Data-Dependent Jitter Estimation Using Single Pulse Analysis Method

Eakhwan Song, Junho Lee, Jingook Kim, Dong Gun Kam, Chunghyun Ryu and Joungho Kim
Terahertz Interconnection and Package Laboratory
Dept. of EECS,KAIST,371-1 Gusung,Yusung,Daejeon 305-701, Korea
E-mail: sehnx@eeinfo.kaist.ac.kr;teralab@ee.kaist.ac.kr, Phone: +82-42-879-9872, Fax: +82-42-869-8058,

Abstract

A simple analysis method for the estimation of data-dependent jitter (DDJ) in high-speed digital system using single pulse response is proposed. A frequency dependent model of a transmission line is considered to charaterize lossy channel properties, and the single pulse response is obtained with the model. The estimated DDJ in the single-ended transmission line is experimentally verified with the timing jitter measurements.

1. Introduction

Timing margin in high-speed data channel becomes more tight as bit rate goes higher. Timing jitter pares down the timing margin and makes it difficult to set timing budget for the receiver. Timing jitter is composed of random jitter (RJ) and deterministic jitter (DJ) [1]. RJ is the timing noise that cannot be predicted, because it has no discernable pattern due to its natural random characteristics. DJ is the jitter caused by non-random events and caused by several major sources: periodic noise in power supplies, duty cycle distortion, and so on. Especially, the timing jitter that is correlated with the bit sequence in a data stream is called data-dependent jitter (DDJ). That means the response of the current bit is affected by the responses of the previous bits [2]. With higher bit rate and dispersive lossy channel, the pattern-dependency of the DDJ becomes increased. As a result, DDJ issues are rising up in recent high-speed serial interface design.

When designing high-speed data channels, it is important to estimate and analyze DDJ to guarantee sufficient timing margin for the receiver detecting data signals. The most common process to predict DDJ is to use commercial simulation tool. However, this way takes much times for good timing resolution and does not give choice of the channel model except the ones which are provided by the tool. So the analytic approach methods are needed for the estimation of the DDJ.

There have been proposed some analytic DDJ predicting methods, which have common problem that the bandwidth of the channel should be known in advance for the estimation of DDJ [2]. Thus bandwidth-limiting filter is used to fix the bandwidth of the channel for a mathematical model, it is hard to say these methods are meaningful at the high-speed data channel of real cases. Because of this limitation, we physically characterized the channel based on the transmission line theory and then proposed the single pulse analysis for the estimation of DDJ. The single pulse analysis gives fast and precise results for predicting DDJ.

In this paper, we present physically analyzed frequency-dependent model of the transmission line, and then show intuitive approach to determine DDJ with the single pulse response which is derived from the model. Finally, we validate the proposed method with measurement results.

2. Frequency-dependent Modeling of Transmission Line

Especially copper based interconnection without low-k substrate material suffers from timing jitter at multi-gigabit wideband signals through frequency-dependent loss. Both skin-effect and dielectic loss are the representative causes to degrade the quality of the signals which commonly used in current high-speed digital system. These losses make signal distorted, and this distortion is shown up as inter-symbol interference (ISI) which is the source of DDJ. And through the process of converting the frequency domain into the time domain, some sources of error can be generated; non-causality, bandwidth limit and others. So well-considered frequency-dependent model is needed and frequency-dependent loss is also important.

In a tansmission line thorem, the transfer function of a transmission line is:

$$H(\omega) = e^{-l \cdot \gamma(\omega)} \tag{1}$$

$$\gamma(\omega) = \sqrt{(R + j\omega L)(j\omega C)} \tag{2}$$

where $\gamma(\omega)$ is propagation constant and R, L, and C are resistance, inductance and capacitance per unit length. Each element is presented as:

$$R = \sqrt{R_{DC}{}^2 + R_{AC}{}^2} \tag{3}$$

$$L = \mu_r L_0 \approx L_0 \tag{4}$$

$$C = \varepsilon_r C_0 = (\varepsilon_r' - j\varepsilon_r'')C_0 \tag{5}$$

where L_0, C_0 are unit inductance,capacitance in an air instead of dielectric. The skin-effect can be seen in R_{AC} which is proportion to a square root of frequency. For more accuracy, proximitty effect caused by ac return current also has to be considered [3]. Conductance G does not seem to appear above, but it is already considered by the imaginary part of the complex relative permittivity ε_r. There is close correlation between ε_r and dielectric relaxation time. The most noteworthy feature of the dielectric relaxation of ordered material such as FR4 is the Debye model [4]:

$$\varepsilon_r = \sum_i \frac{a_i}{1 + j\omega\tau_i} \quad \begin{pmatrix} a = strength \\ \tau = relaxation\ time \end{pmatrix} \tag{6}$$

The imaginary part of ε_r is actually concerned with conductance of the transmission line which determines the leakage current through the substrate, and the real part varies unit capacitance with proportion to frequency.

Substituting (2)-(6) to (1), we can get final transfer function which works in both skin-effect region and dielectric-loss-limit region with only geometric parameters of transmision line.

3. Single Pulse Response

DDJ can be derived from the analysis of single pulse response. The response of a single pulse after consecutive 0' or 1' is considered as the worst pattern on lossy transmission line [Fig. 1].

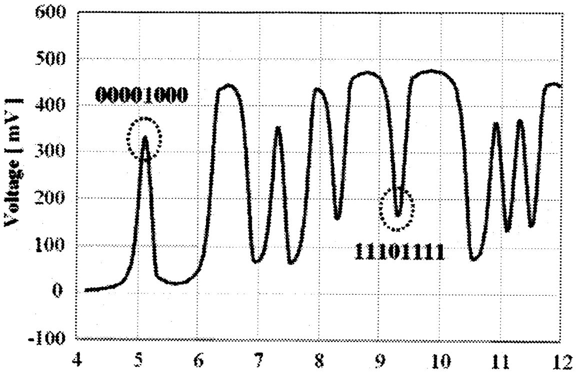


Fig. 1. 5Gb/s bit sequence after flying 50cm microstrip line

It is hard to get the single puse response as closed-form because the frequency-dependent charateristics of the channel are so complex. One method dealing with this problem is to use fast-fourier transtormation (FFT) and the inverse of it (IFFT) and then numerically performs conversion of one domain into the other. This has advantages for flexibility of the model. If we got more suitable frequency-dependent model, we could apply the model easily just by substituting it with old one.

The single pulse is composed of two step functions which have opposite polarity and delay of a bit period. Each step function generated on the transmitter which has RC time constant can be written as [3]:

$$\left.\begin{aligned} g_1(t) &= u(t)erfc\left(\frac{1}{2}\sqrt{\frac{\tau}{t}}\right) \\ g_2(t) &= -u(t)erfc\left(\frac{1}{2}\sqrt{\frac{\tau}{t-t_b}}\right) \end{aligned}\right\} \qquad (7)$$

where τ is time constant and t_b is bit period. Therefore a single pulse input can be modeled as :

$$x(t) = g_1(t) + g_2(t) \qquad (8)$$

This input fuction is fourier tansformed to $X(\omega)$ and mutiplied with the transfer fuction of the transmission line $H(\omega)$ which is defined in (1) to construct the output response. Then final single pulse response in time domain can be derived from IFFT of $H(\omega)X(\omega)$. Fig. 2 shows good correlation of the model with measurement. all caculations were performed using MATLAB.

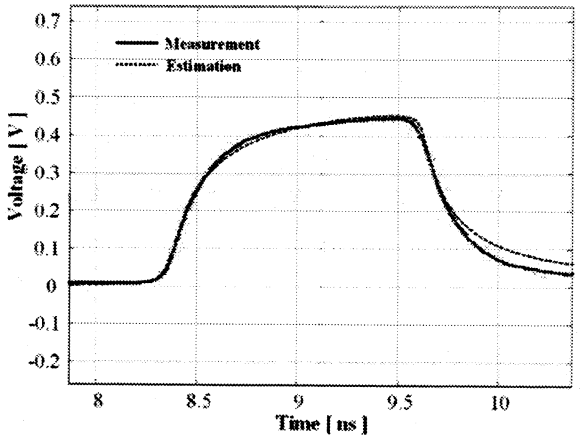


Fig. 2. Comparison of the single pulse waveforms; 0.8Gb/s bit sequence after flying 100cm microstrip line

4. DDJ Estimation with Single Pulse Response

With single pulse response, DDJ can be obtained easily. In Fig. 3, the output of the single pulse rises later than that of a periodic clock signal and falls earlier in the same manner. These discrepancies of the reference-crossing time between the sigle pulse response and periodic clock pattern represent the both side of DDJ in the worst case and the total DDJ becomes the sum of them (9). And it is simply found in Fig. 4 that the difference between bit period and the width of eye also implies total DDJ (10). This relations are shown in Fig. 4.

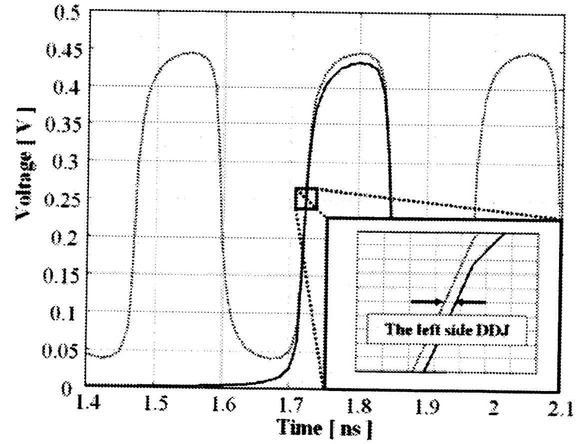


Fig. 3. Comparision of a clock signal and the single pulse response; The largest DDJ is determined by the timing difference of a single pulse with periodic clock signal when these signals cross the reference voltage level (250mV)

$$t_{DDJ} = t_{DDJ\_left} + t_{DDJ\_right} \tag{9}$$

$$= t_{bit} - t_{eye\ width} \tag{10}$$

where, $\begin{cases} t_{DDJ}: \text{ the total DDJ} \\ t_{DDJ\_left}: \text{ the left side DDJ} \\ t_{DDJ\_right}: \text{ the right side DDJ} \end{cases}$

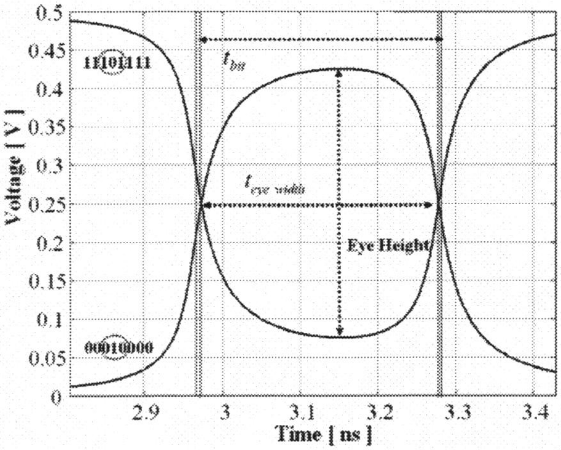


Fig. 4. Single pulse responses at bit rate of 3.2Gb/s

In Fig. 4, the solid and dotted line are the single pulse waveforms after consecutive 1' and 0'. And it is intuitively found that the closed loop between the waveforms is the inner contour of Eye-diagram. This means Eye-mask compliance test can be performed with the analysis of single pulse response as well as estimation of the voltage margin.

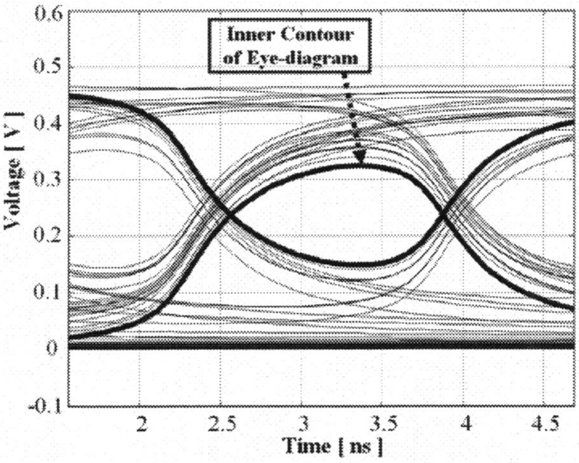


Fig. 5. Eye-diagram construction by combinations of single pulse responses.

In addition, if we knew all patterns of the data and the data channel is passive we could generate eye-diagram. The passive system is regarded as a time-invariant linear system, that means the entire waveform can be constructed by combinations of each single pulse response [Fig. 5]. The eye-diagram also can be obtained by superimposing waveforms every bit period.

When caculating total DDJ in this way, it takes much less time required than that of the commercial tool for simulation because of its simplicity.

5. Experimental Verification

We have performed measurements on 40cm FR4 microstrip line varying bit rate of the data. The input signal was a $2^7 - 1$ pseudo random bit sequence (PRBS) which has rise time of 30ps and 500mV swing from pulse-pattern generator (PPG). The output signal after flying through the microstrip line was observed with Tektronix TDS8000B sampling oscilloscope equipped with 20Gb/s sampling module. To avoid the parastic effectc of connector micoprobe which has a cutoff frequency of 40GHz was used. The mesurement setup is shown in Fig. 6. The bit rate was scanned from 100Mb/s to 6400Mb/s by doubling step.

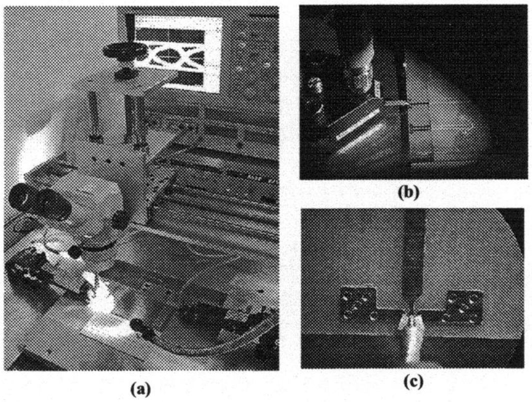
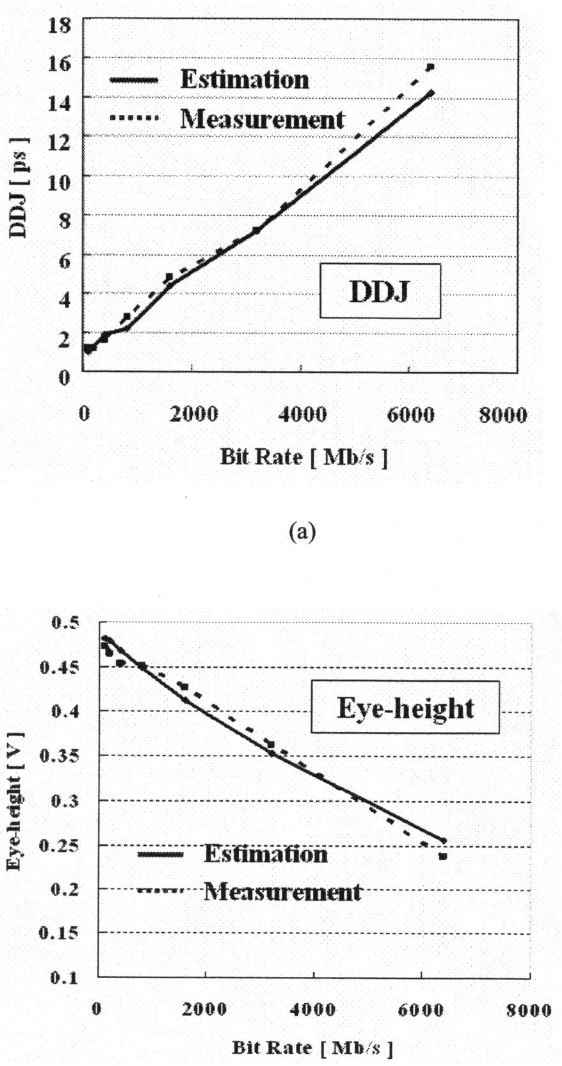


Fig. 6. (a) the entire measurement setup (b) probe contact to DUT (c) a microphotograph of the contact region

Fig. 7 tells the estimated DDJ with the proposed method has a good correlation with the measurement. At 6.4Gb/s bit rate the error is 1.35ps; the estimated DDJ is 14.25ps and the measurement result of DDJ is 15.6ps. The errors at the other bit rate have the values of sub-pico seconds. The numerical comparisons of the estmation and measurement of DDJ are shown in Table. 1.



(a)

(b)

Fig. 7. Comparison of estimated (a) DDJ and (b) Eye-height with the measurement for bit rate variation

Table 1. Estimated and measured DDJ at several bit rates

Bit Rate (Mb/s)	Estimated DDJ (ps)	Measured DDJ (ps)	Estimated Eye-height (V)	Measured Eye-height (V)
100	1	1.2	0.482	0.473
200	1.25	1.2	0.479	0.465
400	1.88	1.6	0.468	0.454
800	2.19	2.8	0.454	0.449
1600	4.38	4.8	0.412	0.427
3200	7.19	7.2	0.353	0.362
6400	14.25	15.6	0.256	0.238

Conclusion

We introduced an analytic method to estimate DDJ in transmission lines with single pulse response, the proposed method based on a physical model of frequency dependent loss due to skin-effect and dielectric loss. And the estimated DDJ and eye-height with the proposed method was verified by the measurement performed in several conditions and found to be quite accurate.

The available bandwidth of the model is broaden by numerical conversion of the domains using fast-Fourier Transform, thus the timing accuracy of the estimated DDJ is improved in high-speed data channel. Because of the simplicity of the method, the time required to caculate takes only a few seconds. And this will be useful for setting a timing budget in high-speed data channel.

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