Interactions between Multiuser Diversity and Spatial Diversity Techniques in an Interference-limited Environment

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Abstract—This paper investigates the interaction between multiuser diversity and spatial diversity in an interference-limited environment based on post-receiver-processing signal-to-interference-plus-noise ratio (SINR) distributions. If opportunistic scheduling is employed, spatial diversity effects limit the achievable multiuser diversity gain. This paper quantifies the interaction by using order statistic theory and shows a spatial diversity technique with a larger SINR variance can be more effective under opportunistic scheduling. Through analysis and simulations, we show that the cyclic delay diversity technique gets the most benefit from the opportunistic scheduling among likely spatial diversity techniques and outperforms space time block coding (STBC), even though STBC is generally considered the most effective transmit diversity technique in a noise-limited environment.

I. INTRODUCTION

Multiple-input multiple output (MIMO) techniques have promised significant performance and capacity gains (refer to [1]–[4] and the reference therein) in wireless communications. In addition to MIMO techniques, multiuser diversity is considered another strong tool to increase spectral efficiency in future wireless communication systems. The idea itself was first proposed in [5] and commercial adoption can be already found in several systems such as 1xEV-DO [6], [7]. The capacity increasing capability of multiuser diversity comes from exploiting the inherent channel variations across users in a system as diversity sources. The total throughput can be significantly improved by serving only the user(s) with the highest instantaneous signal-to-interference-plusnoise ratio (SINR) at a given time.

There are a large number of studies on exploiting multiuser diversity in multiple antenna settings [8]–[15]. However, there are few studies investigating the interaction between spatial diversity and multiuser diversity. In general, it has been just conjectured that there is a fundamental conflict between multiuser diversity and spatial diversity – multiuser diversity is owed to channel variations but spatial diversity generally reduces channel variations of each user. The impact of multiuser diversity on the performance of space-time block codes (STBC) was investigated in [16]. The authors showed that multiuser diversity with no spatial diversity outperforms the schemes that employ both multiuser diversity and

spatial diversity in terms of effective signal-to-noise ratio (SNR) and spectral efficiency. The combined use of spatial diversity and multiuser diversity is studied more generally for various spatial diversity schemes including STBC and coherent beamforming by Larsson [17]. The paper concluded that, in general, spatial diversity with knowledge of channel state information (CSI) increases the total diversity gain in a system exploiting multiuser diversity, although the gain offered by diversity transmission is not as large as in a single user system.

Considering that future cellular systems will operate in increasingly interference-limited environments and are likely to adopt opportunistic transmission scheduling on top of various spatial diversity techniques, the effects of co-channel interference (CCI) and multiuser diversity on various spatial diversity techniques should be carefully studied. The paper [18] investigated the effects of CCI on spatial diversity techniques, but the focus was on SINR analysis so the interaction was not handled in that paper.

In this context, this paper analyzes the interactions between multiuser diversity and spatial diversity in an interference-limited environment. We compare the key characteristics of post-processing signal to interference plus noise ratio (SINR) distributions before and after opportunistic scheduling, and quantify the interaction between multiuser diversity and spatial diversity. Our results show that even though STBC gets more diversity benefit (smaller variance) without opportunistic scheduling, the small SINR variance significantly limits the multiuser diversity gain so it has rather worse performance in terms of both spectral efficiency and error probability than other diversity techniques if opportunistic scheduling is adopted. The investigation is also extended to orthogonal frequency division multiple access (OFDMA) systems by system level simulations.

II. SPATIAL DIVERSITY WITHOUT OPPORTUNISTIC SCHEDULING

The effects of CCI on spatial diversity were investigated in [18]. In this section, we first summarize the results of [18] to investigate the effects of opportunistic scheduling in an interference-limited environment.

A. Analytical Assumptions and Model

We list up the analytical assumptions in [18]. This paper is also based on these assumptions: (1) An interference-limited environment – thermal noise is neglected and the desired signal power is P_0 . (2) K independent and identically distributed (i.i.d.) co-channel interference with identical power P_I . (3) Desired and interfering signals employ the same transmission scheme over the same antenna configuration. – each transmitter has M_t transmit antennas and each receiver unit has M_r receive antennas. (4) Complex Gaussian fading channels are considered – both the desired and interfering signals experience spatially white complex Gaussian fading channels. Each channel coefficient of $M_t \times M_r$ MIMO channel matrices follows $\mathcal{CN}(0,1)$. (5) Channels are static during a symbol period. (6) Matched filter (MF) receivers without knowledge of interfering signals are used.

B. Post-processing SINR Distributions

1) SISO: Probability density function (PDF) of SISO is

$$f_{\gamma}(\gamma) = \frac{P_I}{P_0} \frac{\Gamma(1+K)}{\Gamma(1)\Gamma(K)} \frac{1}{\left(1 + \frac{P_I}{P_0}\gamma\right)^{1+K}}, \quad \gamma \ge 0 \quad (1)$$

with

$$\mathbb{E}[\gamma] = \frac{P_0}{P_I(K-1)} \text{ and } Var[\gamma] = \frac{P_0^2}{P_I^2} \frac{K}{(K-1)^2(K-2)}.$$

where K > 1 and K > 2 for mean and variance, respectively. 2) Receive Diversity (RD) – MRC: PDF is given by

$$f_{\gamma}(\gamma) = \frac{P_I}{P_0} \frac{\Gamma(M_r + K)}{\Gamma(M_r)\Gamma(K)} \frac{(P_I \gamma / P_0)^{M_r - 1}}{(1 + P_I \gamma / P_0)^{M_r + K}}, \quad \gamma \ge 0 \quad (2)$$

with

$$\mathbb{E}[\gamma] = \frac{P_0 M_r}{P_I(K-1)} \text{ and } Var[\gamma] = \frac{P_0^2}{P_I^2} \frac{M_r(M_r + K - 1)}{(K-1)^2(K-2)}.$$

where K > 1 and K > 2 for mean and variance, respectively. 3) Space Time Block Coding (STBC): PDF of STBC is

$$f_{\gamma}(\gamma) = \frac{P_I}{P_0} \frac{\Gamma(M_r M_t + K M_t)}{\Gamma(M_r M_t) \Gamma(K M_t)} \frac{(P_I \gamma / P_0)^{M_r M_t - 1}}{(1 + P_I \gamma / P_0)^{M_r M_t + K M_t}}$$
(3)

with

$$\mathbb{E}[\gamma] = \frac{P_0 M_r M_t}{P_I (K M_t - 1)} \quad \text{and}$$

$$\text{Var}[\gamma] = \frac{P_0^2}{P_I^2} \frac{M_r M_t (M_r M_t + K M_t - 1)}{(K M_t - 1)^2 (K M_t - 2)}.$$

where $KM_t > 1$ and $KM_t > 2$ for mean and variance, respectively.

4) Cyclic Delay Diversity (CDD) or Phase Shifted Transmission: PDF is given by

$$f_{\gamma}(\gamma) = \frac{P_I}{P_0} \frac{\Gamma(M_r + K)}{\Gamma(M_r)\Gamma(K)} \frac{(P_I \gamma / P_0)^{M_r - 1}}{(1 + P_I \gamma / P_0)^{M_r + K}}, \quad \gamma \ge 0 \quad (4)$$

with

$$\mathbb{E}[\gamma] = \frac{P_0 M_r}{P_I(K-1)} \text{ and } Var[\gamma] = \frac{P_0^2}{P_I^2} \frac{M_r(M_r + K - 1)}{(K-1)^2 (K-2)}.$$

where K > 1 and K > 2 for mean and variance, respectively.

5) Coherent Beamforming – Transmit MRC: PDF of coherent beamforming is given by

$$f_{\gamma}(\gamma) = \frac{P_I}{P_0} \frac{\Gamma(M_t + K)}{\Gamma(M_t)\Gamma(K)} \frac{(P_I \gamma / P_0)^{M_t - 1}}{(1 + P_I \gamma / P_0)^{M_t + K}}, \quad \gamma \ge 0 \quad (5)$$

with

$$\mathbb{E}[\gamma] = \frac{P_0 M_t}{P_I(K-1)} \text{ and } Var[\gamma] = \frac{P_0^2}{P_I^2} \frac{M_t(M_t + K - 1)}{(K-1)^2(K-2)}.$$

where K > 1 and K > 2 for mean and variance, respectively.

The results are summarized in Table I. According to these results, STBC is more sensitive to the number of interferers in terms of the mean and the variance since the number of effective interferers becomes KM_t as shown in the denominator. When there are three independent and identically distributed (i.i.d.) interferers with the same power P_I , cumulative density functions (CDF) of spatial diversity techniques are given in Fig. 1. The power ratio between the desired signal power P_0 and P_I is 7.5dB.

III. THE EFFECTS OF MULTIUSER DIVERSITY ON SPATIAL DIVERSITY TECHNIQUES

Without multiuser diversity, a smaller SINR variance typically results in better spatial diversity performance if the mean values of SINR are the same or similar. However, if opportunistic scheduling is adopted, a smaller SINR variance may limit multiuser diversity gain. This section studies this conjecture and quantifies the effects of max-SINR scheduling exploiting multiuser diversity on spatial diversity techniques in an interference-limited environment.

If we adopt a max-SINR scheduler which selects the user with the highest SINR to investigate the effects of multiuser diversity, the post-processing SINR of the selected user is $\gamma_{\text{eff}} = \max_{u=1,\cdots,\ U} \gamma_u$ where γ_u is the SINR of user u and U is the total number of users. Each γ_u follows the derived distribution in Section II for a given diversity technique. Using order statistics, PDF of γ_{eff} is given by

$$f_{\gamma_{\text{eff}}}(\gamma_{\text{eff}}) = U f_{\gamma}(\gamma_{\text{eff}}) F_{\gamma}(\gamma_{\text{eff}})^{U-1}$$
 (6)

where $F_{\gamma}(\cdot)$ is the cumulative density function (CDF) of γ .

Fig. 2 shows the CDFs of the spatial diversity techniques when the numbers of users are 30 (U=30). Three i.i.d. interferers with the same power P_I are considered and $P_0/P_I=7.5 \mathrm{dB}$. Compared to CDF's in Fig. 1 without opportunistic scheduling, 1×2 RD, 2×1 coherent BF, and 2×2 CDD outperform 2×2 STBC in the whole CDF region. Even SISO and 2×1 CDD are better than 2×2 STBC at the high percentage CDF region. The technique with a higher variance takes more advantages of opportunistic scheduling so the mean value after scheduling ($\mathbb{E}[\gamma_{\mathrm{eff}}]$) becomes larger.

Even though it is difficult to obtain an exact closed-form expression of the mean value after scheduling in general, an upper bound on $\gamma_{\rm eff}$ can be obtained by using order statistic theory on an upper bound on the maximum random variable [19] as

$$\mathbb{E}\left[\max_{1\leq u\leq U}\gamma_u\right] \leq \mathbb{E}[\gamma] + \frac{U-1}{\sqrt{2U-1}}\sqrt{\operatorname{Var}[\gamma]}.\tag{7}$$

This theoretical upper bound effectively quantifies the interaction between spatial diversity and multiuser diversity. The mean value after scheduling ($\mathbb{E}[\gamma_{\rm eff}]$) increases with not only the number of

users but also the SINR variance of each user $\mathrm{Var}[\gamma]$. As a result, the improvement of the mean value after scheduling $(\mathbb{E}[\gamma_{\mathrm{eff}}])$ is pronounced for the schemes with higher variances as confirmed in Table III. This table also shows that the theoretical upper bound is reasonably tight with sufficient number of users.

The increased mean values by multiuser diversity are directly translated into performance improvement. Average symbol error rate (SER) for QPSK and spectral efficiency according to the average received SINR are shown in Fig. 3 and Fig. 4, respectively. The spectral efficiency is calculated by ($SE = \log_2(1+\gamma_{\rm eff})$) and the symbol error probability for QPSK is given by

$$SER = \overline{N}_e \mathbb{E} \left[Q \left(\sqrt{\frac{d_{min}^2 \gamma_{\text{eff}}}{2}} \right) \right]$$
 (8)

where \overline{N}_e and d_{min} are the average number and the minimum distance of the nearest neighbors of the given signal constellation, respectively. The number of users is 30 and three i.i.d. interferers are considered. Owing to the increased mean values after opportunistic scheduling, 1×2 RD, 2×1 coherent BF, and 2×2 CDD outperform 2×2 STBC in terms of both SER and spectral efficiency. Although SISO and 2×1 CDD have worse SER than 1×2 STBC for a fixed modulation, they have slightly higher spectral efficiency even than 2×2 STBC if we relax the fixed modulation cosntraint.

To more effectively demonstrate the interaction between multiuser diversity and spatial diversity, spectral efficiencies of various spatial diversity techniques versus the number of users are shown in Fig. 5 where three i.i.d. interferers with the same power P_I are considered and $P_0/P_I=5$ dB. Spectral efficiencies of 1×2 RD, 2×2 CDD, and 2×1 coherent BF grow faster than 2×2 STBC owing to the larger SINR variances. Even SISO and 2×1 CDD have higher spectral efficiencies than 2×2 STBC if the number of users is greater than 20.

For CDD, it should be also noted that as the number of users increases there is more likely to be the user whose desired signals are co-phased such that $h_{11}+e^{-j\theta_0}h_{12}=(|h_{11}|+|h_{12}|)e^{-j\phi}$. These chances are neglected in analysis, but they contribute to the performance enhancement of CDD in frequency selective user scheduling.

IV. ORTHOGONAL FREQUENCY DIVISION MULTIPLE ACCESS (OFDMA) SYSTEMS

In this section, we investigate the interaction between multiuser diversity and spatial diversity in OFDMA systems over flat fading channels. In OFDMA systems, each resource block consisting of multiple sub-carriers over several OFDM symbols is allocated to users by a specific scheduling algorithm. An effective SINR characterizes the signal quality of a resource block for scheduling in OFDMA systems and there are several ways to calculate an effective SINR [20]–[26]. In this paper, we adopt the exponential-effective-SINR (EESM) in 3GPP LTE [24] and proportional fair scheduling [6], [9].

If channel quality indicator (CQI) feedback information from mobile units is available, adaptive modulation and coding selection (MCS) and opportunistic scheduling can be adopted using the CQI feedback information. If a resource block consists of contiguous sub-carriers, then more fluctuations across resource

blocks result in more benefit from frequency selective opportunistic scheduling. In this scenario, statistical characteristics of an effective SINR (EESM) become very similar to those of each SINR in a single carrier system since the sub-carriers within a resource block are highly correlated.

The performance of frequency selective proportional fair scheduling depends on EESM fluctuations across resource blocks. For similar mean values of EESM, a larger EESM variance yields better performance and the scheme with lower SINR correlations gets more benefit from frequency selective opportunistic scheduling. The EESM fluctuations across resource blocks in the contiguous sub-carrier case can be well captured by SINR cross correlation between different sub-carriers.

A. Cross Correlation Properties between Resource Blocks

For analytical tractability, we only derive correlation between desired signals on different resource blocks instead of correlation between the exact effective SINRs on different resource blocks. Considering that the desired signal is independent of interfering signals, the correlation property between desired signals on different resource blocks effectively characterizes the frequency diversity effect for a particular spatial diversity technique in an OFDMA system. This section focuses on the three most interesting spatial diversity techniques – 1×2 RD, 2×2 STBC, and 2×2 CDD.

Let $\mathbb{E}[|h_{mn}^{(i)}|^2|h_{pq}^{(j)}|^2]=\rho$ where $h_{mn}^{(i)}$ denotes the channel coefficient from the nth transmit antenna to the mth receive antenna on the ith resource block. The value of ρ equals to 1 if $|h_{mn}^{(i)}|^2$ and $|h_{pq}^{(j)}|^2$ are uncorrelated and goes to 2 if they are closely correlated. Note that $\mathbb{E}[|h_{nm}^{(i)}|^2|h_{pq}^{(j)}|^2]=1$ for different i and j if $n\neq p$ or $m\neq q$ since we assume that antennas are uncorrelated.

The cross correlation between the desired signals on different resource blocks of 1×2 RD is given by

$$\rho_{\text{RD}} = \left(\mathbb{E} \left[\left(|h_1^{(i)}|^2 + |h_2^{(i)}|^2 \right) \left(|h_1^{(j)}|^2 + |h_2^{(j)}|^2 \right) \right] \\
- \mathbb{E} \left[|h_1^{(i)}|^2 + |h_2^{(i)}|^2 \right] \mathbb{E} \left[|h_1^{(j)}|^2 + |h_2^{(j)}|^2 \right] \right) \\
/ \left(\sqrt{\text{Var} \left[|h_1^{(i)}|^2 + |h_2^{(i)}|^2 \right]} \sqrt{\text{Var} \left[|h_1^{(j)}|^2 + |h_2^{(j)}|^2 \right]} \right) \\
= \rho - 1 \tag{9}$$

since $\mathbb{E}[|h_{nm}^{(i)}|^4] = 2$ and $\mathbb{E}[|h_{nm}^{(i)}|^2] = 1$ for $h_{nm}^{(i)} \sim \mathcal{CN}(0,1)$. For 2×2 STBC, the cross correlation between the desired

signals on different resource blocks is

$$\rho_{\text{STBC}} = \left(\mathbb{E} \left[\left(|h_{11}^{(i)}|^2 + |h_{12}^{(i)}|^2 + |h_{21}^{(i)}|^2 + |h_{22}^{(i)}|^2 \right) \right. \\ \left. \cdot \left(|h_{11}^{(j)}|^2 + |h_{12}^{(j)}|^2 + |h_{21}^{(j)}|^2 + |h_{22}^{(j)}|^2 \right) \right] - 16 \right) \\ \left. / \sqrt{\text{Var} \left[|h_{11}^{(i)}|^2 + |h_{12}^{(i)}|^2 + |h_{21}^{(i)}|^2 + |h_{22}^{(i)}|^2 \right]} \right. \\ \left. / \sqrt{\text{Var} \left[|h_{11}^{(j)}|^2 + |h_{12}^{(j)}|^2 + |h_{21}^{(j)}|^2 + |h_{22}^{(j)}|^2 \right]} \right. \\ = \rho - 1. \tag{10}$$

Finally, the cross correlation between the desired signals on

different resource blocks of 2×2 CDD is given by

$$\rho_{\text{CDD}} = \left(\mathbb{E} \left[\left(\left| h_{11}^{(i)} + e^{-j\phi} h_{12}^{(i)} \right|^2 + \left| h_{21}^{(i)} + e^{-j\phi} h_{22}^{(i)} \right|^2 \right) \right. \\
\left. \cdot \left(\left| h_{11}^{(j)} + e^{-j\theta} h_{12}^{(j)} \right|^2 + \left| h_{21}^{(j)} + e^{-j\theta} h_{22}^{(j)} \right|^2 \right) \right] - 16 \right) \\
\left. / \sqrt{\text{Var} \left[\left| h_{11}^{(i)} + e^{-j\phi} h_{12}^{(i)} \right|^2 + \left| h_{21}^{(i)} + e^{-j\phi} h_{22}^{(i)} \right|^2 \right]} \right. \\
\left. / \sqrt{\text{Var} \left[\left| \left| h_{11}^{(j)} + e^{-j\theta} h_{12}^{(j)} \right|^2 + \left| h_{21}^{(j)} + e^{-j\theta} h_{22}^{(j)} \right|^2 \right]} \right. \\
= \frac{1}{2} (\rho - 1). \tag{11}$$

since
$$h_{11}^{(i)} + e^{-j\phi}h_{12}^{(i)} \sim \mathcal{CN}(0,2)$$
 so $\mathbb{E}\left[\left|h_{11}^{(i)} + e^{-j\phi}h_{12}^{(i)}\right|^2\right] = 2$ and $\mathbb{E}\left[\left|h_{11}^{(i)} + e^{-j\phi}h_{12}^{(i)}\right|^4\right] = 8$.

The cross correlation properties of the spatial diversity techniques show that 2×2 CDD has the lowest cross correlation between SINRs on different resource blocks among other spatial diversity techniques. Owing to this reduced cross correlation of post-processing SINRs across resource blocks, CDD effectively exploits frequency diversity gain in multi-carrier systems. Along with the SINR analysis in a single carrier system, the cross correlation properties characterize the performance of spatial diversity techniques in OFDMA systems well.

B. Spectral Efficiencies

This section presents system level simulation results based on the 3GPP contribution [27] and shows that our theoretical analysis can be consistently applied to predict the observed trends. Matched filter receivers are considered in the system level simulations and other simulation parameters and environments are the same as those of [27].

Fig. 6 shows the CDF of EESM in flat fading channels across resource blocks before frequency selective opportunistic scheduling. In the figure, the mean values of 1×2 RD, 2×2 STBC, and 2×2 CDD are 6.3 dB, 6.6 dB, and 6.4 dB, respectively, while the variances of 1×2 RD, 2×2 STBC, and 2×2 CDD are 46.1 dB, 37.9 dB, and 46.5 dB, respectively. Because the sub-carriers within a resource block are contiguous, the sub-carriers are typically highly correlated and hence the statistical characteristics of EESM of a specific resource block are similar to the post-processing SINR in a single carrier system. Correspondingly, the mean values of the spatial diversity techniques are similar and STBC has the lowest variance as in a single carrier system.

The CDF of EESM of scheduled users in flat fading channels is shown in Fig. 7. The distributions are quite changed after opportunistic scheduling in an OFDMA system. The mean value of 2×2 CDD after scheduling becomes 9.5 dB while the mean values of 1×2 RD and 2×2 STBC after scheduling are 6.5 dB and 6.9 dB, respectively. Since the proportional fair scheduling exploits not only the variation of each EESM but also EESM fluctuations across resource blocks, the mean value of 2×2 CDD after scheduling becomes the largest even though the SINR variances of 1×2 RD and 2×2 CDD are almost the same before proportional fair scheduling. These results confirm the theoretical observation that CDD induces more fluctuations across EESMs as

shown in Subsection IV-A and hence takes the most benefits from frequency selective user scheduling. The increased mean value of CDD owing to frequency selective multiuser diversity directly yields higher spectral efficiency. The average spectral efficiencies of 1×2 RD, 2×2 STBC, and 2×2 CDD are 1.17bps/Hz/sector, 1.21bps/Hz/sector, and 1.56bps/Hz/sector, respectively.

V. CONCLUSION

This paper has investigated the interaction between multiuser diversity and spatial diversity techniques in an interference-limited environment. An effective spatial diversity technique generally reduces SINR variance, but the reduced variation of SINR limits the multiuser diversity gain. We have quantified the interaction by using order statistic theory on an upper bound on the maximal random variable. The analytical and simulation results have shown that the cyclic delay diversity technique outperforms STBC in terms of both spectral efficiency and error probability and gets the most benefit under opportunistic scheduling even though STBC generally outperforms other transmit diversity techniques in a noise-limited environment.

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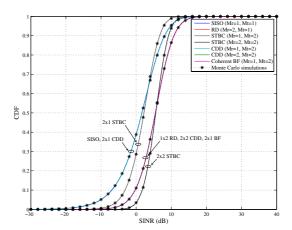


Fig. 1. CDF of post-processing SINRs where there are three (K=3) i.i.d. interferers with identical power P_I ($P_0/P_I=7.5 \mathrm{dB}$).

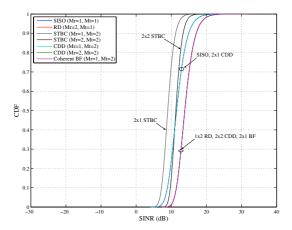


Fig. 2. CDF of maximum SINR when the number of users is 30 (U=30). Three (K=3) i.i.d. interferers with identical power P_I ($P_0/P_I=7.5 {\rm dB})$ are considered.

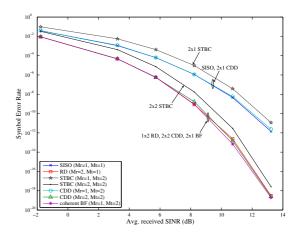


Fig. 3. Average symbol error rate (SER) for QPSK modulation versus average received SINR when the number of users is 30 (U=30). Three (K=3) i.i.d. interferers with identical power are considered. Average received SINR = $P_0/(3P_I)$.

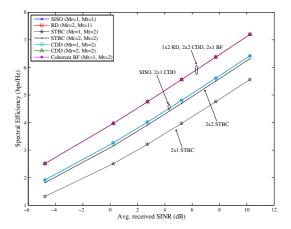


Fig. 4. Spectral efficiency versus average received SINR when the number of users is 30 (U=30). Three (K=3) i.i.d. interferers with identical power are considered. Average received SINR = $P_0/(3P_I)$.

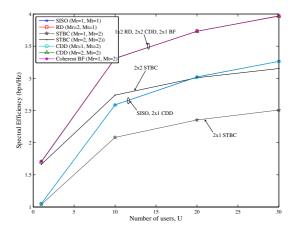


Fig. 5. Spectral efficiency versus average received SINR versus the number of users. Three (K=3) i.i.d. interferers with identical power are considered. Average received SINR = $P_0/(3P_I)$.

TABLE I $\label{eq:summary} \text{Summary of Analytical Results } (M_r \times M_t, K \text{ interferers with identical power } P_I, \text{ desired power } P_0)$

	$\mathbb{E}[\gamma]$	$\mathrm{Var}[\gamma]$	$f_{\gamma}(\gamma)$
SISO	$\frac{P_0}{P_I(K-1)}$	$\frac{P_0^2}{P_I^2} \frac{K}{(K-1)^2(K-2)}$	$\frac{P_I}{P_0} \frac{\Gamma(1+K)}{\Gamma(1)\Gamma(K)} \frac{1}{\left(1 + \frac{P_I}{P_0} \gamma\right)^{1+K}}$
$RD \ (M_t = 1)$	$\frac{P_0 M_r}{P_I (K-1)}$	$\frac{P_0^2}{P_I^2} \frac{M_r(M_r + K - 1)}{(K - 1)^2(K - 2)}$	$\frac{P_I}{P_0} \frac{\Gamma(M_r + K)}{\Gamma(M_r)\Gamma(K)} \frac{(\frac{P_I \gamma}{P_0})^{M_r - 1}}{\left(1 + \frac{P_I \gamma}{P_0}\right)^{M_r + K}}$
STBC	$\frac{P_0 M_r M_t}{P_I (K M_t - 1)}$	$\frac{P_0^2}{P_I^2} \frac{M_r M_t (M_r M_t + K M_t - 1)}{(K M_t - 1)^2 (K M_t - 2)}$	$\frac{P_I}{P_0} \frac{\Gamma(M_r M_t + K M_t)}{\Gamma(M_r M_t) \Gamma(K M_t)} \frac{\left(\frac{P_I \gamma}{P_0}\right)^{M_r M_t - 1}}{\left(1 + \frac{P_I \gamma}{P_0}\right)^{M_r M_t + K M_t}}$
CDD	$\frac{P_0 M_r}{P_I (K-1)}$	$\frac{P_0^2}{P_I^2} \frac{M_r(M_r + K - 1)}{(K - 1)^2(K - 2)}$	$\frac{P_I}{P_0} \frac{\Gamma(M_r + K)}{\Gamma(M_r)\Gamma(K)} \frac{(\frac{P_I \gamma}{P_0})^{M_r - 1}}{\left(1 + \frac{P_I \gamma}{P_0}\right)^{M_r + K}}$
Coherent BF $(M_r = 1)$	$\frac{P_0 M_t}{P_I (K-1)}$	$\frac{P_0^2}{P_I^2} \frac{M_t(M_t + K - 1)}{(K - 1)^2(K - 2)}$	$\frac{P_I}{P_0} \frac{\Gamma(M_t + K)}{\Gamma(M_t)\Gamma(K)} \frac{(\frac{P_I\gamma}{P_0})^{M_t - 1}}{\left(1 + \frac{P_I\gamma}{P_0}\right)^{M_t + K}}$

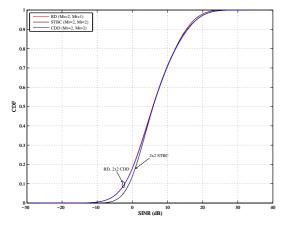


Fig. 6. CDF of EESM for all resource blocks in flat fading channels before frequency selective opportunistic scheduling.

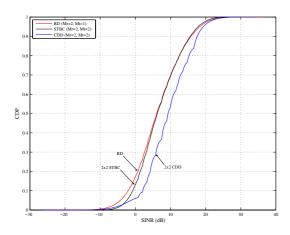


Fig. 7. CDF of EESM in flat fading channels after frequency selective opportunistic scheduling.

 $\label{eq:table II} {\rm TABLE~II}$ 2×2 antenna configuration with $U=1~(K=3~{\rm and}~P_s/P_I=7.5{\rm dB})$

	$\mathbb{E}[\gamma]$ in dB	$Var[\gamma]$ in dB
SISO (reference)	4.47	13.52
$1 \times 2 \text{ RD}$	7.48	17.78
$2 \times 1 \text{ STBC}$	3.53	6.47
2×2 STBC	6.51	10.52
$2 \times 1 \text{ CDD}$	4.47	13.61
$2 \times 2 \text{ CDD}$	7.51	17.70
2×1 BF	7.51	17.63

TABLE III $2\times 2 \text{ antenna configuration with } U=30 \text{ } (K=3 \text{ and }$ $P_s/P_I=7.5 \text{dB})$

	$\mathbb{E}[\max_{U}$	$[x \gamma] (dB)$	$Var[max_U \gamma]$ (dB)
	Simulation	Analy. upper bound	
SISO	12.6	13.1	23.5
$1 \times 2 \text{ RD}$	14.8	15.4	28
$2 \times 1 \text{ STBC}$	9.5	10.1	12
2×2 STBC	11.7	12.3	15.5
$2 \times 1 \text{ CDD}$	12.6	13.1	23.7
$2 \times 2 \text{ CDD}$	14.8	15.4	28
2×1 BF	14.8	15.4	28