Multi-RAT Wireless Network Capacity Optimization under Optimal Spectrum Splitting in LTE-U

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***Abstract*—Efficient utilization of limited spectrum resource has been a major issue in wireless communication systems, which has triggered the design and development of advanced cellular networks. Recently, heterogeneous network (HetNet) with small cells to satisfy the demand for higher data rate transmission and throughput has attracted a lot of attention in the unlicensed spectrum bands environment to overcome the scarcity of licensed spectrum. In this paper, we consider network capacity optimization problem for HetNet comprised of Long-Term Evolution (LTE) small cell and wireless fidelity (Wi-Fi) in LTE unlicensed spectrum (LTE-U) with optimal spectrum splitting ratio. We derive coverage probabilities and average achievable rates for each radio access technologies (RATs) based on the conditions of user equipment (UE) association in multi-RAT wireless network system. Then the optimal unlicensed spectrum splitting ratio can be obtained by maximizing HetNet capacity under the derived constraints on respective coverage probabilities. Simulation results reveal effectiveness of the proposed scheme, where both the spectrum splitting ratio and transmit power ratio are key parameters. This analysis can be easily extended to HetNets with multiple RATs.**

I. INTRODUCTION

The data traffic in cellular network today is booming at an exponential rate as technologies of user equipment (UE) and the corresponding applications are developing at a speed they have never experienced. To satisfy the proliferation of data-hungry devices and applications, heterogeneous cellular network (HCN) architecture with multi radio access technologies (RATs) has been adapted. Deployment of small cell access point (AP) with lower transmit power, such as femtocell, picocell and microcell, can successfully offload data from macro base stations (BSs) without inter-cell interference [1]. At the same time, the scarcity of licensed spectrum for cellular network and severe co-channel interference between small cells and macrocell BSs force wireless industries take the utilization of unlicensed spectrum bands, like Long-Term Evolution in unlicensed spectrum (LTE-U) into account. As the most important candidates, industrial, scientific and medical (ISM) radio bands such as 2.4 GHz and 5 GHz bands where wireless fidelity (Wi-Fi) operated have been considered [2]. In [3], a fair and quality of service (QoS) based unlicensed spectrum splitting strategy between femtocell and Wi-Fi networks was proposed even though inter-femtocell/inter-Wi-Fi is not considered. It is well known that the utilization of Wi-Fi systems on offloading data traffic from macrocell BSs has achieved astonishing performance [4]. The coverage probabilities investigated in all prior works are only for single-RAT cellular networks and their results cannot completely characterize the coverage of a multi-RAT network [5] and as two of the most important performance metrics to evaluate the performance of coexistence of different RATs, coverage probabilities and throughputs of all the coexisting subnetworks were defined and optimized over channel number and density ratio in [5]. With the main challenge for LTE-U that LTE operation could take over the bands and force Wi-Fi to move to silent mode due to the carrier sense multiple access with collision avoidance (CSMA/CA) feature, elaborate design of LTE-U system is essential [6], [7]. The main taxonomy of enabling mechanisms for LTE/Wi-Fi coexistence according to the radio access technology are flexible spectrum access, channel selection, blank subframes and transmit power control [8]. Therefore, an appropriate design of spectrum allocation has significant meaning to the heterogeneous network (HetNet) in LTE-U.

In this work, we provide an efficient spectrum resource allocation scheme for HetNet in LTE-U to maximize network capacity with an optimal unlicensed spectrum splitting ratio under the constraints of coverage probability for each RAT. Firstly, we construct interworking of HCN and Wi-Fi system in LTE-U by using a weighted Voronoi diagram. Secondly, we analyze the conditions for UE association and derive the probability distribution function of distance between UE and the associated AP. Thirdly, we derive the coverage probability and average achievable rate for UE in the downlink with associated AP and formulate a HetNet capacity optimization problem over an unlicensed spectrum splitting ratio with constraints on coverage probabilities. The aggregate characteristic function of Voronoi diagram in HetNet is adopted and analyzed in this part. Simulation results not only validate the effectiveness of the proposed algorithm, but also investigate the sensitivity of the coverage probability and HetNet capacity in terms of the unlicensed spectrum splitting ratio and the transmit power ratio between small cell and Wi-Fi APs.

The rest of this paper is organized as follows. Section II

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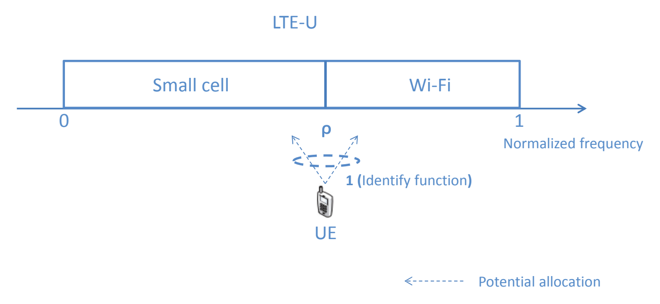

Fig. 1. An illustration for system model of HetNet in LTE-U with multi-RAT.

describes the HetNet system model in a weighted Voronoi tessellation. Section III analyzes the conditions for UE association, derives coverage probability and average achievable rate, and formulates the HetNet capacity optimization problem in terms of unlicensed spectrum splitting ratio. In Section IV, we address the algorithm to maximize HetNet capacity with constraints on coverage probabilities. A set of simulations are presented in Section V to show the superiority of the proposed algorithm. At last, Section VI presents conclusion.

II. SYSTEM MODEL

We consider a HCN and Wi-Fi coexisted HetNet including one macro BS, small cell and Wi-Fi APs, and UE. The HetNet is represented by a two dimension graph $\mathbf{G}(\{m, \mathbf{S}, \mathbf{W}\}, \mathbf{U})$. In $\mathbf{G}$, there are three classes of APs: m is the macro BS; $\mathbf{S}$ is the set of small cell APs; and $\mathbf{W}$ is the set of Wi-Fi APs. $\mathbf{U}$ is the set of UE. $\mathbf{S}$, $\mathbf{W}$ and $\mathbf{U}$ follow Poisson Point Process (PPP) distribution, respectively. A system model for HetNet with unlicensed spectrum splitting ratio in LTE-U is illustrated in Fig. 1.

A. Node Function

Different with the conventional Poisson-Voronoi (PV) system of homogeneous network [9], the PV system of HetNet has irregular tessellation since different classes of APs in the HetNet have different average transmit powers, supported data rates and AP densities [1]. Then a PV partition for HetNet coverage $\mathbf{V}$ with mutually disjoint interiors $\mathbf{V}_i, i \in \mathbf{S} \cup \mathbf{W}$ is defined as [10]

$$\mathbf{V} = \cup_{i \in \{\mathbf{S} \cup \mathbf{W}\}} \mathbf{V}_i, \tag{1}$$

and

$$\mathbf{V}_i = \{u \in \mathbf{V} | h(i,u) \ge h(k,u)\} \qquad \forall i,k \in \{\mathbf{S} \cup \mathbf{W}\}, k \neq i \tag{2}$$

where $u \in \mathbf{U}$ and h is a given non-negative node function [11]. Then UE u is served by AP i and indicated by u^i. As we known, the serving AP i can be small cell AP or Wi-Fi AP and we assume the AP set that i belongs to is $\mathbf{I}$ and the other one is $\mathbf{J}$, e.g. $\mathbf{I}$ is $\mathbf{S}$ and $\mathbf{J}$ is $\mathbf{W}$, or the reverse.

In Voronoi tessellation for homogeneous networks, node function h as shown in (2) is related with distance. In HetNet, cell selection based on maximum mean SINR is more reasonable, which is exactly the same with the maximum received power in single-input single-output (SISO) HetNet [13]. Then we propose a new node function for HetNet as follow

$$h(i,q) = p_{iq}^r, \tag{3}$$

where p_{iq}^r is the received power from AP i at UE q.

B. Channel Model

We use Rayleigh fading model not only for the desired signal but also for the interference in this paper, which results in a random channel power gain in exponential distribution with average unit mean. Then the received power p_{iq}^r at UE q from AP i is shown as

$$p_{iq}^r = p_{iq}^t g_{iq} d_{iq}^{-\alpha}, \tag{4}$$

where p_{iq}^t is the transmit power from AP i. The random variable g_{iq} accounts for multipath fading follows an exponential distribution with mean $\frac{1}{\mu}$. $d_{iq} = \sqrt{|x_i - x_q|^2 + |y_i - y_q|^2}$ is the distance between nodes i and q, where (x_i, y_i) and (x_q, y_q) denote the location between nodes i and q in the Voronoi tessellation. α is the path loss index. For simplicity, we assume that each transmitter uses the maximal transmit power and the received SINR mainly depends on the channel gain [14].

Based on the above assumptions, every node in the PV system is covered by at least one Voronoi cell. For two adjacent Voronoi cells, like $\mathbf{V}_i$ and $\mathbf{V}_j$, are separated by their common edge, which is composed of nodes as shown as

$$p_{iq}^r = p_{jq}^r. \tag{5}$$

Using same α for each channel, the distance relationship between adjacent Voronoi cells and the nodes on their common edge is shown as

$$\frac{d_{iq}}{d_{jq}} = (\frac{p_{iq}^t}{p_{jq}^t})^{\alpha^{-1}}. \tag{6}$$

III. PROBLEM FORMULATION

A. UE Association

In SISO HetNet with a weighted Voronoi tessellation, each UE is allocated to a Voronoi cell based on the received power as shown in (3). A set of UE allocated to AP i, which means the UE aggregate in Voronoi cell $\mathbf{V}_i$, is shown as

$$\mathbf{U}^i \overset{\text{def}}{=} \cup\{u^i : u \in \mathbf{U} \cap \mathbf{V}_i\}. \tag{7}$$

To analyze UE aggregate characteristics for each RAT in HetNet, we introduce a non-negative function $f(u)$ defined on $\mathbf{R}^2$ which is shown as [11]:

$$f(u^i) = \sum_{u \in \mathbf{U}} f(u) \mathbf{1}\{u \in \mathbf{V}_i\}. \tag{8}$$

where $\mathbf{1}$ is the identify function between AP i and UE u and shown as

$$\mathbf{1} = \begin{cases} 1, & \text{if } u \in \mathbf{V}_i, \\ 0, & \text{otherwise}. \end{cases} \tag{9}$$

Then,

$$\mathbf{E}\{f(u^i)\} = \lambda_0 \mathbf{E}\{\int f(u)\mathbf{1}\{u \in \mathbf{V}_i\}\}, \tag{10}$$

where λ_0 is the density of UE in HetNet.

Based on the above assumptions, the conditions that UE u is associated with AP i are shown as follow

$$\begin{aligned} &\{u \in \mathbf{V}_i\} \\ &\overset{\text{def}}{=} \{N^I = 0, N^J = 0, p^r_{iu} > p^r_{ju}\} \\ &= \{N^I = 0, N^J = 0, d_{ju} > (\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}} d_{iu}\}, \end{aligned} \tag{11}$$

where the N^I and N^J are the numbers of APs from I and J using unlicensed spectrum in the circles with radius d_{iu} and d_{ju} centered in u, respectively. In (11), UE u is allocated to Voronoi cell $\mathbf{V}_i$ if and only if the following conditions are satisified:

① There is no APs from $\mathbf{I}$ in the circle that centered in u with radius d_{iu}.

② There is no APs from $\mathbf{J}$ in the circle that centered in u with radius d_{ju}.

③ AP j shows disadvantage than AP i with bias parameter $(\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}}$.

In the perspective of distance, the conditions are that the serving AP i is the nearest AP among all the APs in $\mathbf{I}$ at a distance d_{iu} to UE i and shows advantage in distance than the nearest AP j from $\mathbf{J}$ in a distance d_{ju} to UE i with bias parameter $(\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}}$.

Based on the above analysis, the null probability of d_{iu} is shown as follow

$$\begin{aligned} &\mathbf{P}\{N^I = 0, N^J = 0, d_{ju} > (\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}} d_{iu}\} \\ &= e^{-(\lambda_I \pi {d_{iu}}^2 + \lambda_J \pi \{(\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}} d_{iu}\}^2)}, \end{aligned} \tag{12}$$

where λ_I and λ_J are densities of APs with unlicensed spectrum from I and J, respectively. If we allocate same bandwidth to each communication channel in HetNet, then λ_I and λ_J can express densities of unlicensed spectrum allocated to AP classes I and J.

Therefore, with the assumption that d_{iu} and d_{ju} are the distances of the closet APs from each RAT to UE u, the cumulative distribution function (cdf) of d_{iu} is

$$\begin{aligned} \mathbf{P}\{d_{ju} \leq (\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}} d_{iu}\} &= \mathbf{F}_{d_{ju}}((\frac{p^t_{iu}}{p^t_{ju}})^{-\alpha^{-1}} d_{iu}) \\ &= 1 - e^{-\phi \pi {d_{iu}}^2}, \end{aligned} \tag{13}$$

where $\phi = (\lambda_I + \lambda_J (\frac{p^t_{iu}}{p^t_{ju}})^{-2\alpha^{-1}})$. Therefore, the pdf can be found as

$$f_r(r) = e^{-\phi \pi r^2} 2\pi\phi r, \tag{14}$$

if we use r to replace d_{iu}.

B. Coverage Probability

The coverage probability of a randomly located UE in the HetNet is conditioning on the tagged AP being at a distance r from the UE as follow

$$\begin{aligned} p^c_i(\lambda_I, \lambda_J, \beta, \alpha) &= \mathbf{E}_r[\mathbf{P}[SINR > \beta | r]] \\ &= \int_{r>0} \mathbf{P}[SINR > \beta | r] f_r(r) dr \\ &= \int_{r>0} \mathbf{P}[\frac{p^t_{iu} g r^{-\alpha}}{\sigma^2 + \mathbf{In}_r} > \beta | r] f_r(r) dr \\ &= \int_{r>0} \mathbf{P}[g > \beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r) | r] f_r(r) dr, \end{aligned} \tag{15}$$

where SINR is the received signal-to-interference-plus-noise ratio and β is the threshold for it. σ^2 and In_r are white Gaussian noise and co-channel interference power, respectively.

With the fact that $g \sim exp(\mu)$ [15], we can get

$$\begin{aligned} &\mathbf{P}[g > \beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r) | r] \\ &= \mathbf{E}_{\mathbf{In}_r}[g > \beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r) | r, \mathbf{In}_r] \\ &\overset{\text{(a)}}{=} \mathbf{E}_{\mathbf{In}_r}[exp(-\mu\beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r)) | r] \\ &= e^{-\mu\beta {p^t_{iu}}^{-1} r^\alpha \sigma^2} \mathcal{L}_{\mathbf{In}_r}(\mu\beta {p^t_{iu}}^{-1} r^\alpha) \end{aligned} \tag{16}$$

and the proof for (a) is shown as

$$\begin{aligned} &\mathbf{P}[g > \beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r)] \\ &= 1 - \mathbf{P}[g \leq \beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r)] \\ &= 1 - (1 - exp(-\mu\beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r))) \\ &= exp(-\mu\beta {p^t_{iu}}^{-1} r^\alpha (\sigma^2 + \mathbf{In}_r)). \end{aligned} \tag{17}$$

Then we have

$$\begin{aligned} \mathcal{L}_{\mathbf{In}_r}(\mu\beta {p^t_{iu}}^{-1} r^\alpha) &= \mathbf{E}_{\Phi,g}[\prod_{i \in \Phi} \mathbf{E}_g[exp(\mu\beta {p^t_{iu}}^{-1} r^\alpha)]] \\ &\overset{\text{(b)}}{=} \mathbf{E}_\Phi[\prod_{i \in \Phi} \frac{1}{1 + \beta {p^t_{iu}}^{-1} r^\alpha v^{-\alpha}}] \\ &= exp(-2\pi\phi \int_r^\infty \frac{\beta}{\beta + (\frac{v}{r})^\alpha} v dv) \end{aligned} \tag{18}$$

with $r^2 \rightarrow v$ and Φ is the interference set for the downlink communication between AP i and UE u.

The proof for (b) is shown as

$$\begin{aligned} \mathbf{E}_g[exp(\mu\beta {p^t_{iu}}^{-1} r^\alpha)] &= \int exp(\mu\beta {p^t_{iu}}^{-1} r^\alpha) \mu exp(-\mu g) d_g \\ &= \frac{1}{1 + \beta {p^t_{iu}}^{-1} r^\alpha v^{-\alpha}}. \end{aligned} \tag{19}$$

C. Average Achievable Rate

The average ergodic achievable rate c_I for UE u in the downlink communication from any AP i in I can be achieved by using the same procedure as used for coverage probability in (III-B) and the result is shown as

$$\begin{aligned} c_I(\lambda_I, \lambda_J, \alpha) &= \mathbf{E}[B \log_2(1 + SINR)] \\ &= \int_{r>0} e^{-\phi\pi r^2} \omega 2\pi\phi r dr, \end{aligned} \tag{20}$$

where

$$\omega = \int_{t>0} e^{-\mu {p_{iu}^t}^{-1} r^\alpha \sigma^2 (2^{\frac{t}{B}} - 1)} \mathcal{L}_{\mathbf{In}_r}(\mu {p_{iu}^t}^{-1} r^\alpha (2^{\frac{t}{B}} - 1)) dt \tag{21}$$

with B as the channel bandwidth and

$$\begin{aligned} &\mathcal{L}_{\mathbf{In}_r}(\mu {p_{iu}^t}^{-1} r^\alpha (2^{\frac{t}{B}} - 1)) \\ &= exp(-2\pi\phi \int_r^\infty (1 - \frac{1}{1 + {p_{iu}^t}^{-1} (\frac{r}{v})^\alpha (2^{\frac{t}{B}} - 1)}) v dv). \end{aligned} \tag{22}$$

IV. Optimization Problem

A. *HetNet Capacity Optimization*

Considering $f(u) = 1$ in (10), we can get the expected number of UE N for AP i in Voronoi tessellation as follow [11], [12]:

$$\mathbf{E}[N_i] = \frac{\lambda_0}{\lambda_I + \lambda_J (\frac{p_i^t}{p_j^t})^{-2\alpha^{-1}}}. \tag{23}$$

We assume the density proportions of unlicensed spectrum for small cell and Wi-Fi APs are ρ and $(1 - \rho)$, respectively, and $\lambda = \lambda_I + \lambda_J$. Then, $p_i^c(\lambda_I, \lambda_J, \beta, \alpha)$, $c_i(\lambda_I, \lambda_J, \alpha)$ can change into $p_i^c(\lambda, \rho, \beta, \alpha)$,$c_i(\lambda, \rho, \alpha)$ and it is easy to get $p_j^c(\lambda, \rho, \beta, \alpha)$,$c_j(\lambda, \rho, \alpha)$ with slight modifications.

We also assume the coverage of a macro BS is Ar. Then the aggregated achievable ergodic rate of HetNet $C_t(\rho)$ can be expressed as (24) with assumption that AP classes I and J are small cell and Wi-Fi APs, respectively.

$$\begin{aligned} C_t(\rho) = Ar\rho\lambda_0 & \frac{1}{\rho + (1 - \rho)(\frac{p_s^t}{p_w^t})^{-2\alpha^{-1}}} c_I(\lambda, \rho, \alpha) \\ + Ar(1 - \rho)\lambda_0 & \frac{1}{(1 - \rho) + \rho(\frac{p_w^t}{p_s^t})^{-2\alpha^{-1}}} c_J(\lambda, \rho, \alpha), \end{aligned} \tag{24}$$

where $c_I(\lambda, \rho, \alpha)$ and $c_J(\lambda, \rho, \alpha)$ are from (20).

Then the HetNet capacity optimization problem with unlicensed spectrum splitting ratio can be expressed as

$$\begin{aligned} \max_{\rho} \quad & C_t(\rho) \\ \text{s.t.} \quad & p_s^c \geq p_{thr,s}^c, \\ & p_w^c \geq p_{thr,w}^c, \\ & 0 < \rho < 1, \end{aligned} \tag{25}$$

where $p_{thr,s}^c$ and $p_{thr,w}^c$ are the coverage probability thresholds for a typical UE associated with small cell and Wi-Fi APs, respectively. Note that the HetNet becomes homogeneous network when ρ is 0 or 1, which is out of the scope of this paper even though it is easy to extend by using the same procedure.

Algorithm 1: Proposed algorithm for HetNet capacity optimization.

```
Input: $G(\{m, S, W\}, U)$, $\lambda$
Output: $\rho*$ (Optimal unlicensed spectrum splitting ratio)

1  Initialize Blacklist
2  begin
3      Get maximum $C_t(\rho)$ by using global optimization techniques and the
       corresponding $\rho$.
4      With obtained $\rho$, get $p_s^c(\rho)$ and $p_w^c(\rho)$.
5      if $p_s^c \geq p_{thr,s}^c, p_w^c \geq p_{thr,w}^c$, then
6          Stop.
7      else
8          Add $C_t(\rho)$ and the corresponding $\rho$ in the Blacklist and return to
           Step 3.
9      end
10 end
```

B. *Optimization Method*

The HetNet capacity optimization problem in (25) can be solved by using global optimization techniques. Then based on the optimal unlicensed spectrum splitting ratio ρ^*, unlicensed spectrum can be optimally allocated to small cell and Wi-Fi APs.

The method for HetNet capacity optimization with an optimal unlicensed spectrum splitting ratio is shown in Algorithm 1.

V. NUMERICAL RESULTS

This paper considers HetNet within one macro BS cell coverage with dimensions $0.5km \times 0.5km$. In the simulation, density of UE λ_0 is 10000 per km^2 and density of AP with unlicensed spectrum λ is 0.8 per km^2, which includes small cell and Wi-Fi APs. The transmit power for small cell is set to $10mW$ while that for Wi-Fi is from $10mW$ to $200mW$. The propagation loss factor α is considered to be 4 while the bandwidths for small cell and Wi-Fi APs communication channels are considered to be $1MHz$. The coverage probability thresholds $p_{thr,s}^c$ and $p_{thr,w}^c$ are given as 0.3 and 0.35, respectively. It is worth to mention that based on the different scenario requirement of each RAT, the coverage probability thresholds $p_{thr,s}^c$ and $p_{thr,w}^c$ can be setted to different values.

Fig. 2 and Fig. 3 show the coverage probabilities for UE associated with small cell APs and Wi-Fi APs as a function of unlicensed spectrum splitting ratio ρ with different power ratios, respectively. We can see with the transmit power increasing, the coverage probability for UE associated with small cell APs decreases while that for UE associated with Wi-Fi APs increases. Since these coverage probabilities are achieved by analyzing numerical results, they show some fluctuations which can be avoided by improving the accuracies with longer running time.

Fig. 4 shows HetNet capacity as a function of unlicensed spectrum splitting ratio ρ with different power ratios. The HetNet capacity achieves maximum value when ρ is optimal and that value increases when the power ratio increases with the optimal value of ρ approaching to 1. It shows that HetNet capacity does not change with different value of ρ when transmit power is 1, which is due to there is no difference on the network structure for different AP density ratio in this case.

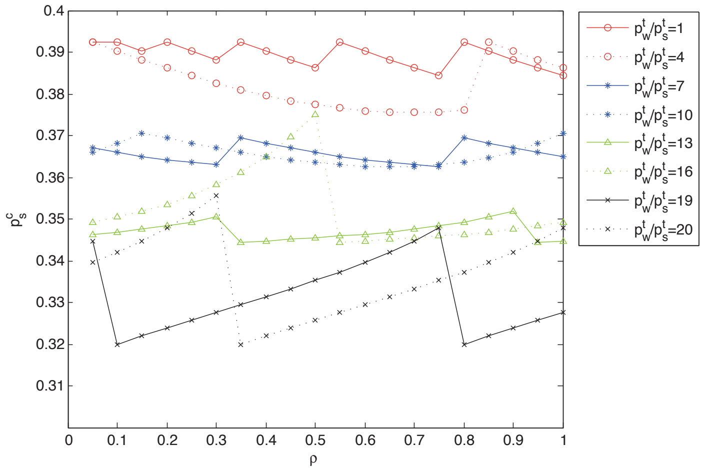

Fig. 2. Coverage probability for UE associated with small cell APs as a function of ρ.

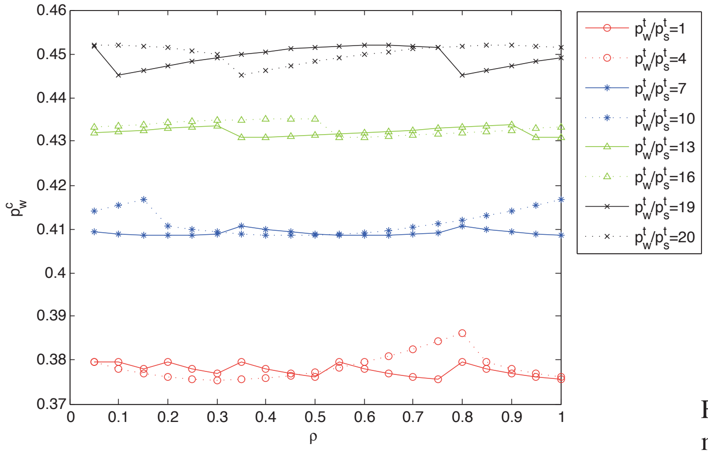

Fig. 3. Coverage probability for UE associated with Wi-Fi APs as a function of ρ.

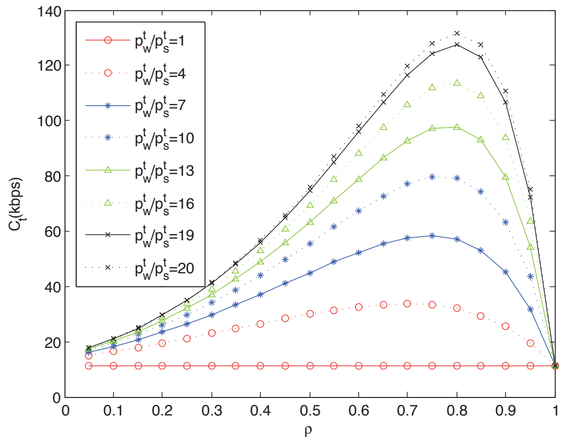

Fig. 4. HetNet capacity as a function of ρ.

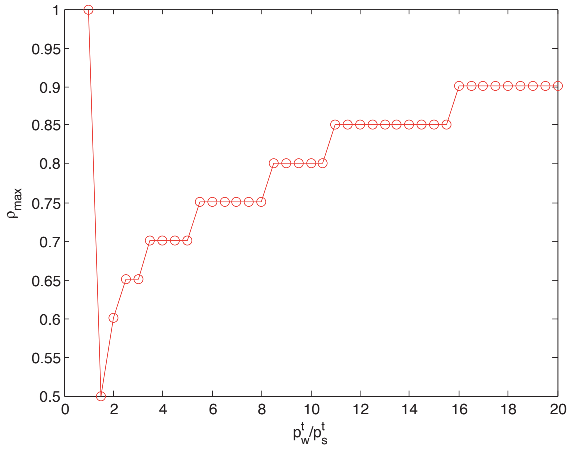

Fig. 5. Optimal ρ with maximum HetNet capacity for transmit power ratios from 1 to 20.

With the transmit power increasing, the maximum HetNet capacity is getting bigger, which reveals the fact that optimal AP density ratio is more important to the HetNet systems with bigger transmit power ratio.

Fig. 5 reveals that the value of optimal ρ gets increasing when transmit power ratio increases. The HetNet capacity keeps same with different value of ρ when transmit power ratio is 1 as shown in Fig. 4 and the optimal value for ρ is randomly chosen to be 1 in this figure, which is still bring into correspondence with the results in other cases.

In Fig. 6, the optimized result of the proposed method is compared with that of Monte Carlo method in terms of maximum HetNet capacity versus the ratio of transmit powers from 1 to 20. In the figure, the HetNet capacity with optimal ρ obtained by using the proposed method shows superior result than ρ in other values.

In Fig. 7, the optimized result of the proposed method is compared with that method, which splits unlicensed spectrum based on the network traffic in small cell and Wi-Fi APs, which is the most accepted method in related works, in terms of maximum HetNet capacity versus the ratio of transmit powers from 1 to 20. In this paper, the density of UE and bandwidth for each channels are constant, then the network traffic for small cell and Wi-Fi APs is only related with coverage probabilities, which are already obtained in (III-B). In the figure, the proposed method also achieved better HetNet capacity with optimal ρ than that when the unlicensed spectrum is shared based on the network traffic by small cell and Wi-Fi APs.

VI. CONCLUSION

In this paper, we proposed an algorithm to maximize HetNet capacity by optimally allocating unlicensed spectrum to small

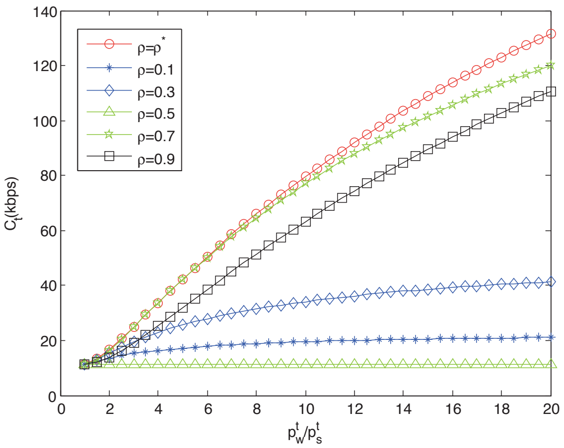

Fig. 6. Comparison of HetNet capacity with that of the Monte Carlo method.

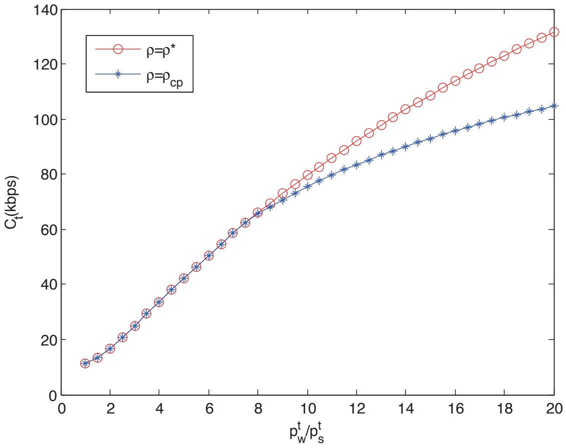

Fig. 7. Comparison of HetNet capacity with that method of the spectrum splitting based on the network traffic.

cell and Wi-Fi APs in HCN and Wi-Fi coexisting LTE-U environment. We adopted a weighted Voronoi tessellation to construct HetNet system model. Under the conditions of UE association, we derived the coverage probabilities and average achievable rates for UE in each RAT to formulate the HetNet capacity optimization problem by using the characteristic of Voronoi diagram. Then, the optimization problem was solved by global optimization techniques with an optimal unlicensed spectrum splitting ratio. Simulation results demonstrated the improved effectiveness of the proposed algorithm, and emphasized the sensitivity and importance of the unlicensed spectrum splitting ratio and the transmit power ratio to the HetNet capacity. This analysis can be simply extended to HetNets with more than two RATs.

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