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Construction and Building Materials 18 (2004) 753–766

Construction and Building MATERIALS

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Design moment variations in bridges constructed using a balanced cantilever method

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Received 25 March 2003; received in revised form 6 April 2004; accepted 8 April 2004
Available online 31 July 2004

Abstract

This paper introduces simple, but effective, equations to calculate the dead load and cantilever tendon moments in reinforced concrete (RC) bridges constructed using the balanced cantilever method (FCM). Through time-dependent analyses of RC bridges considering the construction sequence and creep deformation of concrete, structural responses related to the member forces are reviewed. On the basis of the compatibility condition and equilibrium equation at every construction stage, basic equations which can describe the moment variation with time in balanced cantilever construction are derived. These are then extended to take into account the moment variation according to changes in the construction steps. By using the introduced relations, the design moment and its variation over time can easily be obtained with only the elastic analysis results, and without additional time-dependent analyses considering the construction sequences. In addition, the design moments determined by the introduced equations are compared with the results from a rigorous numerical analysis with the objective of establishing the relative efficiencies of the introduced equations.

Keywords: Balanced cantilever method; RC bridges; Construction sequence; Creep; Dead load moment; Cantilever tendon moment

1. Introduction

In accordance with the development of industrial society and global economic expansion, the construction of long span bridges has increased. Moreover, the construction methods have undergone refinement, and they have been further developed to cover many special cases, such as progressive construction of cantilever bridges and span-by-span construction of simply supported or continuous spans. Currently, among these construction methods, balanced cantilever construction of concrete box-girder bridges has been recognized as one of the most efficient methods of building bridges as it does not require falsework. This method has great advantages over other kinds of construction, particularly in urban areas where temporary shoring would disrupt traffic and service below, in deep gorges, and over waterways where falsework would be not only expensive but also a hazard.

However, the design and analysis of bridges constructed by the balanced cantilever method (FCM) require the consideration of the internal moment redistribution which takes place over the service life of a structure because of the time-dependent deformation of concrete and changes in the structural system repeated during construction. This means that, to preserve the safety and serviceability of the bridge, an analysis of bridges which considers the construction sequence must be performed. All the related bridge design codes [1,6] have also mentioned the need to consider the internal moment redistribution due to creep and shrinkage of concrete when the structural system is changed during construction.

Several studies have dealt with the general topics of design and analysis of segmentally erected bridges, while a few studies have been directed toward the analysis of

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0950-0618/$ - see front matter
doi:10.1016/j.conbuildmat.2004.04.021

the deflection and internal moment redistribution in segmental bridges [3,7,22]. Bishara and Papakonstantinou [3] investigated the time-dependent deformation of cantilever construction bridges both before and after closure, and Cruz et al. [8] introduced a nonlinear analysis method for the calculation of the ultimate strength of bridges. Articles on the design, analysis and construction of segmental bridges have been published by many researchers, and detailed comparisons have been made between analytical results and responses measured in actual structures [11,12,17,20].

Moreover, development of sophisticated computer programs for the analysis of segmental bridges considering the time-dependent deformation of concrete has been followed [10]. Most analysis programs, however, have some limitations on wide use because of complexities in practice applications. Consequently, a simple formula for estimating the internal moment redistribution due to creep and shrinkage of concrete, which is appropriate for use by a design engineer in the primary design of bridges, has been continuously required. Trost and Wolff [22] introduced a simple formula which can simulate internal moment redistribution with a superposition of the elastic moments occurring at each construction step. A similar approach has been presented by the Prestressed Concrete Institute (PCI) and the Post-Tensioning Institute (PTI) on the basis of the force equilibrium and the rotation compatibility at the connecting point [4]; however, these formulas do not adequately address the changing structural system because of several adopted simplifying assumptions.

In this paper, simple, but effective, formulas are introduced which can calculate the internal moment redistribution in segmental bridges after completion of construction. With previously developed computer programs [13–17], many parametric studies for bridges erected by the FCM are conducted, and correlation studies between the obtained numerical results with those by the introduced formulas are included to verify the applicability of the introduced formulas. Finally, a reasonable guideline to determine the internal design moments, which are essential in selecting a proper initial section, is proposed.

2. Construction sequence analysis

2.1. Construction sequences

Every nonlinear analysis algorithm consists of four basic steps: the formulation of the current stiffness matrix, the solution of the equilibrium equations for the displacement increments, the stress determination of all elements in the model, and the convergence check. The preceding papers [13–17] presented an analytical model to predict the time-dependent behavior of bridge structures. Experimental verification and correlation studies between analytical and field testing results were conducted to verify the efficiency of the proposed numerical model. The rigorous time dependent analyses in this paper are performed with the introduced analytical model. Details of the analytical model can be found in the preceding papers [13–17]. In advance, all the material properties related to a tendon, from the definition of stress–strain relation to the formulation of relaxation, can also be found elsewhere [13,14].

Balanced cantilever construction is the term for when a phased construction of a bridge superstructure starts from previously constructed piers cantilevering out to both sides using the cantilever tendons. Each cantilevered part of the superstructure is tied to a previous one by concreting a key segment and post-tensioning tendons. The same erection process is repeated until the structure is completed. After continuing all spans in the structure, the continuous tendons are finally installed along the spans; consequently the internal moment is continuously changed according to the construction sequence and the changing structural system during construction. This means that the cantilever tendons and the dead load dominantly affect the internal moment variation because these two force components accompany the secondary moments caused by the concrete creep deformation with the changing structural system. On these backgrounds, to review structural responses due to changes in the construction sequence, three different cases, FCM1, FCM2 and FCM3, shown in Fig. 1, are selected for this paper.

For time-dependent analysis of bridges that considers the construction sequence, a five-span continuous bridge is selected as an example structure. This bridge has a total length of 150 m with an equal span length of 30 m, and maintains a prismatic box-girder section along the span length. The assumed material and sectional properties are taken from a real bridge and are summarized in Table 1, and the tendon properties are also presented in Table 2. The creep deformation of concrete is considered on the basis of the ACI model with an ultimate creep coefficient of $\phi_{cr}^{\infty} = 2.35$ [2].

As shown in Fig. 1, the time interval between each construction step is assumed to be 50 days. FCM 1 is designed to describe the construction sequence in which construction of all the cantilever parts of the superstructure is finished first at the reference time $t = 0$ day. The continuity of the far end spans and center span follows at $t = 50$ days, and then the construction of the superstructure is finally finished at $t = 100$ days by concreting the key segments at the midspans of the second and fourth spans. FCM 2 describes the continuity process marching from the far end spans to the center span, and FCM 3 describes the step-by-step continuity of the proceeding spans from a far end span.

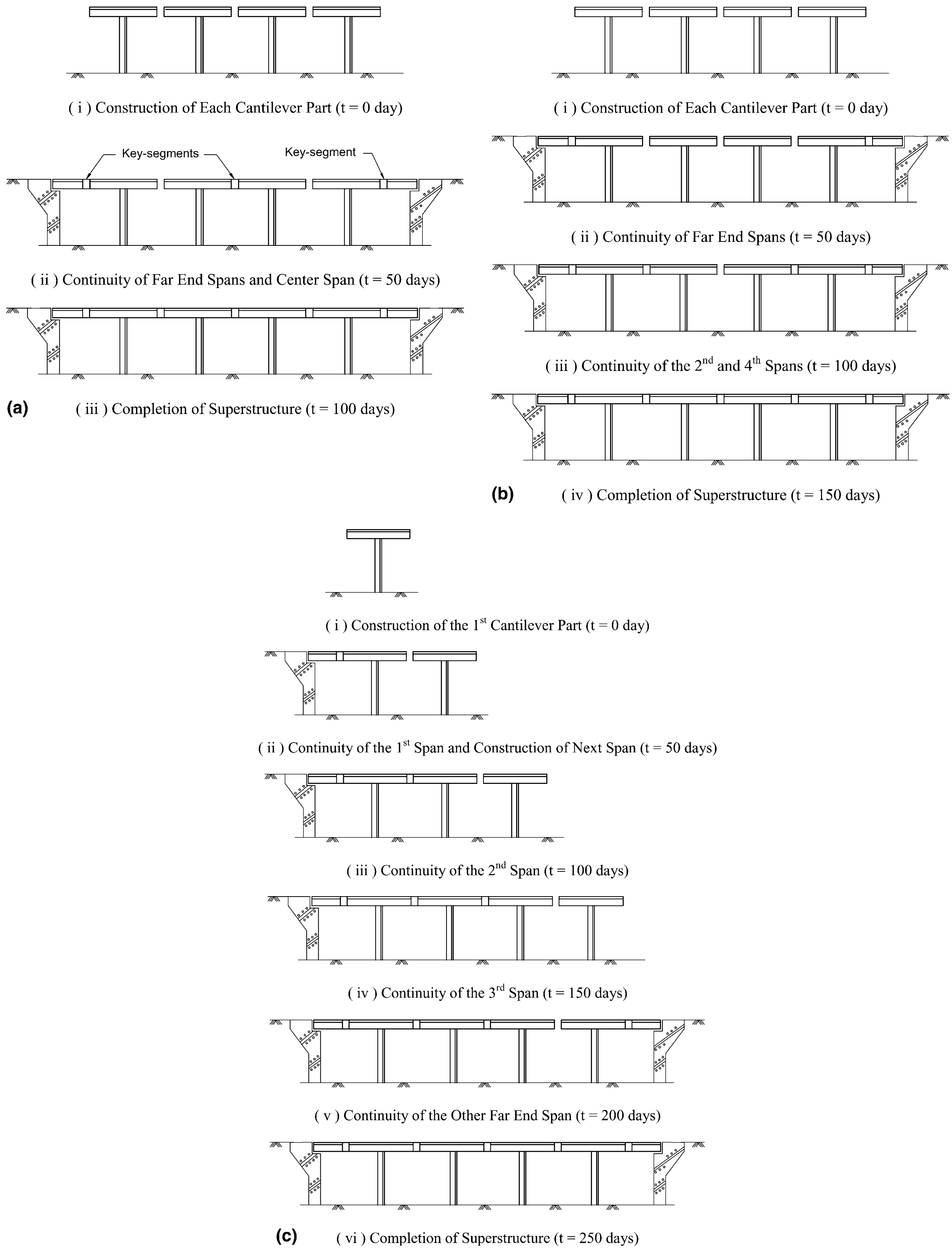

Fig. 1. Construction sequences in balanced cantilever bridges (a) Construction sequence of FCM 1: (i) construction of each cantilever part ($t = 0$ day); (ii) continuity of far end spans and center span ($t = 50$ days); (iii) completion of superstructure ($t = 100$ days). (b) Construction sequence of FCM 2: (i) construction of each cantilever part ($t = 0$ day); (ii) continuity of far end spans ($t = 50$ days); (iii) continuity of the 2nd and 4th spans ($t = 100$ days); (iv) completion of superstructure ($t = 150$ days); (c) Construction sequence of FCM 3. (i) construction of the 1st cantilever part ($t = 0$ day); (ii) continuity of the 1st span and construction of next span ($t = 50$ days); (iii) continuity of the 2nd span ($t = 100$ days); (iv) continuity of the 3rd span ($t = 150$ days); (v) continuity of the other far end span ($t = 200$ days); (vi) completion of superstructure ($t = 250$ days).

Table 1
Material and sectional properties used in application

A_C	$\rho_{sc} = \rho_{st}$	W_D	f'_c	f_{sy}	E_S
4.5 m^2	0.62 %	10.3 ton/m	400 kg/cm^2	4000 kg/cm^2	2.1×10^6 kg/cm^2

Table 2
Tendon properties used in application

P_i	e	A_P	f_{py}	μ	k
117 ton	1.3 m	10 cm^2	14,765 kg/cm^2	0.25	0.13×10^4

2.2. Dead load moment variation

The dead load moments corresponding to each construction sequence at typical construction steps are shown in Figs. 2–4, where total structure (TS) means that all the spans are constructed at once at the reference time $t = 0$ day. After construction of each cantilever part, the negative moment at each pier reaches $M = wl^2/8 = 1160$ ton m, ($l = 30$ m), and this value is maintained until the structural system changes by the connection of an adjacent span. The connection of an adjacent span, however, causes an elastic moment redistribution because the structural system moves from the cantilevered state to the over-hanging simply supported structure (see Fig. 1(a)). Nevertheless, there is no internal moment redistribution by creep deformation of concrete in a span if the structural system maintains the statically determinate structure. As shown in Fig. 1, the statically indeterminate structure begins at $t = 100$ days in all the structural systems (FCM 1, FCM 2, FCM 3). Therefore, it is expected that the dead load bending moments in the structures start the time dependent moment redistribution after $t = 100$ days.

Comparing the obtained numerical results in Figs. 2–4, the following can be inferred: (1) the time-dependent moment redistribution causes a reduction of negative moments near the supports and an increase of positive moments at the points of closure at the midspans; (2) the final moment at an arbitrary time t after completing the construction converges to a value within the region bounded by two moment envelopes for the final statically determinate stage at $t = 100$ days and for the initially completed five-span continuous structure (TS in

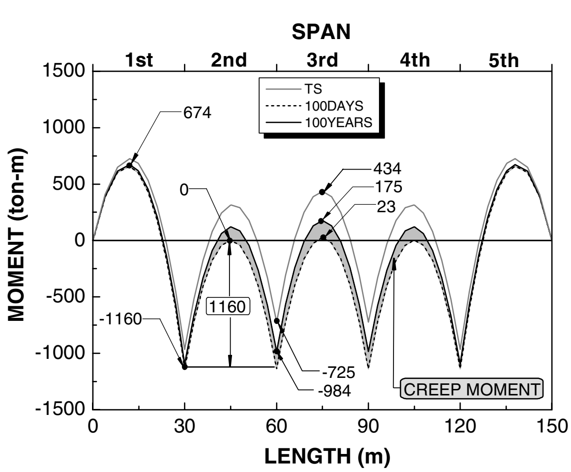

Fig. 2. Moment redistribution in FCM 1.

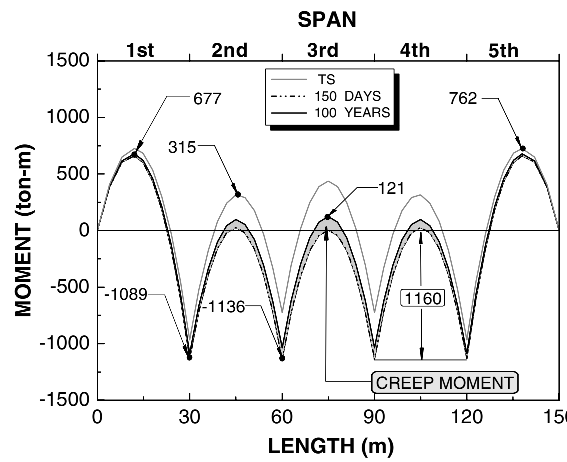

Fig. 3. Moment redistribution in FCM 2.

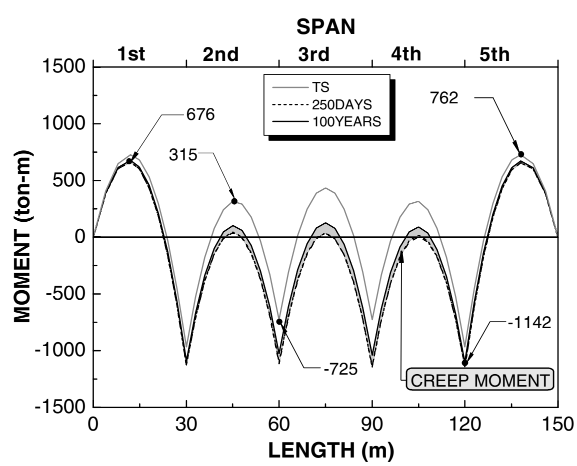

Fig. 4. Moment redistribution in FCM 3.

Figs. 2–4); and (3) the final moments in the structure depend on the order that the joints are closed in the structures, which means that the magnitude of the moment redistribution due to concrete creep may depend on the construction sequence, even in balanced cantilever bridges.

Under dead load as originally built, elastic displacement and rotation at the cantilever tips occur. If the midspan is not closed, these deformations increase over time due to concrete creep without any increase in the internal moment. On the other hand, as the central joints are closed, the rotations at the cantilever tips are restrained while introducing the restraint moments. Moreover, this restraint moment causes a time-dependent shift or redistribution of the internal force distribution in a span. If the closure of the central joints is made at the reference time $t = 0$ day, then the final moments M_t will converge with the elastic moment of the total structure (TS in Figs. 2–4). However, the example structure maintains the statically determinate structure which does not cause internal moment redistribution until $t = 100$ days, so that only the creep deformation after $t = 100$ days, which is a relatively small quantity of time, affects the time-dependent redistribution of the internal moment. Therefore, the moment distribution at time t represents a difference from that of the total structure. In particular, as shown in Figs. 2–4, the difference is relatively large at the internal spans. This means that the moment redistribution caused in proportion to the elastic moment difference between the statically determinate state and the five-span continuous structure will be concentrated at the internal spans. Fig. 5, which represents the creep moment distribution of FCM 1 bridge, shows that the creep moments at the center span are about 3.5 times for the negative moment and about 7.0 times for the positive moment larger than those of the end spans.

Fig. 6 shows the final dead load moment distribution of the example structures constructed by FCM 1, FCM 2 and FCM 3 at $t = 100$ years. As this figure shows, the difference in construction steps does not have a great influence on the final moment distributions, but there is remarkable difference in the final dead load moments between the initially completed continuous bridge (TS in Figs. 2–4 and 6) and the balanced cantilever bridges. Balanced cantilever bridges represent relatively smaller values for the positive moments and larger values for the negative moments than those of a five-span continuous structure (see Figs. 2–4 and 6). This difference is induced from no contribution of the creep deformation of concrete up to $t = 100$ days at which the structural system is changed to the statically indeterminate state.

2.3. Cantilever tendon moment variation

The same example structures are also reanalyzed by considering the cantilever tendons, as shown in Fig. 7. The time interval for the continuity of each segment is assumed to be seven days, and each cantilevered part of the superstructure is assumed to be tied to a previous one by concreting a key segment after 50 days, as mentioned in Fig. 1. The bending moments due to the cantilever tendons and self-weight are shown in Figs. 8–10.

As shown in these figures, the negative moments at the first interior support dramatically decrease from the value of $M_D = 1160$ ton m when only the dead load acts (see Figs. 2–4) to the value of $M \cong 480$ ton m (see Figs. 8–10) because of the positive moments by the cantilever tendons located at the upper flange of a section. This means that an optimum layout of cantilever tendons can give an effective moment distribution in a structure with an increase of the positive moments at mid-spans and a decrease of the negative moments at interior supports. In advance, for the same reasons mentioned in the case

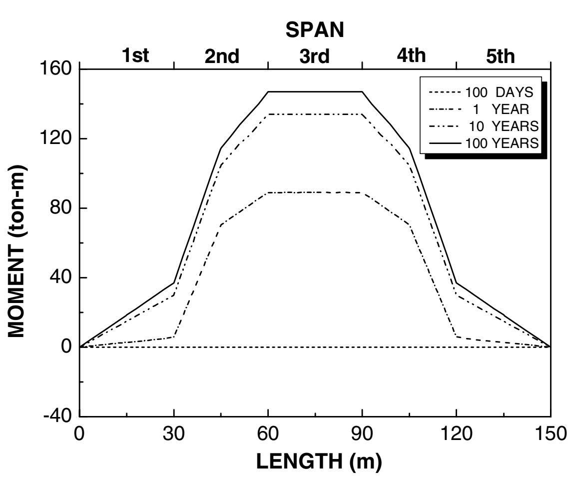

Fig. 5. Creep moment distribution of FCM 1 bridge.

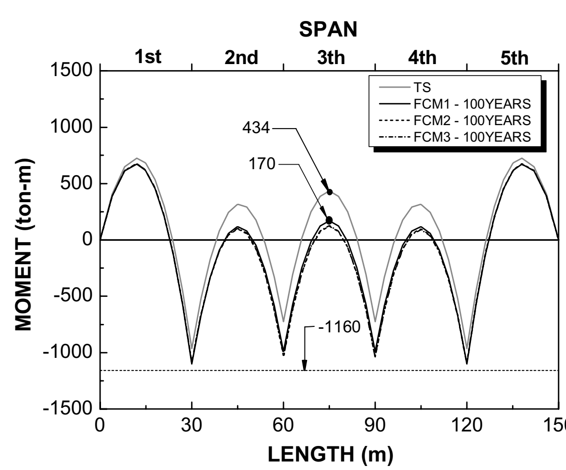

Fig. 6. Internal moment distribution at $t = 100$ years.

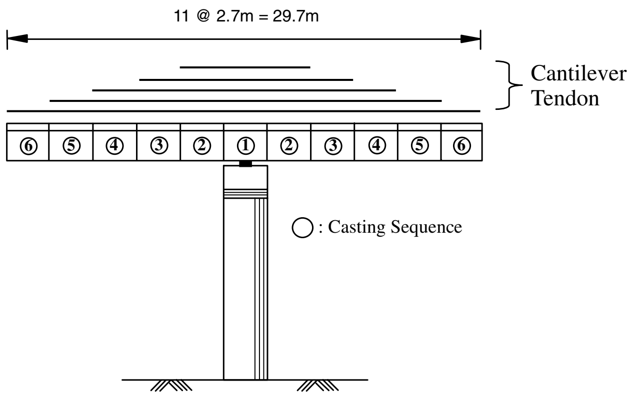

Fig. 7. Construction sequence of each segment.

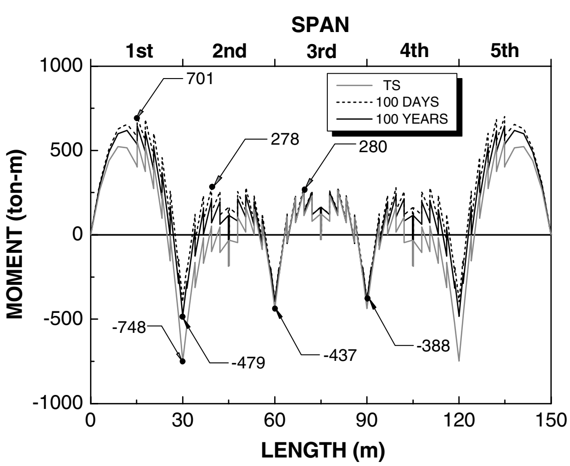

Fig. 8. Moment redistribution in FCM 1.

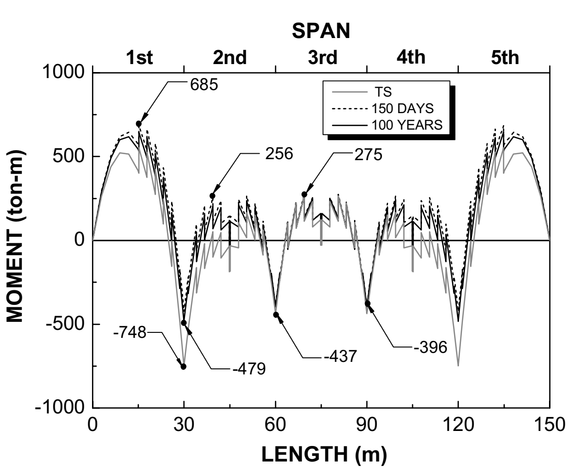

Fig. 9. Moment redistribution in FCM 2.

of the dead load moment, the moment distribution at time t represents a difference from that of the total structure (TS in Figs. 8–10), but the difference in construction steps does not have a great influence on the final moment distribution. Specifically, the internal moments by the cantilever tendons are also affected by the construction sequence as in the case of the dead load moment. However, differently from the creep effect of concrete in the case of the dead load moments which represent an increase of the positive moments at mid-span and a decrease of the negative moments at interior supports with time, the moment variations by the cantilever tendon seem to be dominantly governed by the relaxation which accompanies a decrease of both the positive and negative moments with time.

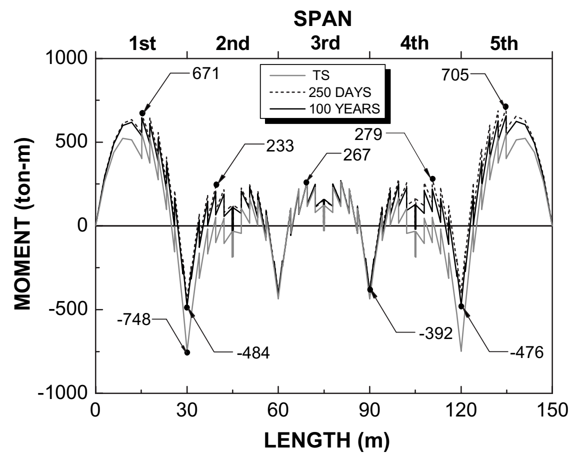

Fig. 10. Moment redistribution in FCM 3.

From the results obtained for the time-dependent behavior of balanced cantilever bridges, it can be concluded that the prediction of more exact positive and negative design moments requires the use of sophisticated time dependant analysis programs [5,10,15,17], which can consider the moment variations according to the construction sequence. To be familiar with those programs in practice, however, is time consuming and has many restrictions caused by complexity and difficulty in use because the adopted algorithms, theoretical backgrounds and the styles of input files are different from each other. Accordingly, the introduction of simple but effective relations, which can estimate design moments on the basis of elastic analysis results without any time-dependent analysis, are highly demanded in the preliminary design stage of balanced cantilever bridges.

3. Determination of design moments

3.1. Calculation of creep moment

Unlike temporary loads such as live loads, impact loads and seismic loads, permanent loads such as the dead load and prestressing force are deeply related to the long-term behavior of a concrete structure, and it is these loads that govern the time-dependent behavior of a structure. Of these two, it is the dead load that includes the self-weight continuously acting on a structure during construction. Thus the moment and deflection variations arising from changes in the structural system are heavily influenced by the dead load. The design moments of a structure can finally be calculated by the linear combination of the factored dead and live load moments. Since the dead load moment depends on the construction method because of the creep deformation of concrete, determination of the dead load moment through time-dependent analysis that considers the construction sequence must be accomplished to obtain an exact design moment.

The time-dependent behavior of a balanced cantilever bridge can be described using a double cantilever with an open joint at the point B, as in Fig. 11. When the uniformly distributed load of q is applied on the structure, the elastic deflection of $\delta = ql^4/8EI$ and the rotation angle of $\alpha = ql^3/6EI$ occurs at the ends of the cantilevers (see Fig. 11(b)), where l and EI refer to the length of the cantilever and the bending stiffness, respectively. If the joint remains open, then the deflection at time t will increase to $\delta \cdot (1 + \phi_t)$ and the rotation angle to $\alpha \cdot (1 + \phi_t)$, where ϕ_t is the creep factor at time t. However, if the joint at the point B is closed after application of the load, an increase in the rotation angle $\alpha \cdot \phi_t$ is restrained, and this restraint will develop the moment M_t, as shown in Fig. 11(c). The moment M_t, if acting in the cantilever, causes the elastic rotation at point B, defined as $\beta = M_t l/EI$, and also accompanies the creep deformation. Since the creep factor increases by $d\phi_t$ during a time interval dt, the variations in the angles of rotation will be $\alpha \cdot d\phi_t$ and $d\beta$ (the elastic deformation) + $\beta \cdot d\phi_t$ (the creep deformation) for α and β, respectively.

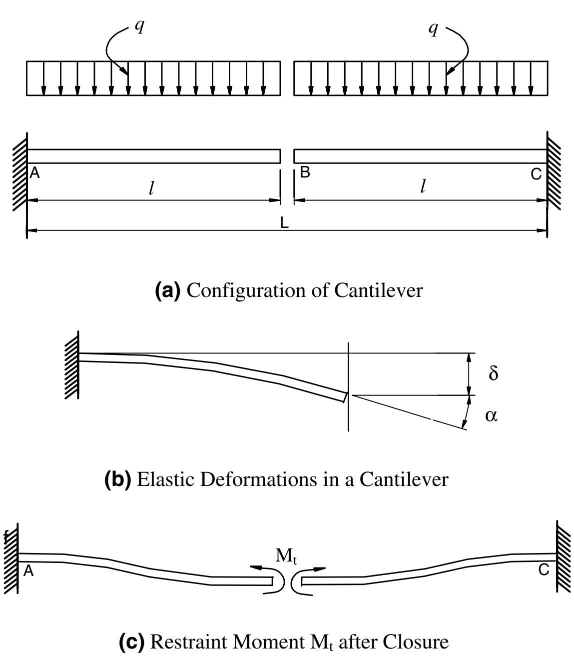

Fig. 11. Deformation of cantilevers before and after closure: (a) configuration of cantilever; (b) elastic deformations in a cantilever; (c) restraint moment M_t after closure.

From these relations and the fact that there is no net increase in discontinuity after the joint is closed, the compatibility condition for the angular deformation $(\alpha \cdot d\phi_t = d\beta + \beta \cdot d\phi_t)$ can be constructed. The integration of this relation with respect to ϕ_t gives the restraint moment M_t [4]:

$$M_t = ql^2 \frac{(1 - e^{-\phi_t})}{6} = qL^2 \frac{(1 - e^{-\phi_t})}{24}, \tag{1}$$

where ϕ_t means the creep factor at time t and $L = 2l$.

From Eq. (1), it can be found that for a large value of ϕ_t, the restraint moment converges to $M_t = qL^2/24$, which is the same moment that would have been obtained if the joint at the point B had been closed before the load q was applied. This illustrates the fact that moment redistribution due to concrete creep following a change in the structural system tends to approach the moment distribution that relates to the structural system obtained after the change.

Referring to Fig. 12, which shows the moment distribution over time, the following general relationship may be stated [4]:

$$M_{cr} = M_{III} - M_I = (M_{II} - M_I)(1 - e^{-\phi_t}), \tag{2}$$

where M_{cr} is the creep moment resulting from change in the structural system, M_I the moment due to loads before a change of structural system, M_{II} the moment due to the same loads applied on the changed structural system, and M_{III} is the restraint moment M_t.

The derivation of Eq. (2) is possible under the basic assumption that the creep deformation of concrete starts from the reference time, $t = 0$ day. If it is assumed that the joint is closed after a certain time, $t = C$ days, while maintaining the same assumptions adopted in the derivation of Eq. (2), then the structure can be analyzed by means of the rate-of-creep method (RCM) [9], and the obtained creep moments in Fig. 12 can be represented by the following expression [21]:

$$M_{cr} = (M_{II} - M_I)(1 - e^{-(\phi_t - \phi_C)}). \tag{3}$$

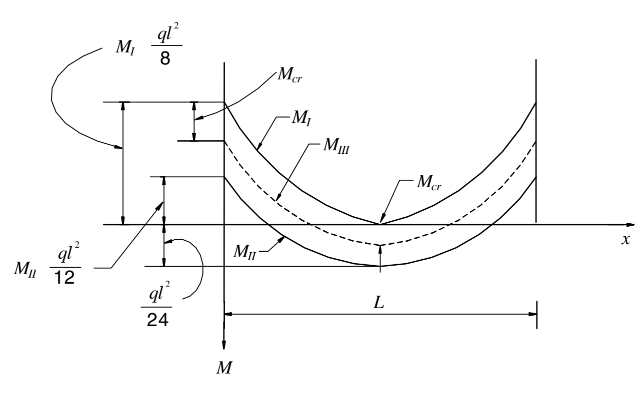

Fig. 12. Moment distribution over time.

Specifically, in balanced cantilever bridges, the restraint moment grows continuously from the time at which the structural system is changed ($t = C$ days), and its magnitude is proportional to $(1 - e^{-(\phi_t - \phi_C)})$ [4,9,21].

Generally, construction of a multi-span continuous bridge starts at one end and proceeds continuously to the other end. Therefore, change in the structural system is repeated whenever each cantilever part is tied by concreting a key-segment at the midspan. Moreover, the influence of the newly connected span will be delivered into the previously connected spans so that there are some limitations in direct applications of Eq. (3) to calculate the restraint moment at each span because of the many different connecting times of $t = C$ days. To solve this problem and for a sufficiently exact calculation of the final time-dependent moments, Trost and Wolff [22] proposed a relation on the basis of the combination of elastic moments ($\sum M_{S,i}$; equivalent to M_I in Eq. (3)) occurring at each construction step (see Fig. 13), and the moment obtained by assuming that the entire structure is constructed at the same point in time (M_E; equivalent to M_{II} in Eq. (3))

$$M_T = \sum M_{S,i} + \left(M_E - \sum M_{S,i}\right)\frac{\phi_t}{1 + \rho\phi_t}, \quad (4)$$

where ϕ_t and ρ represent the creep factor and corresponding relaxation factor, respectively.

This relation has been broadly used in practice because of its simplicity. In particular, the exactness and efficiency of this relation can be expected in a bridge constructed by incremental launching method (ILM) or movable scaffolding system (MSS), that is, in a span-by-span constructed bridge. However, there are still limitations in direct applications of Eq. (4) to balanced cantilever bridges because this equation excludes the proportional ratio, $(1 - e^{-(\phi_t - \phi_C)})$ in Eq. (3), which represents the distinguishing characteristic of the FCM.

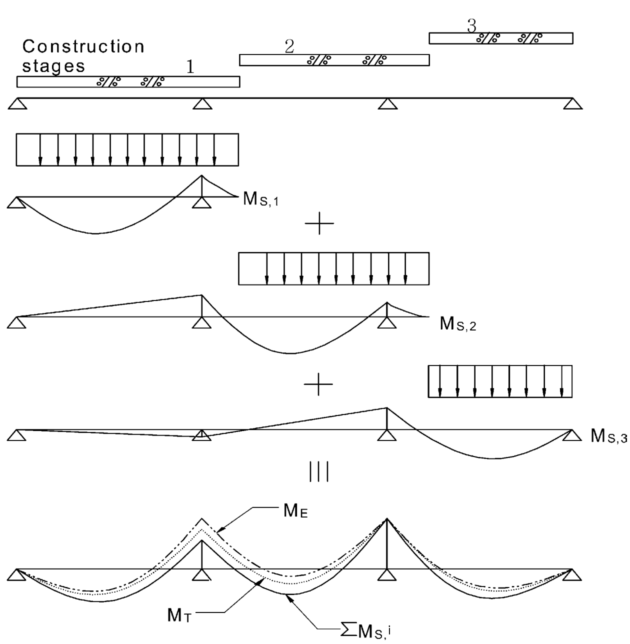

Fig. 13. Combination of $M_{S,i}$.

3.2. A Proposed relation for dead load moment

The difference in the internal moments ($M_E - \sum M_{S,i}$) in Eq. (4), which is equivalent to $M_{II} - M_I$ in Eq. (3)) is not recovered immediately after connection of all the spans but gradually over time, and the occurred internal restraint moments at time t also decrease with time because of relaxation accompanied by creep deformation. From this fact, it may be inferred that Eq. (4) considers the variation of the internal restraint moments on the basis of a relaxation phenomenon. When a constant stress σ_0 is applied at time t_0, this stress will be decreased to $\sigma(t)$ at time t (see Fig. 14). Considering the stress variation with the effective modulus method (EMM), the strain $\varepsilon(t)$ corresponding to the stress $\sigma(t)$ can be expressed by $\varepsilon(t) = \sigma_0/E_0 \cdot (1 + \phi_t)$. Moreover, the stress ratio becomes $\sigma(t)/\sigma_0 = 1/(1 + \phi_t)$ and the stress variation $\Delta\sigma(t) = \phi_t/(1 + \phi_t) \cdot \sigma_0$. That is, the stress variation is proportional to $\phi_t/(1 + \phi_t)$. If the age adjusted effective modulus method (AEMM) is based on calculation to allow the influence of aging due to change of stress, the stress variation can be expressed by $\Delta\sigma(t) = \chi\phi_t/(1 + \chi\phi_t) \cdot \sigma_0$, where χ is the aging coefficient [19].

With the background for the time-dependent behavior of a cantilever beam effectively describing the internal moment variation in balanced cantilever bridges, and by maintaining the basic form of Eq. (4) suggested by Trost and Wolff [22] considering the construction sequence while calculating the internal moments at an arbitrary time t, the following relation is introduced in this paper

$$M_T = \sum M_{S,i} + \left(M_E - \sum M_{S,i}\right)(1 - e^{-(\phi_t - \phi_C)}) \cdot f(\phi_t), \quad (5)$$

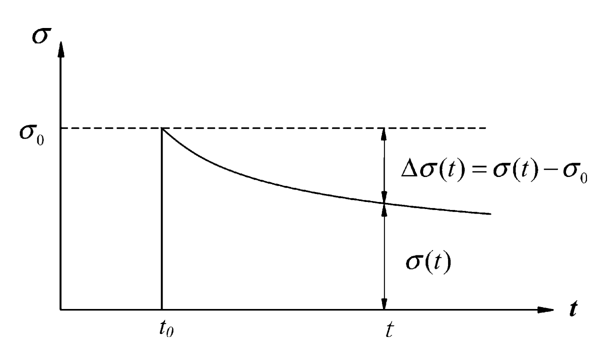

Fig. 14. Stress variation due to relaxation.

where $f(\phi_t) = \chi\phi_t/(1 + \chi\phi_t)$. χ is the concrete aging coefficient which accounts for the effect of aging on the ultimate value of creep for stress increments or decrements occurring gradually after application of the original load. It was found that in previous studies [5,15,16] an average value of $\chi = 0.82$ can be used for most practical problems where the creep coefficient lies between 1.5 and 3.0. An approximate value of $\chi = 0.82$ is adopted in this paper. In addition, if the creep factor ϕ_t is calculated on the basis of the ACI model [2], $f(\phi_t) = \chi\phi_t/(1 + \chi\phi_t)$ has the values of 0.62, 0.64 and 0.65 at 1, 10 and 100 years, respectively.

Comparing this equation (Eq. (5)) with Eq. (4), the following differences can be found: (1) to simulate the cantilevered construction, a term, $(1 - e^{-(\phi_t - \phi_C)})$ describing the creep behavior of a cantilevered beam is added in Eq. (5) (see Eq. (3)); and (2) the term $\phi_t/(1 + \rho\phi_t)$ in Eq. (4) is replaced by $f(\phi_t) = \chi\phi_t/(1 + \chi\phi_t)$ in Eq. (5) on the basis of the relaxation phenomenon.

To verify the effectiveness of the introduced relation, the internal moment variations in FCM 1, FCM 2 and FCM 3 bridges (see Fig. 1), which were obtained through rigorous time-dependent analyses, are compared with those by the introduced relation. The effect of creep in the rigorous numerical model was studied in accordance with the first-order algorithm based on the expansion of a degenerate kernel of compliance function [13–17]. Figs. 15–17, representing the obtained results at $t = 1$, 10 and 100 years after completion of construction, show that the relation of Eq. (4) proposed by Trost and Wolff gives slightly conservative positive moments even though they are still acceptable in the preliminary design stage. On the other hand, the relation introduced in Eq. (5) effectively simulates the internal moment variation over time regardless of the construction sequence and gives slightly larger positive moments than those obtained by the rigorous analysis along the spans. Hence, the use of Eq. (5) in determining the positive design moments will lead to more reasonable designs of balanced cantilever bridges. Besides, the underestimation of the negative moments, which represents magnitudes equivalent to the overestimation of the positive moments, will be induced. The negative design moments, however, must be determined on the basis of the cantilevered state because it has the maximum value in all the construction steps as mentioned in Fig. 2. This means that the negative design moment has a constant value of $M = 1160$ ton. m in this example structure and is calculated directly from the elastic moment of a cantilevered beam.

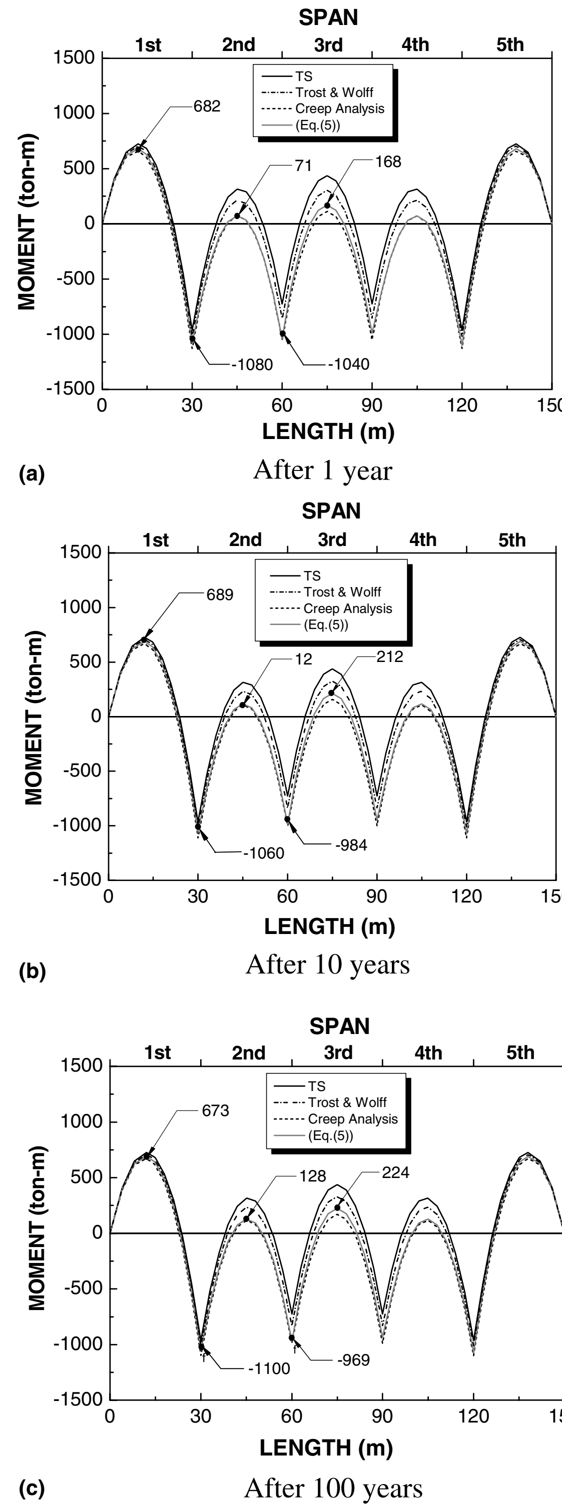

Fig. 15. Moment variations of FCM 1 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

3.3. A Proposed relation for cantilever tendon moment

As mentioned before, post-tensioning cantilever tendons are installed to connect each segment during construction, and the prestressing forces introduced will also be redistributed from the cantilevered structural system to the completed structural system due to concrete creep and the relaxation of tendons. Therefore, a simple equation to determine the cantilever tendon

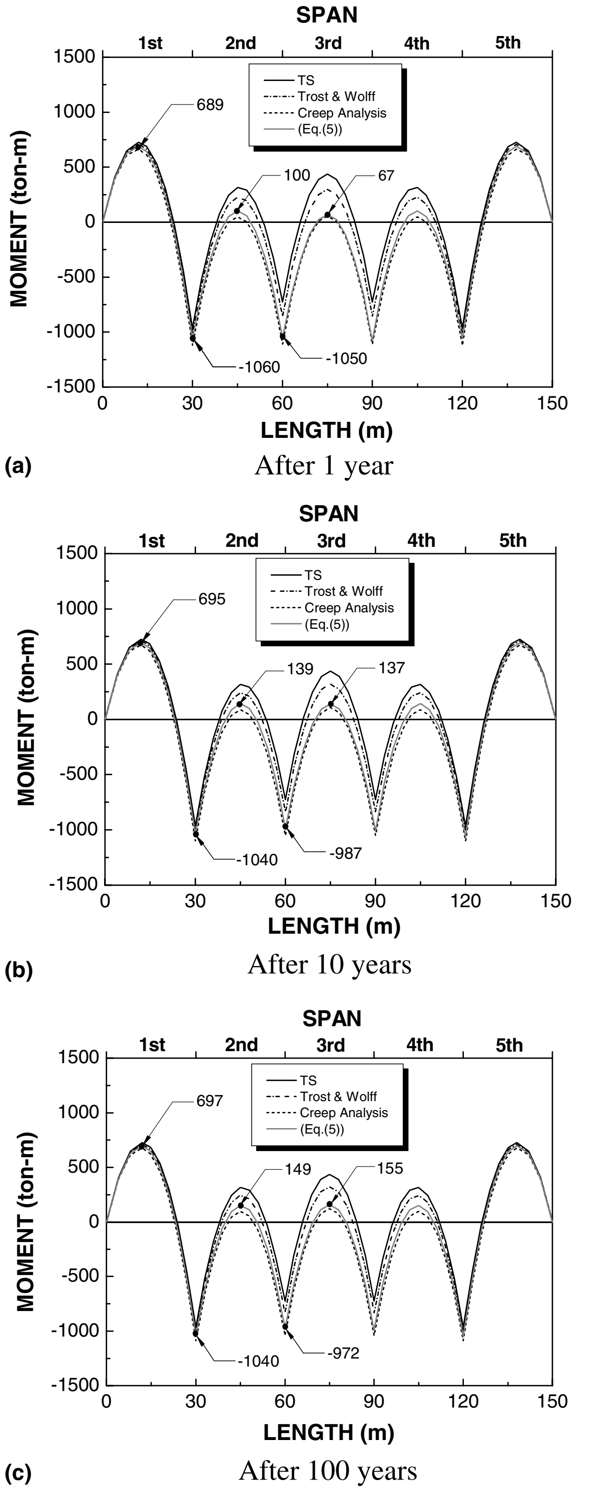

Fig. 16. Moment variations of FCM 2 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

moment needs to be introduced as in the case of the dead load moment. If the equation is constructed on the basis of the tendon moments calculated by subtracting the dead load moments from the total moments produced by both the cantilever tendons and the dead load, then the design moment required in the preliminary

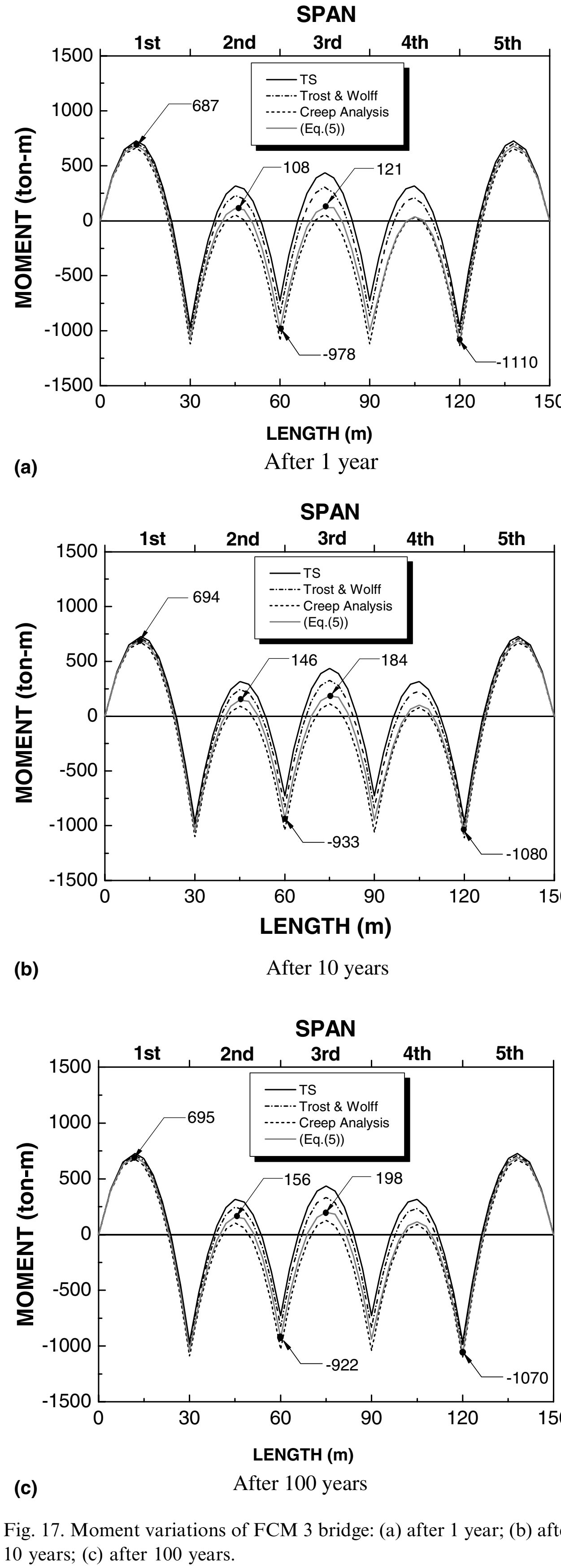

Fig. 17. Moment variations of FCM 3 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

design stage to determine an initial section can be calculated through the superposition of moments by both simple equations for the dead load moment and the cantilever tendon moment.

Figs. 18–20 show the cantilever tendon moments for the three structural systems of FCM 1, FCM 2 and FCM 3 from Fig. 1. These moment distributions are determined by subtracting the dead load moments from the total moments calculated from a sophisticated time dependent analysis program [10,13,14] by considering the dead load and the cantilever tendons. These figures show that the cantilever tendon moments represent a remarkable difference between the initially completed continuous bridge (TS in Figs. 18–20) and the balanced cantilever bridges. This difference seems to be induced without the contribution of the creep deformation of concrete for up to $t = 100$ days at which the structural system is changed, as mentioned in the case of the dead load moment. Differently from the case of the dead load moment, however, the cantilever tendons also accompany the stress relaxation from $t = 0$ day regardless of the construction sequence. This means that relatively larger moment differences between the initially completed continuous bridge and the balanced cantilever bridge and more complex time-dependent behavior may be caused in the case of cantilever tendon moments.

The cantilever tendons also accompany the moment decrease due to stress relaxation even in a structure without any change in the structural system, as well as the moment variation according to changes in the structural system. Based on this aspect, the combination of elastic moments $\sum M_{S,i}$ mentioned in Eq. (5) representing the dead load moment distribution needs to be revised by $\sum M_{S,i} \cdot R(t)$ in the case of the cantilever tendons to take into account the relaxation of the cantilever tendon force. In advance, the stress variation caused by changes in the structural system is affected by the creep deformation of concrete and also influenced by

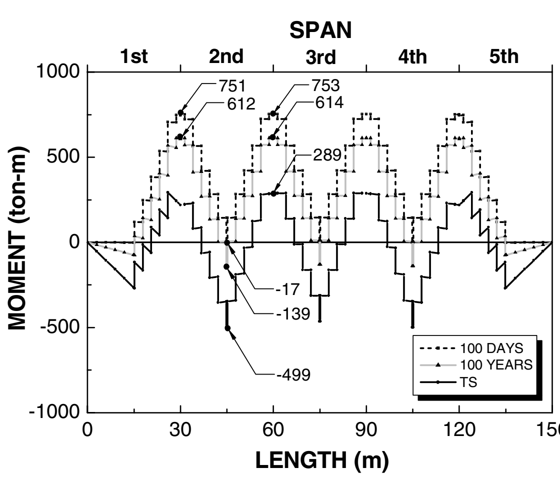

Fig. 18. Cantilever tendon moment redistributed in FCM 1 bridge.

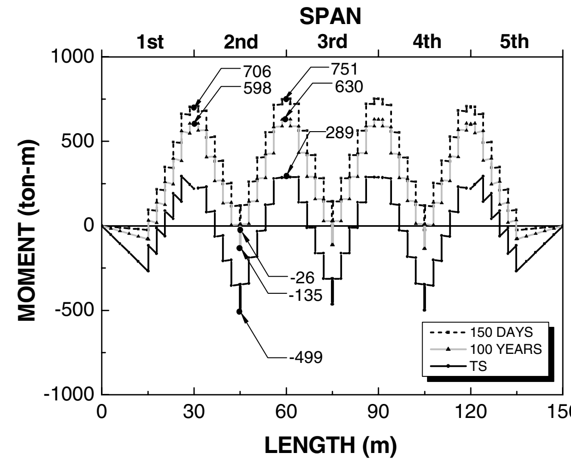

Fig. 19. Cantilever tendon moment redistributed in FCM 2 bridge.

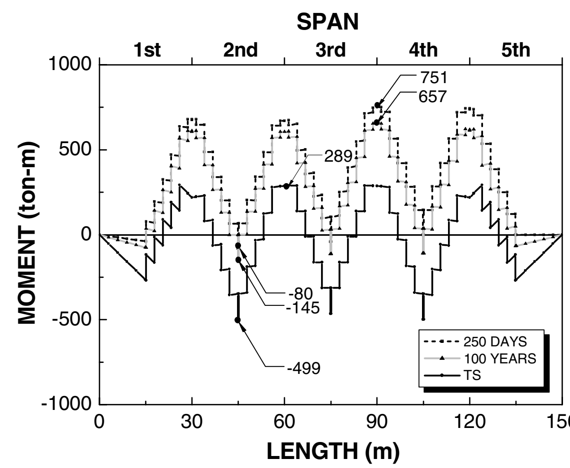

Fig. 20. Cantilever tendon moment redistributed in FCM 3 bridge.

the relaxation of the concrete stress itself. Considering these factors, the two terms of $(1 - e^{-(\phi_t - \phi_C)})$ and $f(\phi_t) = \chi\phi_t/(1 + \chi\phi_t)$ were implemented in Eq. (5). However, the cantilever tendon moment distribution has been basically determined by subtracting the dead load moments from the total moments by both the cantilever tendons and the dead load in this paper. Consequently, the cantilever tendon moment variation at time t due to the relaxation of concrete stress may be influenced by $1 - \chi\phi_t/(1 + \chi\phi_t) = 1/(1 + \chi\phi_t)$ instead of $f(\phi_t) = \chi\phi_t/(1 + \chi\phi_t)$ in Eq. (5). In the light of these issues, the cantilever tendon moment distribution can be inferred from Eqs. (4) and (5) as

$$M_T = \sum M_{S,i} \cdot R(t) + \left(M_E - \sum M_{S,i} \cdot R(t)\right) (1 - e^{-(\phi_t - \phi_C)}) \cdot \frac{1}{1 + \chi\phi_t}, \quad (6)$$

where $R(t)$ means the relaxation of tendon force with time and can be calculated by the relaxation proposed by Magura et al. [18].

To verify the effectiveness of the introduced relation, the internal moment variations by the cantilever tendons in FCM 1, FCM 2 and FCM 3 bridges, which were obtained by subtracting the dead load moments from the total moments determined through rigorous time-dependent analyses, are compared with those by the introduced relation of Eq. (6) in Figs. 21–23. As shown in these figures, the numerical results by the rigorous analysis represent little difference from those by Eq. (6). Generally, the prestressing losses due to the relaxation of

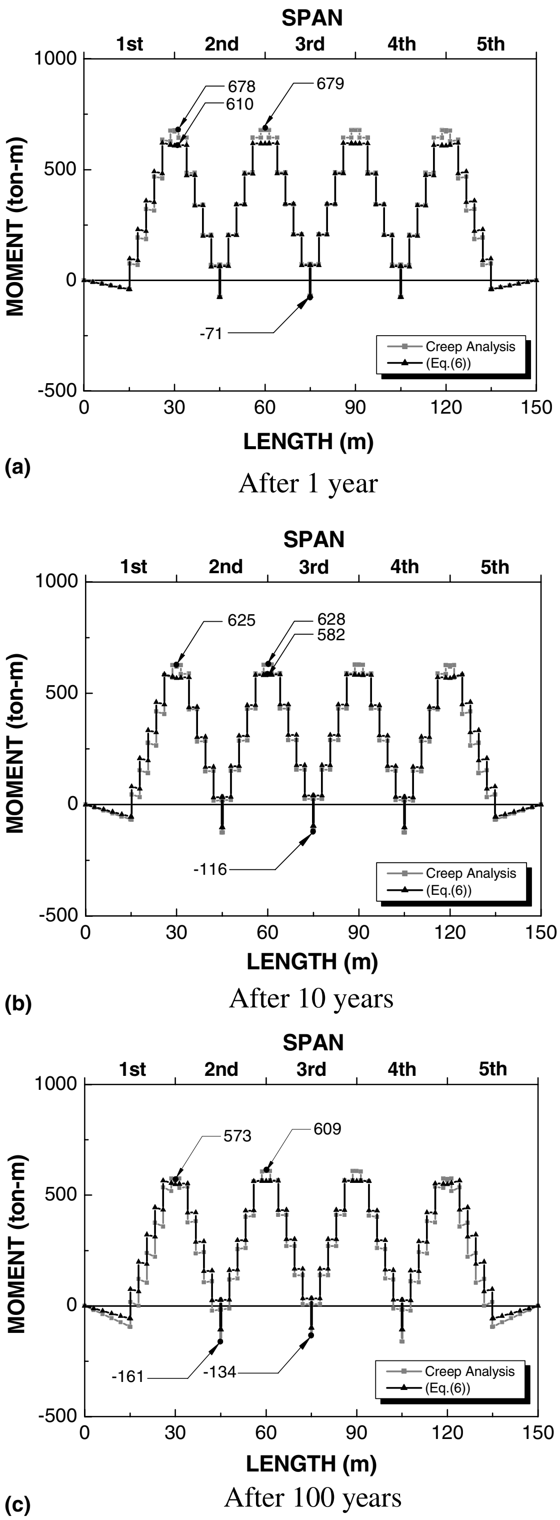

Fig. 21. Cantilever tendon moments in FCM 1 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

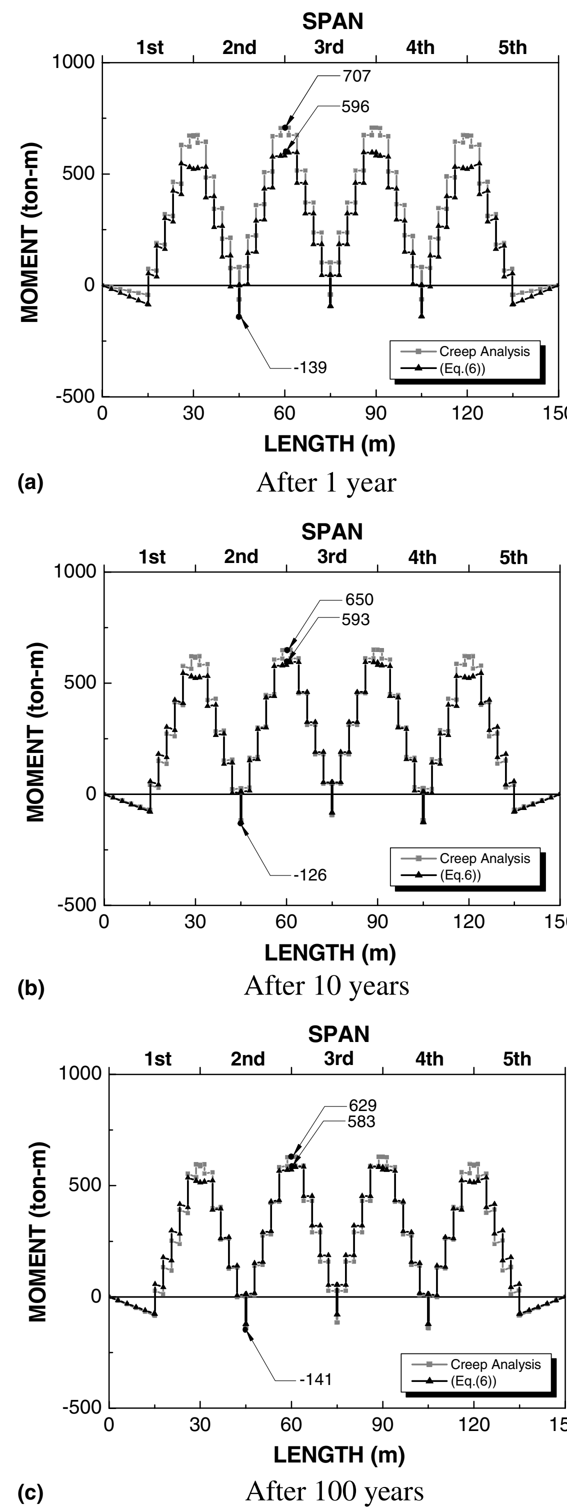

Fig. 22. Cantilever tendon moments in FCM 2 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

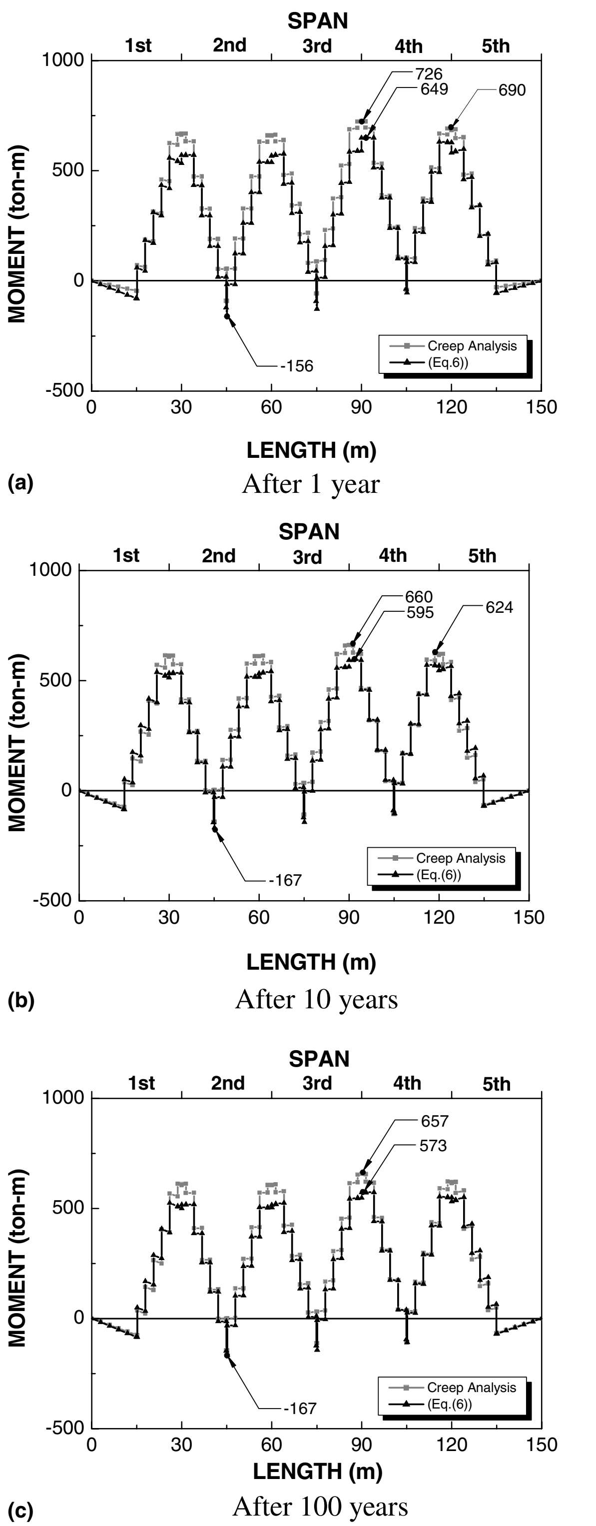

Fig. 23. Cantilever tendon moments in FCM 3 bridge: (a) after 1 year; (b) after 10 years; (c) after 100 years.

tendons and the prestressing force itself also influence the moment variation by the changes in the structural system because all these force components contribute to the moment variation basically caused by the creep deformation of concrete. Nevertheless, the prestressing losses are not taken into consideration in describing the moment variation in terms of the creep deformation. Only the relaxation phenomenon is considered in Eq. (6). The differences in numerical results, therefore, seem to be caused by ignoring the creep deformation of concrete induced by the prestressing losses. The differences in the maximum and minimum moments, however, still maintain acceptable ranges for use in the preliminary design stage as in the case of the dead load moment.

4. Conclusions

Simple, but effective, relations which can simulate the internal moment variation due to the creep deformation of concrete, relaxation of cantilever tendons, and the changes in the structural system during construction are proposed, and a new guideline to determine the design moments is introduced in this paper. The positive design moment for a dead load can be determined by the introduced relation, while the negative design moment for a dead load must be calculated directly from the elastic moment of a cantilevered beam in balanced cantilever bridges. In advance, if the cantilever tendons, which may affect the internal moment redistribution during construction, need to be considered in calculating the internal moments and the corresponding normal stresses at an arbitrary section, it may be achieved by Eq. (6), even though, the calculated results represent slightly conservative values.

Moreover, since the internal moments by other loads, except the dead load and the cantilever tendon force, are not affected by the construction sequence, these can be calculated on the basis of the complete continuous structure, and the calculation of the final factored design moment can also be followed by the linear combination of moments for each load. In addition, if a rigorous time dependent analysis is conducted with the initial section determined on the basis of the initial design moments obtained by using Eqs. (5) and (6), then a more effective design of balanced cantilever bridges can be expected.

Acknowledgements

The research reported in this paper was made possible by the financial support from the Smart Infra-Structure Technology Center funded by the Korea Science and Engineering Foundation. The authors would like to express their gratitude to this organization for the financial support.

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