Transmit beamforming with Imperfect CSIT in Spectrum Leasing for Physical-Layer Security

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Abstract—In spectrum leasing, primary users (PUs) lease the part of their spectral resources to secondary users (SUs) in exchange for appropriate remuneration. In this paper, we consider spectrum leasing via cooperation for physical-layer security that the secondary cooperation exists for improving the primary secrecy rate while maintaining its quality of service (QoS). Unlike the previous researches with the perfect channel state information (CSI) at transmitter (CSIT) assumption, we study when no information regarding the eavesdropper is available at the transmitter. This imperfect CSIT makes the SU’s transmission strategy limited. To find the optimal transmission technique for the SU, we formulate a problem appropriate for the imperfect CSIT case. By using the proposed problem, we design the optimal transmit beamforming for the SU. Also, we analyze the ergodic rate of the secondary link at high signal-to-noise ratio (SNR) when the secondary cooperation focuses only on maximizing the primary secrecy rate for the viable choice of the secondary QoS level. The numerical result shows that the primary secrecy rate by the proposed transmit beamforming is comparable to that based on the perfect CSIT.

Index Terms—Physical security, spectrum leasing, interference channel, imperfect channel state information.

I. INTRODUCTION

Within the communication range, anyone including the eavesdropper can overhear and extract the confidential message of the transmitter due to the broadcast nature of wireless medium. The secrecy capacity between a transmitter and its intended receiver in wire-tap channel (WTC) model was introduced in [1]. Recently, for the improvement of the physical-layer security, an information theoretic view on the impact of the helper on the secrecy capacity and the effective transmission techniques with multiple antennas were studied in [2]–[5]. On the other hand, cognitive radio networks (CRs) have been introduced to improve the spectrum efficiency by allowing the secondary device to opportunistically access underutilized frequency bands. Among the recent researches on CR, spectrum leasing via cooperation techniques was introduced to employ the secondary cooperation for enhancing either rate or outage performance of the primary link [6], [7].

Unlike the previous spectrum leasing techniques for maximizing the primary throughput, [8] proposed an alternative objective of spectrum leasing. In [8], the secondary cooperation is aimed at the improvement of the primary secrecy rate while maintaining its QoS or minimum required rate. Surprisingly, the performance results in [8] showed that the secrecy rate of the primary link can be improved by spectrum leasing via secondary cooperation while the secondary QoS is satisfied.

In [8], the authors assumed the perfect CSI including the information of eavesdropper. However, in practice, any knowledge about the eavesdropper’s CSI is hardly available. As a result, we take the imperfect CSIT assumption that no information regarding the eavesdropper is known at the transmitter. Since this assumption makes the SU disable to generate the artificial interference using the eavesdropper’s CSI, the proposed transmit beamforming vector in [8] needs to be changed accordingly. To find a new transmission strategy for the SU, we first formulate a problem appropriate for the imperfect CSIT case. By using the proposed problem, we design the optimal transmit beamforming for the SU. Also, for the operational choice of the secondary QoS level, we derive the ergodic rate of secondary link in high SNR regime when the secondary cooperation maximizes the primary secrecy rate regardless of the minimum requirement on the SU’s rate.

This paper is organized as follows. Section II provides the system model and preliminaries about the spectrum leasing with the perfect CSIT. In Section III, we study the impact of the imperfect CSIT in spectrum leasing and design the optimal transmit beamforming for the SU. Also we analyze the ergodic rate of the secondary link in the helper case with imperfect CSIT at high SNR. In Section IV, we show the simulation results and this paper is concluded in Section V.

Notation: The notation for this paper is as follows: \(\|a\|\): Euclidean norm of \(a\); \([A]^+\): max\((A, 0)\); \([\cdot]^*\): conjugate transpose; \(P_X \triangleq X(X^*X)^{-1}X^*\): orthogonal projection onto the column space of \(X\); and \(P_{X^\perp} \triangleq I - P_X\): orthogonal projection onto the orthogonal complement of the column space of \(X\).

II. SYSTEM MODEL AND PRELIMINARIES

A. System model

We consider a communication system with primary and secondary transmitter/receiver pairs and a passive eavesdropper. We suppose that the secondary transmitter has \(N\) multiple antennas while all other nodes have a single antenna. The system model is illustrated in Fig. 1. We consider an imperfect CSIT scenario where the primary and the secondary transmitters are unaware of the eavesdropper’s operating parameters while the CSI about other channels is perfectly known to them. All receivers including the eavesdropper suffer from...
the interference caused by the undesired transmission. We assume that they treat the interference from the unintended transmitter as an additive noise [9]. This assumption can be justified by the fact that in most practical applications the receivers would be devised with a simple structure. We also assume that the transmission consists of scalar coding followed by beamforming. We denote the power constraints at the primary transmitter as an additive noise. This assumption can be justified by the fact that in most practical applications the secondary link is given by

$$P_{\text{opt}} = \max \left( 1 + \frac{|h_{pp}|^2}{\sigma_p^2 + |w^* h_{sp}|^2} \right)$$

$$\text{min} \left( 1 + \frac{|h_{pe}|^2}{\sigma_e^2 + |w^* h_{se}|^2} \right)$$

where $w$ denotes an $N \times 1$ beamforming vector at the secondary transmitter satisfying $|w|^2 \leq 1$, $h_{pi}$ or $h_{sj}$ are complex-valued channel coefficient or $N \times 1$ channel vector between nodes for $i, j \in \{p, s, e\}$, and $\sigma_p^2$ and $\sigma_e^2$ are variances of the zero-mean Gaussian noise at the primary receiver and the eavesdropper, respectively. Also, the achievable rate of the secondary link is given by

$$R_s(w) = \log \left( 1 + \frac{|w^* h_{ss}|^2}{\sigma_s^2 + |h_{ps}|^2} \right),$$

where $\sigma_s^2$ is the variance of the zero-mean Gaussian noise at the secondary receiver.

B. Spectrum leasing with perfect CSIT

In this part, we introduce the spectrum leasing with the perfect CSIT in [8]. [8] introduced the optimal beamforming vector, $w_{\text{opt}}$, for the SU to guarantee the QoS of the secondary link as well as to maximize the primary secrecy rate under its own transmit power constraint. Accordingly, $w_{\text{opt}}$ can be obtained by solving the following Problem $P1$:

$$P1: \max_w R_p(w) \quad \text{s.t.} \quad ||w||^2 \leq 1, R_s(w) \geq R_{\text{min}},$$

where $R_{\text{min}}$ is the minimum required rate by the SU. $R_{\text{min}}$ is defined by

$$R_{\text{min}} = \alpha R_{\text{max}},$$

where $\alpha \in [0, 1]$ and $R_{\text{max}} = \log \left( 1 + \frac{|h_{ps}|^2}{\sigma_p^2 + |h_{pe}|^2} \right)$ which is the maximum achievable rate of the secondary link. The parameter $\alpha$ represents the QoS level requested by the secondary link from the lowest ($\alpha = 0$) to the highest ($\alpha = 1$).

In [8], the optimal solution of Problem $P1$ was given by a parametrization with two real numbers irrespective of the number of antennas as follows:

$$w_{\text{opt}} = v_{\text{max}}(Z), Z = \lambda_1 h_{se} h_{se}^* + \lambda_2 h_{ss} h_{ss}^* - \lambda_3 h_{sp} h_{sp}^*,$$

$$\lambda_k \in [0, 1] \text{ and } \sum_{k=1}^3 \lambda_k = 1,$$

where $v_{\text{max}}(Z)$ denotes the principal eigenvector of $Z$.

III. Spectrum leasing with imperfect CSIT

In practice, it is impracticable to obtain any knowledge about the eavesdropper. Accordingly, we consider Problem $P1$ with the imperfect CSIT assumption in this work. To find the optimal transmission strategy for the SU, we first formulate the problem appropriate for the imperfect CSIT case. We also derive the ergodic rate of secondary link at high SNR for the realistic decision about the secondary QoS when the secondary cooperation focuses only on improving the primary secrecy regardless of its QoS. Then, we finally design the optimal transmit beamforming for the SU in spectrum leasing with imperfect CSIT by using the proposed problem formulation.

A. Impact of imperfect CSIT

The imperfect CSIT assumption implies that the eavesdropper’s CSI is not available at the transmitter. This makes (5) based on the perfect CSIT impossible for the SU to implement. Consequently, a new transmission strategy for the SU needs to be established for the imperfect CSIT case.

The optimal $w$ of Problem $P1$ is designed to maximize the primary secrecy rate under the transmit power and QoS constraint of the SU. However, the imperfect CSIT assumption makes it unrealizable for the SU to generate the artificial interference using the eavesdropper’s CSI. Therefore, the best strategy for the SU is to distribute the artificial interference along the spatial dimensions that will produce the minimum of the interference for the PU subject to its QoS constraint. In other words, the SU allocates the smallest possible fraction of the available transmit power to achieve its minimum required rate. Simultaneously, the remaining power should be allocated to maximize the primary secrecy rate by jamming the eavesdropper in the way of minimizing the interference for the PU. Hence, we can formulate a new problem for the imperfect CSIT case as

$$P2: \min_w \frac{|w^* h_{sp}|^2}{\text{s.t.} \quad ||w||^2 \leq 1, R_s(w) \geq R_{\text{min}}.$$
In the rest of the paper, by using Problem \textbf{P2} we design the optimal beamforming vector for the SU and compare the results with the solution based on the perfect CSI.

It is noteworthy that Problem \textbf{P2} seems similar with the opportunistic spectrum sharing in CRs or MISO interference channel (IFC) (see \cite{10} and \cite{11} references therein). We would like to emphasize that, however, the objective of our work is clearly different to theirs. The fundamental challenge of \cite{10} is to maximize the achievable throughput of the secondary networks under the interference power constraint for the PU. Also, the goal of MISO IFC is to maximize the sum rate of the network to find the achievable rate region for users. On the other hand, Problem \textbf{P2} aims to minimize the interference under the transmit power and QoS constraint for the SU. This is for maximizing the secrecy rate of the primary link while maintaining the secondary QoS without the eavesdropper’s CSI.

\subsection{B. Helper case: no QoS constraint}

In this section, we consider the helper case of no QoS constraint, i.e., $R_{\text{min}} = 0$ where the eavesdropper’s CSI is not available at the transmitter. By analyzing the ergodic rate of the secondary link in this simple scenario, we can be aided in determining the practical QoS level for the SU.

When the secondary transmitter acts as the helping interferer with the imperfect CSI, the secondary cooperation focuses only on maximizing the primary secrecy rate without the concerns about its QoS. In this case, the best option available at the secondary transmitter is to uniformly spread the information along the spatial dimensions that will create no interference for the primary receiver and to use the remaining degrees of freedom for maximizing the interference at the eavesdropper. This is the zero-forcing (ZF) beamforming.

As we stated in Section III-A, by using the imperfect CSI, the utmost efforts of the SU to maximize the primary secrecy rate is to minimize the interference for the PU by controlling its QoS level. Therefore, for deciding an appropriate QoS for the SU, we analyze the achievable rate of the secondary link when $\textbf{w}^\text{ZF}$ is employed at the secondary transmitter.

\textbf{Proposition 1:} When the SU performs ZF beamforming transmission, the ergodic achievable rate of the secondary link at high SNR is given by

$$\lim_{\sigma^2 \rightarrow 0} E_{h_{ss}, h_{ps}}[R_s] = \frac{\Psi(N - 1) + \gamma + 1/(N - 1)}{\log 2}, \quad (7)$$

where $\Psi(x) = d\log \Gamma(x)/dx = \Gamma(x)/\Gamma(x)$ is Psi(DiGamma) function and $\gamma$ is Euler’s constant.

\textbf{Proof:} The effective channel gain received at the secondary receiver with $\textbf{w}^\text{ZF}$ is given by

$$|\textbf{w}^\text{ZF}^* h_{ss}|^2 = \frac{|P^*_h h_{ss}|^2}{|P^*_h h_{ss}| h_{ss}^*} = h_{ss}^* P^*_h h_{ss}, \quad (8)$$

where $\textbf{w}^\text{ZF}$ denotes the ZF beamforming vector and is defined by $\textbf{w}^\text{ZF} = P^*_h h_{ss}/|P^*_h h_{ss}|$. Since $P^*_h h_{ss}$ has a unit norm, $P^*_h h_{ss}$ can be written by $\textbf{U}^* \textbf{U}$ where $\textbf{U}$ is a $N \times (N - 1)$ semi-unitary matrix. The upperity of $\textbf{U}$, $\textbf{U}^* h_{ss}$ is a vector of length $N - 1$ with i.i.d zero mean and unit variance Gaussian elements. Therefore (8) can be rewritten such as

$$x \triangleq h_{ss}^* P^*_h h_{ss} = ||\textbf{U}^* h_{ss}||^2 \quad (9)$$

and $x$ is distributed with $N - 1$ complex degrees of freedom, which follows gamma distribution of $p(x) = 2^{-(N-1)x} \exp(-\frac{x}{2})/\Gamma(N-1) \sim \text{Gamma}(N-1,2)$. Also $|h_{ps}|^2$ is a standard exponentially distributed,

$$y \triangleq |h_{ps}|^2 \sim \frac{1}{2} \exp(-\frac{y}{2}). \quad (10)$$

Hence the achievable rate of the secondary link with $\textbf{w}^\text{ZF}$ is given by

$$R_s(\textbf{w}^\text{ZF}) = E_{x,y} \left[ \log_2 \left( 1 + \frac{x}{\sigma^2 + y} \right) \right]. \quad (11)$$

Note that $x$ and $y$ are independent. At high SNR, (11) becomes

$$\lim_{\sigma^2 \rightarrow 0} E_{x,y} \left[ \log_2 \left( 1 + \frac{x}{\sigma^2 + y} \right) \right] = E_{x,y} \left[ \log_2 \left( 1 + \frac{x}{y} \right) \right]. \quad (12)$$

Finally, the achievable secondary rate with $\textbf{w}^\text{ZF}$ can be obtained by taking the expectation of (12) with respect to $x$ and $y$ as below.

$$E_{x,y} \left[ \log_2 \left( 1 + \frac{x}{y} \right) \right] = E_x \left[ \frac{1}{\log 2} \left( \log x - \log 2 + \gamma + e^\frac{x}{2} E_i(1, \frac{x}{2}) \right) \right]$$

$$= \frac{1}{\log 2} \left( E_x \left[ \log \log x - \log 2 \right] + E_x \left[ \gamma + e^\frac{x}{2} E_i(1, \frac{x}{2}) \right] \right)$$

$$= \frac{1}{\log 2} \left( \Psi(N-1) + E_x \left[ \gamma + e^\frac{x}{2} E_i(1, \frac{x}{2}) \right] \right)$$

$$\leq \frac{1}{\log 2} \left( \Psi(N-1) + \gamma + 1/(N-1) \right), \quad (13)$$

where $E_i$ is an exponential integral s.t. $E_i(1,x) = \int_x^{\infty} e^{-t}/t \, dt$. In (13), (a) evaluates the expectation of (12) with respect to $y$, (b) follows the property of the gamma distribution such as $E[\log X] = \Psi(N-1) + \log 2$ where $X \sim \text{Gamma}(N-1,2)$, which results in canceling out $\log 2$ and (c) comes from the expectation of the remaining term $\gamma + e^\frac{x}{2} E_i(1, \frac{x}{2})$ with respect to $x$.

From Proposition 1, we can predict the QoS level that ZF beamforming is able to provide for the secondary link at high SNR, which is summarized as the following.

\textbf{Remark 1:} When ZF beamforming vector is employed by the secondary transmitter, the ergodic $\alpha$ at high SNR is

$$\lim_{\sigma^2 \rightarrow 0} E_{h_{ss}, h_{ps}}[\alpha] = \frac{\Psi(N - 1) + \gamma + 1/(N - 1)}{\Psi(N) + \gamma + 1/N}. \quad (14)$$

Since $\alpha = \frac{R_s}{R_{\text{max}}}$ in (4) and $|h_{ss}|^2 \sim \text{Gamma}(N,2)$, the ergodic $\alpha$ at high SNR when the SU employs ZF beamforming
is
\[
\lim_{\sigma' \to 0} E_{h_{ss}, h_{pp}}[\alpha] = \lim_{\sigma' \to 0} E_{h_{ss}, h_{pp}} \left[ \frac{\log_2 \left( 1 + \frac{|w_{ZF}^e h_{ss}|^2}{\sigma^2 + |h_{pp}|^2} \right)}{\log_2 \left( 1 + \frac{|w_{ZF}^e h_{ss}|^2}{\sigma^2 + |h_{pp}|^2} \right)} \right]
\]
according to Proposition 1. The following values show the ergodic \( \alpha \) for \( N = 2 \) and \( N = 3 \) by Remark 1, respectively.

\[
\frac{\Psi(1) + \gamma + 1}{\Psi(2) + \gamma + 1/2} \simeq 0.6667, \quad \frac{\Psi(2) + \gamma + 1/2}{\Psi(3) + \gamma + 1/3} \simeq 0.8182
\]

For more than two transmit antennas, the ergodic \( \alpha \) at high SNR is higher than 0.8. Hence, we can realize that \( w_{ZF} \) provides a high QoS for the secondary link.

C. Spectrum leasing case: with QoS constraint

In this section, we propose the optimal beamforming vector for the SU in spectrum leasing with imperfect CSIT. The following proposition shows that the solution of Problem P2 can be expressed into a linear combination of maximum ratio transmission (MRT) and ZF beamforming vectors.

**Proposition 2:** The optimal beamforming vector \( w_{opt} \) for Problem P2 is given by

\[
w_{opt} = \frac{\lambda w_{ZF} Z^*}{\lambda w_{ZF} Z^* + (1 - \lambda) w_{MRT} Z^*} ||w_{MRT} Z^*||
\]

where \( 0 \leq \lambda \leq 1, w_{ZF} = P_{h_{sp}} h_{ss} / ||P_{h_{sp}} h_{ss}|| \) and \( w_{MRT} = \delta_{ss} / ||h_{ss}|| \).

**Proof:** The proof requires the main results related to power gain region introduced in [12]. By *Theorem 1* in [12], the optimal solution of P2 exists on the outer boundaries of power gain region in specific direction \( e = \{e_1, e_2\} \) for \( e_k \in [+1, -1] \) and we can achieve the optimal beamforming vector such as

\[
w_{opt} = \nu_{\max}(Z), Z = e_1 \lambda h_{ss} h_{sp}^* + e_2 (1 - \lambda) h_{sp} h_{sp}^*, \quad \text{where} \quad 0 \leq \lambda \leq 1
\]

where \( \nu_{\max}(Z) \) denotes the principal eigenvector of \( Z \). The detail proof of *Theorem 1* can be found in [12]. Here, the power gain region related to P2 is

\[
\Omega = \left\{ \left( |w^* h_{ss}|^2, |w^* h_{sp}|^2 \right) \bigg| ||w||^2 \leq 1 \right\}. \quad (17)
\]

According to P2, the secondary transmitter with imperfect CSI is interested in minimizing its power gain at the primary receiver and also interested in maximizing the power gain on its receiver for satisfying the QoS. If we attempt to characterize the set of efficient transmission strategies for the secondary user, the outer boundary of \( \Omega \) in direction \( e = \{+1, -1\} \) is relevant to the optimal beamforming vector, \( w_{opt} \). The proof goes by contradiction.

Suppose that \( w_{opt} \) is not the boundary point of the set \( \Omega \) in direction \( e = \{+1, -1\} \). Then, there exists the beamforming vector, \( w_{bound} \) such that lies on outer boundary of \( \Omega \) in direction \( e = \{+1, -1\} \). Also \( w_{bound} \) satisfies one of the following properties by definition of the boundary region.

1. \( |w_{opt} h_{sp}| > |w_{bound} h_{sp}| \), \( w_{opt} h_{ss} = w_{bound} h_{ss} \)
2. \( w_{opt} h_{sp} = w_{bound} h_{sp}, w_{opt} h_{ss} < w_{bound} h_{ss} \)

For 1), \( w_{bound} \) decreases the interference power at the primary receiver and satisfies the minimum required rate of SU. Since the existence of such \( w_{bound} \) contradicts the optimality of \( w_{opt} \), the optimal beamforming vector must be the boundary point of \( \Omega \) in direction \( e = \{+1, -1\} \). However, for 2) \( w_{bound} \) increases the secondary rate while generating the same interference power at the primary receiver with that of \( w_{opt} \).

This implies that if \( w_{opt} \) satisfies all constraints in P2, \( w_{bound} \) also satisfies all constraints. Hence, both \( w_{opt} \) and \( w_{bound} \) can be the solution of P2 only in this case. This is because the SU’s QoS constraint does not mean that \( w_{opt} \) maximize the power gain on secondary receiver, so the other beamforming vectors in the power gain region can satisfy all constraints in P2. Until now, we proved that the optimal beamforming vector can be expressed in terms of the parametrization of the Pareto boundary of power gain region. The interrelationship between this parametrization techniques and the linear combination of \( w_{ZF} \) and \( w_{MRT} \) can be found in [11]. Finally, by using Corollary 2 in [11] the proof is concluded.

IV. SIMULATION RESULTS

In this section, the analytic results in the helper case with imperfect CSIT are investigated by the simulation. Then, we compare the primary secrecy rate by the proposed transmit beamforming using the imperfect CSI with that based on the perfect CSIT in spectrum leasing. In all simulations, the channel coefficients were assumed to be independent zero-mean Gaussian random variables with unit variance.

Fig. 2 and Fig. 3 show the guaranteed QoS level and the secrecy rate of the primary link, respectively, when the secondary transmitter acts as the helping interferer with imperfect CSI. In Fig. 2, we show \( \alpha \), the ratio of \( R_s(w_{ZF}) \) to \( R_{s, max} \) when \( N = 3 \) and 4. These results are helpful to know how many bits are guaranteed for the secondary link when the SU employs the ZF beamforming vector. From this, we can choose the appropriate secondary QoS level in practice. As shown in Fig. 2, the guaranteed QoS level of the SU approaches the theoretical value obtained by *Remark 1* at higher SNR. It is also observed that for more than two transmit antennas, the ergodic \( \alpha \) is higher than 0.8 at high SNR. In light of this, \( w_{ZF} \) can provide the substantial QoS level for the secondary link. Fig. 3 shows the ergodic secrecy rate of the primary link obtained by \( w_{ZF} \). Note that the difference of the primary secrecy rates with the helper using between the perfect CSI and the imperfect CSI is only about 1 bit per channel use. Therefore, we can find that the lack of CSIT can be compensated by \( w_{ZF} \) to a certain extent.

In Fig. 4, we compare the achievable primary secrecy rate by the proposed transmit beamforming for the SU with various scenarios; spectrum leasing with perfect CSI and only helper with perfect CSI. We use 0.25, 0.5 and 0.8
for the secondary QoS level. Fig. 4 shows that despite the imperfect CSIT assumption, the achievable rate obtained by the proposed transmit beamforming is comparable to that achieved by using the perfect CSIT. However, this difference decreases as the value $\alpha$ increases, i.e., the secondary QoS level is higher. That is because the higher $\alpha$ implies that the optimal transmission strategy approaches to MRT for $R_{s,\max}$, which makes the availability of the eavesdropper’s CSI at the secondary transmitter less influential. Therefore, we can have a benefit of the helping interferer while still serving the QoS for the SU by the proposed beamformer in spectrum leasing with imperfect CSIT.

V. CONCLUSION

We have studied the impact of imperfect CSIT in spectrum leasing via cooperation for physical-layer security in this paper. In particular, we proposed the optimal transmit beamforming composed of the known channels for the SU. By the proposed beamformer, we can improve the primary secrecy while still serving the secondary QoS with imperfect CSIT. Also by analyzing the ergodic rate of the secondary link in helper case with imperfect CSIT, we can be aided in deciding the viable secondary QoS level. As a future work, the spectrum leasing with the channel covariance feedback will be studied.

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