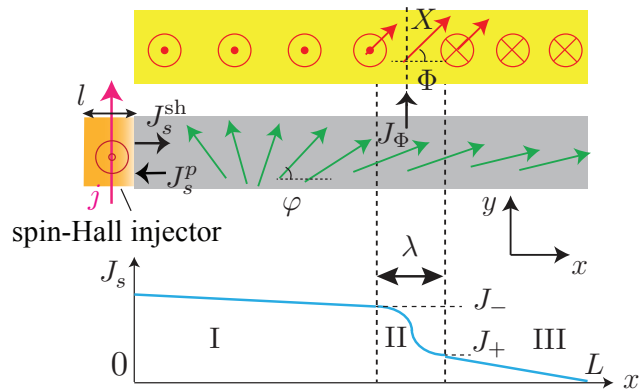


FIG. 2. The model of a domain wall of width λ coupled to a spin superfluid. The domain wall divides the bilayer into three regions: up domain (I), down domain (III) and the domain wall (II). A spin current, J_s^{sh} , is injected on the left by converting a charge current, j , into a spin accumulation via the spin Hall effect. Upon reaching the domain-wall region a portion of this spin current, J_Φ , is absorbed from the easy-plane magnet by the domain wall. The resultant dynamics of the domain wall is characterized by the generalized coordinates X and Φ , parametrizing its position and the associated azimuthal angle. The dynamics of the spin superfluid pumps a spin current, J_s^p , back to the contact. Bottom panel: the corresponding superfluid spin current flowing in the easy-plane magnet, as obtained by plotting $-At\partial_x\phi$.

the easy-plane magnet. In the strong anisotropy and the long-wavelength limit of the spin dynamics, the transport is described by [12]:

$$st\dot{n}_z = -\partial_x J_s - \alpha st\dot{\phi}, \quad (2)$$

with $J_s \equiv -At\partial_x\phi$ and s being the magnitude of the saturated spin density in the easy-plane magnet. The first term on the right-hand side describes the flow of a superfluid spin current (per unit length along the y axis), and the second term describes the transfer of the spin current to the atomic lattice due to a finite Gilbert damping, α , within the easy-plane magnet. In region II, additional spin current, J_Φ , is absorbed by the domain wall. Using the collective coordinate approach [21], the resultant domain-wall dynamics can be written as:

$$\bar{s}\dot{\Phi} - \bar{\alpha}\bar{s}\dot{X}/\lambda = 0 \quad (3a)$$

$$2\bar{s}\dot{X} + 2\bar{\alpha}\bar{s}\dot{\Phi} = J_\Phi, \quad (3b)$$

where the so-called *soft* modes X and Φ represent the location and the spin azimuthal angle at the center of the domain wall, where the z component of the spin density vanishes. Here, λ is the domain-wall width and \bar{s} is the magnitude of the saturated spin density in the easy-axis magnet. Eq. (3b) describes the flow of the spin current within the domain-wall region. Namely, the spin current absorbed by the domain wall, J_Φ , is converted into its motion, giving rise to the term proportional to

\dot{X} . In addition, a portion of the absorbed spin current is transferred to the atomic lattice in the easy-axis magnet, resulting in the term proportional to $\bar{\alpha}$.

In the spirit of the long-wavelength spin dynamics, throughout this Letter, we consider the domain wall as a point-like object satisfying $\lambda \sim \sqrt{A/K} \ll 1/\partial_x \varphi$. In this case, the width of the region II can be neglected, and the discontinuous jump in the spin current flowing in the easy-plane magnet at $x = X$ [see the bottom panel of Fig. 2] should be the same as J_Φ . Consequently, using Eq. (3) we have:

$$J_- - J_+ = J_\Phi = 2\bar{s}\bar{t}(1 + \bar{\alpha}^2)\dot{X}. \quad (4)$$

Here, J_- and J_+ are the spin current flowing in the easy-plane magnet just before and after the domain wall (i.e. region II), respectively. Equipped with this boundary condition, at $x = X$, we are now ready to discuss the motion of the domain wall in response to a spin current injected from the left of the bilayer.

In particular, we consider the open geometry proposed in Ref. 12. See the top panel of Fig. 2 for a schematic. A charge current flowing along the y axis at the metal/easy-plane magnet interface, with a density j (per unit thickness), is converted into a spin current via the spin Hall effect [22]. Within the spin Hall phenomenology [23], the corresponding spin current injected into the bilayer can be written in terms of the so-called spin Hall angle θ [24], the charge of an electron e , and the length (along the x axis) of the metallic contact l , as $J_s^{\text{sh}} = t\hbar j \tan \theta/2el$. In addition, the dynamics induced via such an injection pumps a portion of the spin current, $J_s^p = t\hbar g^{\uparrow\downarrow} \mathbf{n} \times \dot{\mathbf{n}}/4\pi$ [25], back into the metallic contact resulting in the following boundary condition at the left end:

$$J_s|_{x=0} = \vartheta j - t\hbar g^{\uparrow\downarrow} \mathbf{n} \times \dot{\mathbf{n}}/4\pi. \quad (5)$$

Here, $g^{\uparrow\downarrow}$ parametrizes the real part of the spin mixing conductance and we have defined $\vartheta \equiv \hbar \tan \theta/2el$. Finally, for the right interface we assume the usual exchange boundary condition:

$$J_s|_{x=L} = 0. \quad (6)$$

Linear regime.—We proceed to look for dynamic solutions of the form $\dot{\Phi} = \Omega$, $\varphi(x, T) = f(x) + \Omega T$ and $\dot{n}_z = 0$, where T denotes time. Physically, such an ansatz represents the following dynamic state. The spins in the easy-plane magnet rotate globally about the z axis with a linearly decaying spin current in regions I and III [12], and a steady-state motion of the domain wall with $\dot{X} = v$. We highlight that within this ansatz, the domain-wall angle is precessing at the same frequency as the underlying spin superfluid and refer to this dynamic regime as the “locked” phase. Furthermore, in the presence of a moving domain wall, the assumption of having a position independent Ω is not self evident. We justify and discuss

its validity *a posteriori* [17]. Balancing the flow of spin current, via substitution of the ansatz in Eqs. (2), (3) and the boundary conditions Eqs. (4)-(6), yields:

$$v = \frac{\vartheta j t}{2\bar{s}\bar{t}(1 + \bar{\alpha}^2) + \bar{\alpha}t(\gamma^{\uparrow\downarrow} + \gamma_\alpha)/\lambda}. \quad (7)$$

Here, we have used $\mathbf{n} \times \dot{\mathbf{n}} = \Omega \mathbf{z}$ and defined $\gamma_\alpha \equiv \alpha s L$, $\gamma^{\uparrow\downarrow} \equiv \hbar g^{\uparrow\downarrow}/4\pi$. This is one of the central results of the Letter. In the absence of the Gilbert damping, all of the injected spin current is absorbed by the domain wall giving a velocity obtained by the conservation of the angular momentum, i.e. $v = \vartheta j t/2\bar{s}\bar{t}$. While, the loss of the spin current results in a reduction of the velocity from this perfect absorption case. This loss of spin current occurs at two sources: (a) interface to the metal (due to spin-pumping), giving rise to the term proportional to $\gamma^{\uparrow\downarrow}$, and (b) bulk, giving an algebraically decaying velocity with the length of the bilayer.

Nonlinear regime.—At a critical strength of the external drive, the velocity of the domain wall can no longer increase linearly with the injected spin current. This phenomenon is referred to as the Walker breakdown [26] and is observed for both external field and current-induced domain-wall motion [27]. In this section we focus on the analogue of the Walker breakdown phenomenon for the superfluid-mediated spin transfer. For this purpose, we derive an analytical expression of J_Φ within the Landau-Lifshitz phenomenology. The z component of the torque applied on the easy-axis magnet, due to the coupling to the easy-plane magnet, reads as: $\tau_z = -\mathbf{z} \cdot \mathbf{m} \times \delta_{\mathbf{m}} U/\bar{t}$. The spin current absorbed by the domain wall is then given by integrating the torque over the domain-wall region, i.e. $J_\Phi = \bar{t} \int_\lambda \tau_z dx$. Substituting the following parametrization of the Cartesian components of the unit vector fields: $\mathbf{m} \equiv (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ and $\mathbf{n} \equiv (\cos \varphi, \sin \varphi, n_z)$ in U , we get:

$$J_\Phi = \pi g \lambda \sin(\varphi|_X - \Phi), \quad (8)$$

where $\varphi|_X$ is the value of φ at X .

For a given coupling g , there exists a maximum value of the absorbed spin current J_Φ^c , i.e. when $\varphi|_X - \Phi = \pi/2$. This results in a corresponding critical value for the injected spin current, J_c^s , and a critical domain wall velocity [from Eq. (3b)], $v_c = J_c^s/2\bar{s}\bar{t}(1 + \bar{\alpha}^2)$, above which the locked phase can no longer exist. Namely, Φ and $\varphi|_X$ precess at different frequencies, resulting in an oscillatory exchange of the spin current between the domain wall and the spin superfluid (corresponding to the jump $j_- - j_+$, shown in the bottom panel of Fig. 2, oscillating between a positive and a negative value). We refer to this transition as a locked to unlocked breakdown. Consequently, as in the case of the Walker breakdown, the domain wall is expected to drift in an oscillatory fashion, with $\langle v \rangle < v_c$. Substituting the value of critical velocity in Eq. (7), we

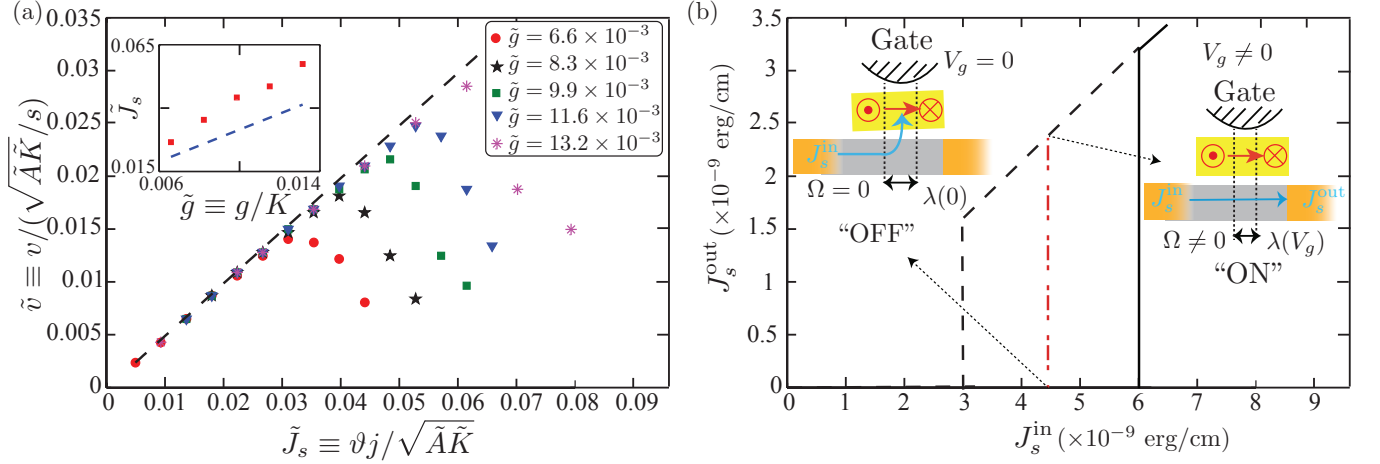


FIG. 3. (a) For a given exchange coupling $\tilde{g} \equiv g/Kt$, two regimes for domain-wall motion are obtained. A steady-state regime with linearly increasing velocity (\tilde{v}) and oscillatory motion above a critical value of injected spin current \tilde{J}_s . Broken line plots the analytical result from Eq. (7). Inset shows that the critical \tilde{J}_s increases linearly with \tilde{g} . Broken line shows the analytical result from Eq. (9). (b) The spin current detected at the right end of the bilayer (J_s^{out}) exhibits a nonlinear behavior in the presence of a pinned domain wall. When the injected spin current, J_s^{in} , is below (above) a critical breakdown current, $J_s^{\text{out}} = 0$ ($J_s^{\text{out}} \neq 0$). Solid and broken curves plot this non-linear characteristics for $\lambda = 10$ nm and $\lambda = 5$ nm, respectively. The nonlinearity can be used to construct a transistor, as indicated by the vertical dash-dot line. Fixing J_s^{in} and changing λ by an external gate switches the device from an OFF ($J_s^{\text{out}} = 0$) to an ON ($J_s^{\text{out}} \neq 0$) state. These OFF and ON states are depicted schematically in the insets.

obtain for the breakdown spin current:

$$J_c^s = \vartheta j_c t = \pi g \lambda \left[1 + \frac{\bar{\alpha}(\gamma^{\uparrow\downarrow} + \gamma_\alpha)}{2\bar{s}(1 + \bar{\alpha}^2)\lambda} \right]. \quad (9)$$

This is the second main result of the model, predicting a linear dependence of the breakdown spin current on λ . Here, we note that the locked to unlocked transition is analogous to the transition of superconducting Josephson junctions from the zero-voltage state to the finite-voltage state [28].

In Fig. 3(a), we compare the analytical results with micromagnetic simulations [17]. As predicted by the model, two regimes are observed in the simulations: (a) linearly increasing domain-wall velocity below a critical value of the injected spin current (J_s^c), and (b) oscillatory drift of the domain wall with a reduced average velocity above J_s^c . Moreover, both the velocity in the linear regime and the value of the critical current for locked to unlocked breakdown agrees well with the simulations.

Spin transistor.—We propose to utilize the domain-wall width dependence of the locked to unlocked breakdown in conjunction with the voltage control of the magnetic anisotropy (VCMA) [29] to construct a spin transistor. For this purpose we consider the case of a strongly pinned domain wall, i.e. with $\dot{X} = \dot{\Phi} = 0$. The pinning of Φ could be achieved by fabricating a nanowire geometry for the easy-axis magnet. In this case, the dipolar interaction forces the domain-wall magnetization to be oriented along the long axis of the nanowire. The domain-wall position can be pinned by engineering “notches”, which

create a local energy minima with respect to X [4]. For an injected spin current $J_s^{\text{in}} \equiv \vartheta j t < J_\Phi^c$, a static solution results for the spin superfluid with the domain wall absorbing all of the spin current injected at the left contact. See the “OFF” schematic in the inset of Fig. 3(b). On the other hand, for $J_s^{\text{in}} > J_\Phi^c$ locked to unlocked breakdown occurs, resulting in a precessing solution for the superfluid. Since $J_\Phi \propto \sin(\varphi_X - \Phi)$, the spin current absorbed by the domain wall averages to zero. Utilizing the inverse spin Hall effect [30], the spin current beyond the domain wall can be detected by adding a right metal contact. See the “ON” schematic in the inset of Fig. 3(b). Focusing on the case when the interfaces dominate over the bulk, i.e. $\gamma^{\uparrow\downarrow} \gg \gamma_\alpha$, half of the spin current is pumped back to the left contact and the other half is detected by the right contact, i.e. $J_s^{\text{out}} = J_s^{\text{in}}/2$. Here, the interfaces are assumed to be symmetric, parametrized by the same $\gamma^{\uparrow\downarrow}$. The λ dependence of J_Φ^c then translates into the following transistor-like action [plotted in Fig. 3(b)]. The “OFF” (“ON”) state of the device is defined as J_s^{out} being zero (nonzero). In the absence of the gate voltage, V_g , the device is biased to be below the locked to unlocked breakdown and hence in the OFF-state. Application of a gate voltage changes λ (by changing \bar{K} via VCMA) and turns the device ON abruptly, via inducing locked to unlocked breakdown. The proposed spin transistor has an added advantage, i.e. the domain wall can be moved to a desired location by applying a magnetic field, making the device reconfigurable.

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- [1] A. V. Chumak, V. I. Vasyuchka, A. A. Serga, and B. Hillebrands, *Nat Phys* **11**, 453 (2015).
- [2] D. Hinzke and U. Nowak, *Phys. Rev. Lett.* **107**, 027205 (2011); P. Yan, X. S. Wang, and X. R. Wang, *Phys. Rev. Lett.* **107**, 177207 (2011); A. A. Kovalev and Y. Tserkovnyak, *EPL (Europhysics Letters)* **97**, 67002 (2012); L. Kong and J. Zang, *Phys. Rev. Lett.* **111**, 067203 (2013); J. Iwasaki, A. J. Beekman, and N. Nagaosa, *Phys. Rev. B* **89**, 064412 (2014); A. A. Kovalev, *Phys. Rev. B* **89**, 241101 (2014); C. Schütte and M. Garst, *Phys. Rev. B* **90**, 094423 (2014).
- [3] A. Kosevich, B. Ivanov, and A. Kovalev, *Physics Reports* **194**, 117 (1990).
- [4] S. S. P. Parkin, M. Hayashi, and L. Thomas, *Science* **320**, 190 (2008); N. Nagaosa and Y. Tokura, *Nature Nanotech* **8**, 899 (2013); *Nature Nanotech* **8**, 883 (2013).
- [5] W. Jiang, P. Upadhyaya, Y. Fan, J. Zhao, M. Wang, L.-T. Chang, M. Lang, K. L. Wong, M. Lewis, Y.-T. Lin, J. Tang, S. Cherepov, X. Zhou, Y. Tserkovnyak, R. N. Schwartz, and K. L. Wang, *Phys. Rev. Lett.* **110**, 177202 (2013).
- [6] M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshibae, J. Zang, M. Mostovoy, Y. Tokura, and N. Nagaosa, *Nature Materials* **13**, 241 (2014).
- [7] D. A. Allwood, *Science* **309**, 1688 (2005).
- [8] S. O. Demokritov, V. E. Demidov, O. Dzyapko, G. A. Melkov, A. A. Serga, B. Hillebrands, and A. N. Slavin, *Nature* **443**, 430 (2006).
- [9] E. B. Sonin, *Journal of Low Temperature Physics* **171**, 757 (2012); D. A. Bozhko, A. A. Serga, P. Clausen, V. I. Vasyuchka, F. Heussner, G. A. Melkov, A. Pomyalov, V. S. L'vov, and B. Hillebrands, *ArXiv e-prints* (2015), arXiv:1503.00482 [cond-mat.quant-gas].
- [10] E. Sonin, *Advances in Physics* **59**, 181 (2010).
- [11] J. König, M. C. Bønsager, and A. H. MacDonald, *Phys. Rev. Lett.* **87**, 187202 (2001); H. Chen, A. D. Kent, A. H. MacDonald, and I. Sodemann, *Phys. Rev. B* **90**, 220401 (2014); H. Skarsvåg, C. Holmqvist, and A. Brataas, *Phys. Rev. Lett.* **115**, 237201 (2015); S. Takei and Y. Tserkovnyak, *Phys. Rev. Lett.* **115**, 156604 (2015); W. Chen and M. Sigrist, *Phys. Rev. B* **89**, 024511 (2014); S. Takei, A. Yacoby, B. I. Halperin, and Y. Tserkovnyak, *ArXiv e-prints* (2015), arXiv:1506.01061 [cond-mat.mes-hall].
- [12] S. Takei and Y. Tserkovnyak, *Phys. Rev. Lett.* **112**, 227201 (2014); S. Takei, B. I. Halperin, A. Yacoby, and Y. Tserkovnyak, *Phys. Rev. B* **90**, 094408 (2014).
- [13] K. Nakata, K. A. van Hoogdalem, P. Simon, and D. Loss, *Phys. Rev. B* **90**, 144419 (2014).
- [14] S. K. Kim, S. Takei, and Y. Tserkovnyak, *Phys. Rev. B* **93**, 020402 (2016); S. K. Kim and Y. Tserkovnyak, *Phys. Rev. Lett.* **116**, 127201 (2016).
- [15] S. Takei, Y. Tserkovnyak, and M. Mohseni, *ArXiv e-prints* (2015), arXiv:1512.04546 [cond-mat.mes-hall].
- [16] At a finite temperature, angular momentum can also be transported by an incoherent channel of thermal magnons within the easy-axis magnet itself [31]. These magnons can also apply torque on the domain wall.
- [17] See Supplemental material for a comparison of FM/FM and AFM/FM case, the validity condition for the assumed ansatz in the linear regime, and the details of the micromagnetic simulations, which includes Refs. [32] and [33].
- [18] Y. Zhang, W. S. Zhao, D. Ravelosona, J.-O. Klein, J. V. Kim, and C. Chappert, *Journal of Applied Physics* **111**, 093925 (2012), <http://dx.doi.org/10.1063/1.4716460>.
- [19] S. Parkin and S.-H. Yang, *Nature Nanotechnology* **10**, 195 (2015).
- [20] H. Wu, C. H. Wan, Z. H. Yuan, X. Zhang, J. Jiang, Q. T. Zhang, Z. C. Wen, and X. F. Han, *Phys. Rev. B* **92**, 054404 (2015).
- [21] A. A. Thiele, *Phys. Rev. Lett.* **30**, 230 (1973); O. A. Tretiakov, D. Clarke, G.-W. Chern, Y. B. Bazaliy, and O. Tchernyshyov, *Phys. Rev. Lett.* **100**, 127204 (2008).
- [22] M. Dyakonov and V. Perel, *Sov. Phys. JETP Lett.* **13**, 467 (1971); M. Dyakonov and V. Perel, *Phys. Lett. A* **35**, 459 (1971); J. E. Hirsch, *Phys. Rev. Lett.* **83**, 1834 (1999).
- [23] Y. Tserkovnyak and S. A. Bender, *Phys. Rev. B* **90**, 014428 (2014).
- [24] A. Hoffmann, *IEEE Transactions on Magnetism* **49**, 5172 (2013).
- [25] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, *Phys. Rev. Lett.* **88**, 117601 (2002).
- [26] N. L. Schryer and L. R. Walker, *J. Appl. Phys.* **45**, 5406 (1974).
- [27] G. Beach, M. Tsoi, and J. Erskine, *Journal of Magnetism and Magnetic Materials* **320**, 1272 (2008).
- [28] M. Tinkham, *Introduction to superconductivity* (Dover Publications, Mineola, N.Y., 2004).
- [29] T. Maruyama, Y. Shiota, T. Nozaki, K. Ohta, N. Toda, M. Mizuguchi, A. A. Tulapurkar, T. Shinjo, M. Shiraishi, S. Mizukami, Y. Ando, and Y. Suzuki, *Nat. Nanotech.* **4**, 158 (2009).
- [30] E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, *Applied Physics Letters* **88**, 182509 (2006), <http://dx.doi.org/10.1063/1.2199473>.
- [31] L. J. Cornelissen, J. Liu, R. A. Duine, J. B. Youssef, and B. J. van Wees, *Nat Phys* **11**, 1022 (2015).
- [32] <http://llgmicro.home.mindspring.com/>.
- [33] S. Ikeda, K. Miura, H. Yamamoto, K. Mizunuma, H. Gan, M. Endo, S. Kanai, J. Hayakawa, F. Matsukura, and H. Ohno, *Nat. Mat.* **9**, 721 (2010).