rf Wien filter in an electric dipole moment storage ring: The "partially frozen spin" effect

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An rf Wien filter (WF) can be used in a storage ring to measure a particle's electric dipole moment (EDM). If the WF frequency equals the spin precession frequency without WF, and the oscillating WF fields are chosen so that the corresponding transverse Lorentz force equals zero, then a large source of systematic errors is canceled but the EDM signal is not. This effect, discovered by simulation, can be called the "partially frozen spin" effect.

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I. INTRODUCTION

The sensitivity of electric dipole moment (EDM) measurements of charged particles can benefit from the high intensities of polarized protons and deuteron beams available in storage rings like RHIC at Brookhaven National Laboratory (BNL) in the USA and COSY at Jülich in Germany. There are a number of different approaches to such measurements, based on different manipulations of the spin precession frequency. The spin precession frequency of a particle in any electromagnetic storage ring equals (under condition $\vec{\beta}\cdot\vec{B} = \vec{\beta}\cdot\vec{E} = 0$)

$$\begin{aligned}\vec{\omega} &= \vec{\omega}_a + \vec{\omega}_{\rm edm}\\ &= -\frac{e}{mc}\left[a\vec{B} + \left(a - \frac{m^2c^2}{p^2}\right)\frac{\vec{E}\times\vec{v}}{c}\right]\\ &\quad -\frac{e}{mc}\left[\frac{\eta}{2}(\vec{E} + \vec{\beta}\times\vec{B})\right]; \end{aligned}\tag{1}$$

$$\omega_{\rm edm} = -\frac{e\eta}{2mc}(\vec{E} + \vec{\beta}\times\vec{B}),\tag{2}$$

$$d\vec{s}/dt = (\vec{\omega}_a + \vec{\omega}_{\rm edm})\times\vec{s}.\tag{3}$$

Equation (3) is a spin rotation equation. $a = (g-2)/2$, and η plays the same role for the particle electric dipole moment as the g factor plays for the magnetic dipole moment:

$$d = \frac{\eta}{2}\frac{e}{mc}s, \qquad \mu = \frac{g}{2}\frac{e}{mc}s,\tag{4}$$

with $s = \hbar$ for spin 1 and $s = \hbar/2$ for spin $\frac{1}{2}$ particles. For muons, protons, and deuterons, respectively,

$$\begin{aligned} d_\mu &= \eta_\mu \times 4.67\times 10^{-14}\ \text{e cm},\\ d_p &= \eta_p \times 5.26\times 10^{-15}\ \text{e cm},\\ d_d &= \eta_d \times 5.26\times 10^{-15}\ \text{e cm}. \end{aligned}\tag{5}$$

Consider first an all-magnetic ring without WF. In such a ring, Eq. (1) becomes

$$\vec{\omega} = -\frac{e}{mc}\left[a\vec{B} + \frac{\eta}{2}(\vec{\beta}\times\vec{B})\right].\tag{6}$$

In this case, $\vec{\omega}_{\rm edm}$ is perpendicular to $\vec{\omega}_a$. Thus, in the presence of a nonzero η, the spin precession axis is tilted by an angle $\eta\beta/2a$ with respect to the direction of the magnetic field $\vec{B}$. This angle is at the microradian level for a deuteron EDM of 10^{-21} e cm and is more than 10 times smaller for a proton EDM of the same magnitude. Because of the EDM tilt, $\omega = \sqrt{\omega_a^2 + \omega_{\rm edm}^2}$. It can be shown (we will not do so here) that when $\omega_a \gg \omega_{\rm edm} \neq 0$, $\omega \approx \omega_a$, the radial spin component acquires a small constant-in-time term proportional to $\omega_{\rm edm}/\omega_a$ in addition to the usual, oscillating term. The vertical spin component acquires, in addition to the usual constant-in-time term, a small term oscillating with frequency ω and amplitude $s_{L0}\omega_{\rm edm}/\omega$, where s_{L0} is the amplitude of the longitudinal spin component. The time average of these oscillations equals zero, so it is rather difficult to measure EDM effects in such a ring. Nevertheless, this was done as a part of the BNL muon g-2 experiment [1]; the achieved limit on the muon EDM is $|d_\mu| < 1.9\times 10^{-19}$ e cm.

Separate EDM experiments with an accuracy $\sim 10^{-29}$ e cm have been proposed at BNL [2,3]. They require construction of dedicated electric rings (for protons) or electromagnetic rings (for deuterons) and are based on the idea of a frozen spin ring [4], in which $\omega_a = 0$ by design. In Eq. (1), above, only the second square bracket term is not zero. Both the radial $(\omega_{\rm edm})_R$ and the longitudinal s_L in Eq. (3) are constant in time, so that the vertical spin component grows, $(d\vec{s}/dt)_V = (\vec{\omega}_{\rm edm}\times\vec{s})_V$, instead of being constant or oscillating together with the magnetic dipole as

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in all-magnetic rings. [In theory, an EDM oscillates also in a frozen spin ring, now, mostly as $\sin(\omega_{edm}t)$; see Eqs. (24)–(29), below. However, we expect that $(\omega_{edm}t) \ll 1$ and therefore that the growth of the vertical spin component will be approximately linear in time.]

Rabi-type resonance experiments have also been proposed [5,6]. In such experiments, $\omega_a \neq 0$ so that the horizontal components of spin perform the usual magnetic moment precession and ω_{edm} oscillates in resonance with it, making $(\vec{\omega}_{edm} \times \vec{s})_V$ of Eq. (3) and hence $(d\vec{s}/dt)_V$ nonzero. To produce this resonance, either the longitudinal velocity, β_L[5], or the electric field [6] of Eq. (2) oscillates with the frequency of s_L.

The new approach proposed and investigated here is neither the frozen spin method, since $\omega_a \neq 0$, nor the Rabi-type spin resonance method, since we do not use a resonance between oscillating $\vec{\omega}_{edm}$ and oscillating $\vec{s}$. Our approach involves slightly modifying an existing storage ring by adding rf Wien filter(s) to its lattice in order to probe the particle EDM with an accuracy of at least 10^{-25} e cm.

II. THE RF WIEN FILTER METHOD

The idea is to measure EDM by combining radial electric and vertical magnetic rf fields in the standard Wien filter configuration, $\vec{E} = \vec{B} \times \vec{v}/c$ [7], oscillating with the spin resonance frequency. Such a configuration eliminates the filter fields' own Lorentz force, thus canceling the source of the largest systematic error associated with the method. Unfortunately, this configuration also cancels the Rabi-type resonance between the ω_{edm} and the spin because, according to Eq. (2), the oscillating part of ω_{edm} equals zero together with the oscillating part of the Lorentz force. When we run spin tracking, however, we observe an unexpected effect: the rf Wien filter running at one of the spin tune resonances, $1 + a\gamma$, appears to be very effective in probing the particle EDM [8]. We will refer to such an rf-WF as *a magic Wien filter* (mWF).

This phenomenon can be qualitatively explained. Without an mWF, the oscillation mode of the spin's planar components is the usual g-2 mode. But in the presence of the mWF's resonant fields, each planar component acquires (among other new modes) a zero-frequency mode. This means that each planar component now has a part that is constant in time ("frozen"). The ω_{edm} is unaffected by the mWF; $\omega_{edm} \propto \vec{\beta}_L \times \vec{B}_V$, where $\vec{B}_V$ is the main, constant part of the magnetic field. As a result, the constant EDM torque $(e\eta/2mc)(\vec{\beta}_L \times \vec{B}_V) \times (\vec{s}_L)_{frozen}$ is able to rotate spin around the radial axis. A parasitic resonance rotation of the spin by the particle magnetic moment does not simultaneously occur, because the designed mWF fields modulate only the magnitude of the g-2 frequency vector without changing its direction, which ideally is perpendicular to the ring plane.

The immediate question arises of how, exactly, such a zero-frequency spin mode can appear. The answer is the following. Since the modulation frequency of the (g-2) frequency equals that very g-2 frequency, the resulting spin g-2 oscillations are obviously a superposition of an infinite number of integer g-2 modes, $N\omega_{a0}$'s, and only of such modes. The main mode is $N = 1$, the initial g-2 mode. The next biggest mode is $N = 0$. This zero g-2 frequency mode is none other than the frozen spin mode whose appearance makes EDM observable. In the original idea of the frozen spin [4], the g-2 spectrum contains only the $N = 0$ mode. Here we have many modes and use one of them, the $N = 0$ mode, for the EDM observation. The mWF method can thus be called the "partially frozen" spin method [9].

To demonstrate the corresponding calculations in the most transparent way, we assume that the EM fields of many mWF's are distributed uniformly along the equilibrium orbit. By design, these fields contain only a vertical magnetic component, b_V, and a radial electric component, e_R; both components oscillate synchronously with the nonperturbed part of the g-2 frequency, ω_{a0}:

$$b_V = b_{V0}\cos\omega_{a0}t, \tag{7}$$

$$e_R = e_{R0}\cos\omega_{a0}t, \tag{8}$$

with the condition

$$e_{R0} = \beta b_{V0}. \tag{9}$$

This condition guarantees that the transverse Lorentz force—radial and, ideally, vertical—produced by the mWF's equals zero. Thus, ideally, particle movement inside and outside of these mWF's is not perturbed. (It should be noted, however, that the free synchrotron and betatron oscillations of the particles lead to dispersion of the ω_{a0} values, and hence to shortening the spin coherence time. This well-known and generally understood problem of any EDM experiment requires a detailed analysis depending on a lattice; we do not make such an analysis here. It should also be noted that a particle's kinetic energy, γmc^2, can slightly oscillate with frequencies $\omega_{a0} \pm \omega_R + k\omega_C$ due to the radial betatron oscillations and the work produced by the radial electric field, $\vec{E}\cdot\vec{v}$. These forced oscillations of energy are averaged to zero by a proper choice of betatron frequencies. Here, we neglect this effect.)

Since the EDM is very small, we first solve our spin equations without EDM. Then we apply this solution to the equation for s_V. The spin equation without EDM but with fields (7) and (8) becomes

$$d\vec{s}/dt = \vec{\omega}_a \times \vec{s}; \qquad \omega_a = \omega_{a0} + \omega_{rf}(t), \tag{10}$$

$\omega_{a0} = a\gamma\omega_C$, in a magnetic ring. $\omega_{rf}(t)$ is defined by substituting b_V and e_R for $\vec{B}$ and $\vec{E}$ of Eq. (1)

$$\omega_{\mathrm{rf}}(t) = -\frac{e}{mc}\left[ab_V + \left(a - \frac{1}{\gamma^2 - 1}\right)(-\beta e_R/c)\right] = -\frac{e}{mc}\frac{1+a}{\gamma^2}b_{V0}\cos\omega_{a0}t. \tag{11}$$

From (10) and (11), we get the following equations for the longitudinal (L) and radial (R) spin components,

$$ds_L/dt = -\omega_{a0}s_R + s_R\frac{eb_{V0}}{mc}\frac{1+a}{\gamma^2}\cos\omega_{a0}t, \tag{12}$$

$$ds_R/dt = \omega_{a0}s_L - s_L\frac{eb_{V0}}{mc}\frac{1+a}{\gamma^2}\cos\omega_{a0}t. \tag{13}$$

Neglecting the oscillating terms, we get the familiar solution corresponding to the above-mentioned $N = 1$ mode,

$$s_L \approx s_{L0}\cos\omega_{a0}t, \qquad s_R \approx s_{L0}\sin\omega_{a0}t. \tag{14}$$

This is the zero approximation for s_L, s_R. For a reason that will be clear later, the phase of these free oscillations is chosen (at injection) to be the same as the phase of the mWF fields. Ideally, the phase must be the same for all particles (i.e., must be a coherent polarization beam).

Substituting this $N = 1$ mode for s_L in the last term of (13), we get $-s_{L0}\frac{eb_{V0}}{mc}\frac{(1+a)}{\gamma^2}\frac{(1+\cos 2\omega_{a0}t)}{2}$, with half of this expression being a constant in time. But ds_R/dt of Eq. (13) cannot be proportional to a constant in time since the spin magnitude cannot be bigger than 1. Therefore, the constant half of our expression must be canceled by the first term on the right side of Eq. (13). It follows from this that s_L possesses an $N = 0$ mode induced by the mWF fields; let us call it s_{Lm} (m for "magic"):

$$s_L \approx s_{L0}\cos\omega_{a0}t + s_{Lm}, \tag{15}$$

$$s_{Lm} = \frac{s_{L0}}{2\omega_{a0}}\frac{eb_{V0}}{mc}\frac{(1+a)}{\gamma^2}. \tag{16}$$

This is our first-order approximation for s_L containing $N = 0$ and $N = 1$ modes. In (16), due to our choice of phase in (14), we have the maximal possible frozen part of the longitudinal spin component induced by the given mWF fields.

Other choices of the spin phases in Eqs. (14) would give us smaller magic s_{Lm} values. Thus, the optimal phase of the field oscillations must be the same as the phase of the coherent oscillations of the particles' longitudinal spin components.

Here we give the complete solution of Eqs. (12) and (13), omitting successive-approximation calculations of other modes:

$$s_L = s_{L0}\cos\Phi, \qquad s_R = s_{L0}\sin\Phi, \tag{17}$$

$$\Phi = \omega_{a0}t - \frac{eb_{V0}}{mc\omega_{a0}}\frac{(1+a)}{\gamma^2}\sin\omega_{a0}t. \tag{18}$$

When the second term at the right side of (18) is much smaller than 1,

$$\cos\Phi \approx \cos\omega_{a0}t + \frac{eb_{V0}}{2mc\omega_{a0}}\frac{(1+a)}{\gamma^2}(1-\cos 2\omega_{a0}t) - \frac{1}{4}\left[\frac{eb_{v0}(1+a)}{mc\omega_{a0}\gamma^2}\right]^2\cos\omega_{a0}t(1-\cos 2\omega_{a0}t) + \cdots. \tag{19}$$

The first-order approximation (15) and (16) follows from the general solution (17) and (18).

III. THE EDM SIGNAL AND SYSTEMATIC ERRORS

In Eq. (2), the EDM rotation frequency connects vertical and longitudinal spin components, so in Eq. (1) all three components are interconnected. However, when investigating very small changes of the vertical spin component, $\Delta s_V \ll 1$, it is sufficient to use the simple approximation in which we deal only with the vertical spin component:

$$\left(\frac{ds_V}{dt}\right)_{\mathrm{EDM}} = \frac{e}{2mc}\eta\{-E_R - e_R(t) + \beta[B_V + b_V(t)]\}s_{L0}\cos\Phi \approx \frac{e\eta}{2mc}s_{L0}(-E_R + \beta B_V)\cos\Phi, \tag{20}$$

where E and B are the off-mWF fields of the ring. From (20) and (19), after averaging the oscillating terms,

$$\left(\frac{ds_V}{dt}\right)_{\mathrm{EDM}} = \eta\frac{eb_V}{4mc}\frac{(1+a)}{\gamma^2}s_{L0}\frac{e(-E_R + \beta B_V)}{mc\omega_{a0}}. \tag{21}$$

In a purely magnetic ring, $e(-E_R + \beta B_V)/mc\omega_{a0} \rightarrow \beta/a$.

To deal with systematic errors directly connected with this method, we must first make the particles' trajectories maximally insensitive to the interconnected mWF EM field amplitudes. This means canceling their Lorenz force in both radial and vertical directions. With neither radial nor vertical Lorentz force, perturbations of the mWF fields will contain only terms

$$e_V(t) = -\beta b_R(t) = -\beta b_{R0}\cos\omega_{a0}t. \tag{22}$$

There remains a small perturbation, acting only inside the mWF's, which imitates the EDM:

$$\left(\frac{ds_V}{dt}\right)_{\mathrm{sys}} = \frac{eb_{R0}}{2mc}\frac{(1+a)}{\gamma^2}s_{L0}. \tag{23}$$

This false signal—a systematic error—is well defined and depends on β differently from the EDM signal. This difference can be used to reduce the error.

It should be noted that the linear-in-time growth of the vertical spin component described by Eqs. (21) and (23) is the linear approximation of the start of a long-wave sinusoid. The validity of the linear approximation (23) is well

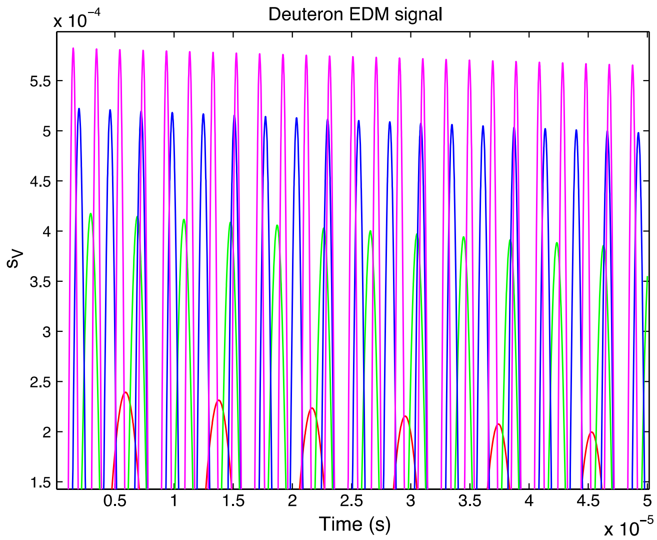

FIG. 1. The vertical spin component, s_V, normalized to 1, vs time for the deuteron, assuming an EDM of 10^{-18} e cm when the magic Wien filter is on, $\theta = 0$. In red, the momentum is 0.7 GeV/c, green 1.4 GeV/c, blue 2.1 GeV/c, and magenta 2.8 GeV/c. The EDM signal is the slope of the vertical spin component as a function of time. The vertical oscillations are at the g-2 frequency, as expected. This figure is similar to Fig. 3 for protons, but contains only a part of the full curve, since the EDM effect for deuterons is bigger than for protons (for the parameters given in the text).

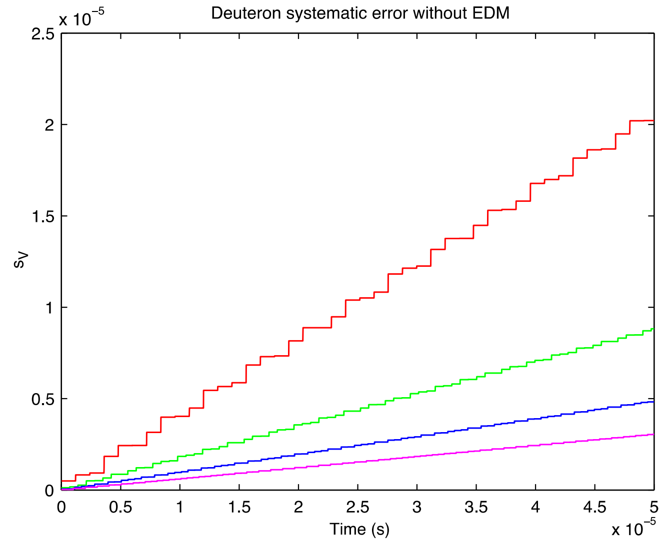

FIG. 2. The vertical spin component, s_V, normalized to 1, vs time for the deuteron, assuming a misalignment angle of 0.1 milliradian when the magic Wien filter is on. In red, the momentum is 0.7 GeV/c, green 1.4 GeV/c, blue 2.1 GeV/c, and magenta 2.8 GeV/c. The EDM-like background signal (systematic error) is the slope of the vertical spin component as a function of time.

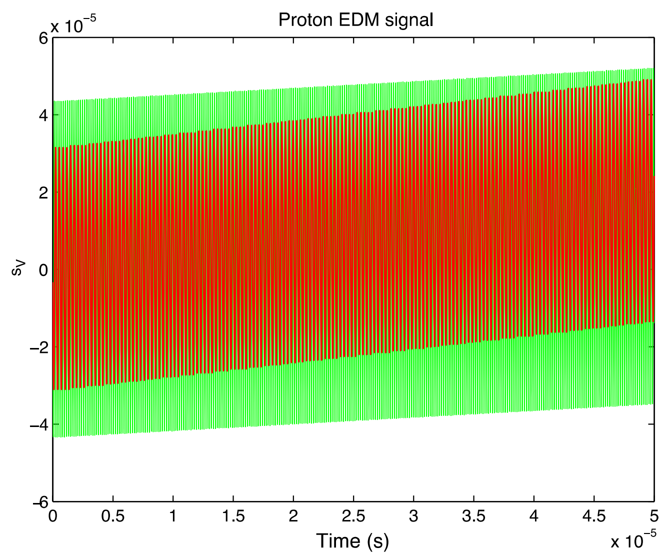

FIG. 3. The vertical spin component, s_V, normalized to 1, vs time for the proton, assuming an EDM of 10^{-18} e cm when the magic Wien filter is on, $\theta = 0$. In red, the momentum is 0.7 GeV/c, and green 1.4 GeV/c. The EDM signal is the slope of the vertical spin component as a function of time. We show only two cases for clarity. The vertical oscillation is at the proton g-2 frequency, which is an order of magnitude larger than the corresponding deuteron frequency.

confirmed by the simulations; see Figs. 1–5. That it is the start of a sinusoid can be shown as follows.

Let us take the ideal case of a particle moving along the equilibrium orbit in the presence of a mWF and an EDM. From Eqs. (1) and (2), $\vec{\omega}_a = (0, 0, \omega_{aV})$, $\vec{\omega}_{\rm edm} = (\omega_{eR}, 0, 0)$, where L, R, V are the longitudinal, radial, and vertical coordinates. So Eq. (3) can be rewritten as

$$\frac{ds_L}{dt} = -\omega_{aV} s_R + \omega_{eR} s_V, \tag{24}$$

$$\frac{ds_R}{dt} = \omega_{aV} s_L, \tag{25}$$

$$\frac{ds_V}{dt} = -\omega_{eR} s_L. \tag{26}$$

[The precise solution of Eqs. (24)–(26) with constant ω_{aV}, ω_{eR} is qualitatively outlined above, after Eq. (6).] In the presence of the mWF, however, ω_{aV} depends on time and these three equations cannot be analytically solved with precision. We can get a solution by taking into account the fact that ω_{eR} is small and constant in time. In the zero approximation, (i.e., $\omega_{eR} = 0$), we get the solution for s_L, s_R, (15)–(18). Substituting solution s_{Lm} for s_L into (26) and then integrating (26), we get the linear approximation for s_V [assuming the initial condition $s_V(0) = 0$]:

$$s_V = -s_{Lm}(\omega_{eR} t), \tag{27}$$

with s_{Lm} from (16). Substituting the solution for s_V (27) into (24) and taking into account only s_{Lm} and terms growing in time, we get the quadratic approximation for s_L:

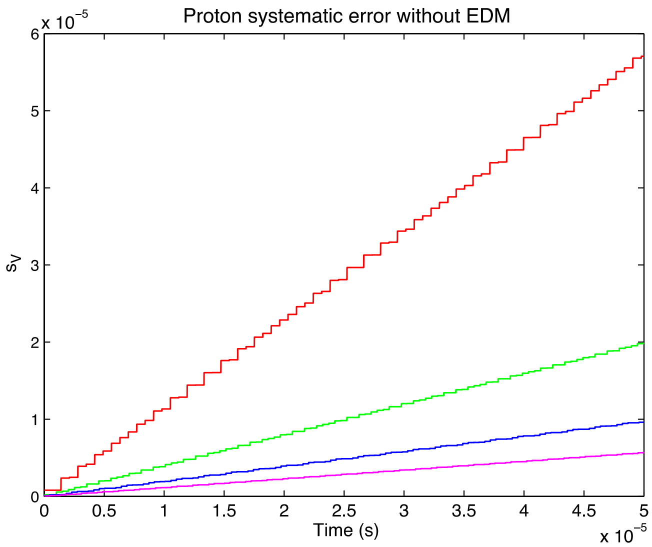

FIG. 4. The vertical spin component, s_V, normalized to 1, vs time for the proton, assuming a misalignment angle of 0.1 milliradian when the magic Wien filter is on. In red, the momentum is 0.7 GeV/c, green 1.4 GeV/c, blue 2.1 GeV/c, and magenta 2.8 GeV/c. The EDM-like background signal (systematic error) is the slope of the vertical spin component as a function of time.

$$s_L = s_{Lm} - \frac{1}{2} s_{Lm} (\omega_{eR} t)^2. \tag{28}$$

Substituting this for s_L in (26), we get the cubic approximation for s_V:

$$s_V = s_{Lm} \left[-\omega_{eR} t + \frac{1}{6} (\omega_{eR} t)^3 \right], \tag{29}$$

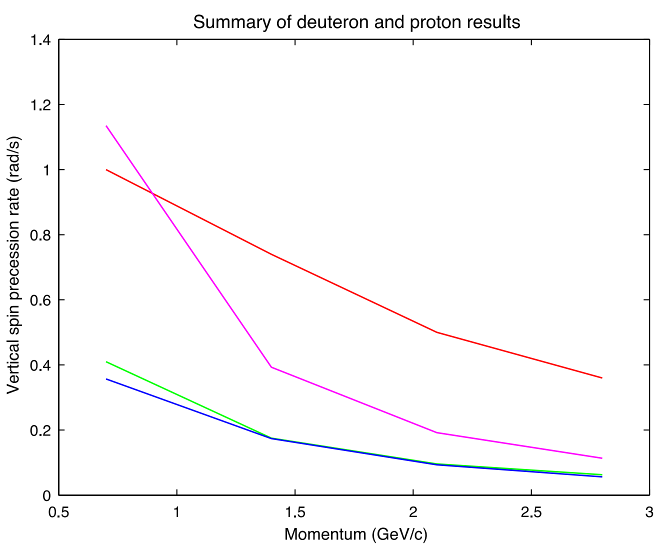

FIG. 5. The deuteron and proton results from tracking. The red corresponds to an EDM of 10^{-18} e cm and green to systematic error due to a misalignment angle of 0.1 milliradian for the deuteron. The blue corresponds to an EDM of 10^{-18} e cm and magenta to systematic error due to a misalignment angle of 0.1 milliradian for the proton.

and so on, with $s_V \to -s_{Lm} \sin(\omega_{eR} t)$ + higher-order terms and terms not growing in time. The presence of the long-wave EDM sinusoid is obvious.

IV. SPIN TRACKING SIMULATION RESULTS

The spin tracking simulation reported here is based on combined beam and spin dynamics equations [10]. It originated in a preliminary simulation [8] whose counterintuitive results were then confirmed and explained by theoretical analysis [9]. The current simulation was designed to thoroughly test that analysis. It showed that the linear approximation just mentioned is valid with sufficiently high accuracy. It likewise confirmed the validity of another theoretical simplification: limiting analysis to uniformly distributed mWF fields, even though mWFs can in practice be distributed in discrete locations along a ring. (Later, this confirmation would itself be confirmed theoretically by a different technique, which showed that both continuous and discrete distributions result in the same EDM signal [11].)

Our formulas for the EDM signal and the systematic error are

$$\left(\frac{ds_V}{dt} \right)_{\mathrm{EDM}} = \frac{e\eta}{4mc} e_{R0} \frac{1+a}{a\gamma^2} s_{L0}, \tag{30}$$

which is equivalent to Eq. (21) and

$$\left(\frac{ds_V}{dt} \right)_{\mathrm{SYS}} = + \frac{e}{2mc} e_{V0} \frac{1+a}{\beta\gamma^2} s_{L0}, \qquad \vec{e}_{V0} = \vec{e}_{R0} \times \vec{\theta}, \tag{31}$$

which is equivalent to Eq. (23). We have one single mWF in our simulation. θ in Eq. (31) is the misalignment angle of this mWF relative to its ideal vertical axis. The differential equations [10] used for tracking are

$$\frac{d\vec{\beta}}{dt} = \frac{e}{mc\gamma} [\vec{E} + \vec{\beta} \times \vec{B} - \vec{\beta}(\vec{\beta} \cdot \vec{E})], \tag{32}$$

$$\frac{d\vec{s}}{dt} = \frac{e\vec{s}}{mc} \times \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - \frac{a\gamma\vec{\beta}}{\gamma+1} (\vec{\beta} \cdot \vec{B}) - \left(a + \frac{1}{\gamma+1} \right) \vec{\beta} \times \vec{E} + \frac{\eta}{2} \left[\vec{E} + \vec{\beta} \times \vec{B} - \frac{\gamma\vec{\beta}(\vec{\beta} \cdot \vec{E})}{\gamma+1} \right] \right\}, \tag{33}$$

with the EDM term added in the spin precession. We use Runge-Kutta integration and a time step size of the order of 0.5 ps.

The (magnetic) ring radius in the simulation is 20 m; the field-focusing index, $n = -(\frac{\partial B}{\partial x}) R/B$, is 0.01. (In a future experiment, a rather weak vertical focusing will probably be needed to better manipulate the spin perturbations.) The rf electric field (RFE) amplitude is 30 MV/m, with the single dipole 10 cm long and the nonperturbed E field in the radial direction. The RFB is always matched such that the Lorentz force equals zero in the mWF (i.e., 30 MV/m

TABLE I. Comparison between tracking results and analytical estimations for the deuteron case, in rad/s.

Momentum [GeV/c]	EDM *tracking*	EDM *analytical*	Systematic error *tracking*	Systematic error *analytical*
0.7	−1.00	−1.00	0.41	0.41
1.4	−0.74	−0.73	0.175	0.17
2.1	−0.50	−0.51	0.096	0.097
2.8	−0.36	−0.35	0.063	0.06

TABLE II. Comparison between tracking results and analytical estimations for the proton case, in rad/s.

Momentum [GeV/c]	EDM *tracking*	EDM *analytical*	Systematic error *tracking*	Systematic error *analytical*
0.7	0.357	0.357	1.135	1.137
1.4	0.174	0.172	0.393	0.396
2.1	0.0934	0.093	0.192	0.195
2.8	0.0566	0.0563	0.1135	0.1135

divided by β). The nonperturbed RFB field direction is vertical.

Figures 1 and 2 show the EDM signal and systematic error for the deuteron with arbitrarily assumed $\eta = 2 \times 10^{-4}$; Figs. 3 and 4 show the corresponding cases for the proton (with the assumed $\eta = 1.876 \times 10^{-4}$). The EDM level is assumed to be 10^{-18} e cm for both deuteron and proton. The misalignment angle θ is (arbitrarily) assumed to be 0.1 milliradian. Time is shown in seconds; the initial condition is always $s_V(0) = 0$; total spin is always normalized to 1. The mWF modulation tune used is $1 + a\gamma$; a is the anomalous magnetic moment. (The initial radial spin component influences only the resonance phase. In this simulation, we have always chosen the phase that gives the biggest EDM signal.)

Figure 5 shows the deuteron and proton results in one graph.

Tables I and II, above, show the agreement between the tracking results from simulation (shown in Fig. 5) and Eqs. (30) and (31). The agreement is excellent. As predicted by (30) and (31), the ratio of the EDM signal to the systematic errors (produced by the mWF's misalignment) for a specific particle is proportional to velocity β. This correlation can be used to correct errors.

V. CONCLUSION

The agreement between our analytical results and results from tracking for the mWF proves that we correctly understand the partially frozen spin phenomenon. Thus, it can be used in a draft design to measure EDM, disregarding details of mWF filter distribution along the ring. Tables I and II and Eqs. (30) and (31) indicate that the best choice of particle momentum will be around or slightly higher than 1 GeV/c, where the EDM signal is not as small as for higher momenta and the ratio EDM/SYS (proportional to velocity β) is also acceptable. By contrast, at $\gamma \gg 1$ the EDM signal goes down as $1/\gamma^2$ does. Moreover, we lose the correlation between systematic error and β, which is an important tool for correcting that error. From independent consideration of the polarimetry problems, it is not advisable to deal with momenta below 0.7 GeV/c.

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