# A dynamic heuristic wavelength assignment algorithm for optical network with wavelength conversion 

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#### Abstract

The blocking performance of wavelength routing WDM optical networks can be enhanced by applying routing and wavelength assignment. In this paper, we consider wavelength assignment problem in the optical network. Specifically, we propose a dynamic heuristic wavelength assignment algorithm, called Longest Segment (LS) algorithm, for WDM networks. In comparison to other proposed algorithms, the blocking performance of LS algorithm is better. In addition, the LS algorithm minimizes the usage of converters by chaining the minimum number of continuous segments which have at least one same available wavelength. Furthermore, the low algorithm' complexity is an another advantage of the algorithm.


Keywords: RWA, WDM, lightpath

## 1. INTRODUCTION

The two most important problems of designing wavelength-routing network are Lightpath Topology Design (LTD) and Routing and Wavelength Assignment (RWA). LTD is the task of designing a lightpath topology interconnecting the IP routers and realizing this topology within the optical layer. RWA is the problem of realizing the lightpath topology within the optical layer. A good RWA algorithm is crucial to decrease the blocking probability of the WDM network. RWA is responsible for selecting a suitable route and wavelengths among the many possible choices for establishing the calls. There are two kinds of RWA problems: offline problem and online problem. Offline problem is the solution of RWA determining the specific set of wavelength on each link to realize the specific lightpath topology. On the other hand, online problem has to be solved for one lightpath connection at a time [1]. For simplicity, RWA is divided into routing sub-problem and wavelength assignment sub-problem. In this paper, we deal with the second sub-problem on the online model.

There have been a number of wavelength assignment algorithms proposed before: Random algorithm [5][6][8], Least Used (LU) algorithm [2], Most - Used (MU) [2] algorithm, Wavelength - Graph-based (WG) algorithm [7] and FirstFit (FF) algorithm [4]-[6]. Among them, LU algorithm gives the worst blocking performance. Random algorithm is better than LU algorithm. FF and MU has better blocking performance than the others. The blocking probability of MU is slightly better than that of FF. However, it requires global information and higher algorithm complexity [9].

In this paper, we propose a dynamic heuristic wavelength assignment algorithm, called Longest Segment (LS) algorithm. When a connection is requested, the algorithm select the combination of consecutive links which have the same available wavelengths. The mathematical analysis and simulation results demonstrate that: 1) LS algorithm achieves much better blocking performance than other algorithms, 2) the usage of wavelength converters is minimized, and 3) the algorithm complexity is $\mathrm{O}(w)$ where $w$ is the number of wavelengths per link. Here, we assume that the capacities of all links are the same. It should be noted that in most previous algorithms, the usage of converters has not been considered.

The paper is organized as follows: in section 2, we present the LS algorithm. Following is the mathematical analysis in section 3. Section 4 shows simulation results for blocking performance of the LS algorithm compared with the FF algorithm. Section 5 presents our conclusions. Finally, section 6 is an appendix.

## 2. LONGEST SEGMENT ALGORITHM

It is clear that the wavelength converters can significantly reduce the blocking probability of the networks. However, their price is still high. Therefore, the problem is how to minimize the number of wavelength converters used while reducing the blocking probability. Our algorithm solves these issues.

In wavelength assignment sub - problem, it is assumed that the route is already selected for a given source - destination pair. The remaining work is only how to assign wavelengths along that route.

Suppose that for a given connection, we have the route to connect the source node and the destination node as in Figure1.


Figure 1. A route for a given connection
In Figure1, $S$ and $D$ indicate the source node and destination node respectively, $v_{1}, v_{2}, \ldots, v_{N}$ are the intermediate nodes from the source to the destination nodes, $e_{1}, e_{2}, \ldots e_{N}$ are the links connecting $S$ and $v_{1}, v_{1}$ and $v_{2}, \ldots, v_{N}$ and $D$, respectively. Note that $v_{1}, v_{2, \ldots}, v_{N}$ can be the source and/or destination nodes of other connections, however, in this case we do not consider them as the either source or destination nodes. In addition, we assume that the capacities of all links are the same and let $w$ be the capacity of each link. Denote $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{w}$ are the $w$ wavelengths in each link.

For each connection on the route from the source node to the destination, we define a segment as follows:

1) the chain of the largest number of the consecutive links which have same the particular one available wavelength.
2) the starting node of the segment is the source node of that connection or a converter and the ending node of the segment is the destination node or another converter.
3) the direction of the segment advances toward destination node and
4) nodes in the segments must be nodes on the route from the source node to the destination node.

When each connection arrives, LS algorithm consequently finds all segments from the source node for each free wavelength and selects the one which has the longest link length until the destination is reached.
The following is the detail of the LS algorithm:
Step 0:

- $\mathrm{i}=0$;
- Starting node of segment $0=$ source node.

Step 1:

- From the starting node of segment $i$, find all candidate segments for segment $i$.
- Segment i is selected as the longest one among those candidate segments.

Step 2:

- If ( the ending node of segment $\mathrm{i}=$ destination node ) Then

Stop.

- Else
- Starting node of segment $(i+1)=$ the ending node of segment i.
- $\mathrm{i}=\mathrm{i}+1$.
- Go back to step 1 .


## 3. MATHEMATICAL ANALYSIS

In this section some mathematical results and their proof are demonstrated. Again, one should recall that, in wavelength assignment problem, the route for a given connection is assumed to be obtained by given routing algorithm.

With a chosen route for a source-destination pair, suppose that there exits at least one wavelength assignment approach to form a lightpath from the source node to the destination node. Then, the following theorems are proved by using contradiction method in mathematics under this assumption.

Theorem 1: The segment always exists.
Proof: Suppose we have a route for a connection as in Figure 2.


Figure 2. The route for the source-destination pair
Assume there is no segment. Let $v_{k}(1 \leq k \leq \mathrm{N})$ be the closest node to $S$, which has converter(s) or be the destination node. Because there is no segment, there does not exist the wavelength $\lambda_{\mathrm{i}}(1 \leq \mathrm{i} \leq w)$ which is available on all links from the source node to node $v_{k}$. As a result, there is no way to go from the source node to node $v_{k}$. Because of this, it is obvious that the destination node cannot be reached. This assessment contradicts with the above assumption that there is at least one wavelength assignment approach to go from the source node to the destination node.
Hence, the first theorem is proved.
Theorem 2: The solution for LS algorithm is always found.
Proof: Assume that the solution for LS algorithm does not exist. That means from the ending node of a given segment, we cannot go to the ending node of another segment in the direction toward the destination node.
Let the ending node of that given segment be $v_{j}$ and $v_{t}(j<t \leq N)$ be the closest node to $v_{j}$ which has converter. If that node does not exist, so $v_{t}$ is the destination node. We will show that the consecutive links starting from link $e_{j+1}$ (starting at node $v_{j}$ ) to link $e_{t}$ (ending at node $v_{t}$ ) form a segment.


Figure 3. Consecutive links from node $v_{j}$ to $v_{t}$
Because there is at least one way from the source to the destination, the link $e_{j+1}$ (between nodes $v_{j}$ and $v_{j+1}$ ) must have one or more available wavelengths. Then, the link $e_{j+2}$ must have the same free available wavelength(s) with link $e_{j+1}$. If not, we cannot go from $v_{j}$ to $v_{j+2}$ and, hence, we cannot go to the destination node. This conflicts with the assumption we have made. Similarly, the link $e_{j+3}$ must have the same available wavelength with link $e_{j}$ (in case $j+3 \leq \mathrm{t}$ ) until links $e_{t}$ is reached.

However, according to above assessment that we cannot go to the ending node of another segment in the direction toward the destination node, then from $e_{j}$ we cannot go to $e_{t}$ because $e_{t}$ is the ending node of the segment from $e_{j}$ to $e_{t}$. This is contradiction. Therefore, the assumption that the solution for the LS algorithm does not exist is incorrect. Hence, the first theorem is proved.

Theorem 3: The number of converters used is minimized.
Proof: Suppose another wavelength assignment algorithm employs less number of converters.

Suppose that the number of converters used in our algorithm is $n$, and the nodes which have converters to be used are $v_{i_{i}}, v_{i_{i}}, \ldots, v_{i_{i}}$. Also, let the number of converters used in another wavelength assignment algorithm which employs less the number of converters be $m$, at the locations $v_{j_{1}}, v_{j_{2}}, \ldots, v_{j_{m}}$. Obviously $n>m$. Note that the consecutive links from source to $v_{i_{1}}$, from $v_{i_{1}}$ to $v_{i_{2}}, \ldots$, and from $v_{i_{n}}$ to the destination are segments defined as before.


Figure 3. Locations of nodes which have converters used (a) in our algorithm and (b) in another algorithm.
In the route of LS algorithm, let:
$a_{1}$ (length unit) be the length from the source node to $v_{i_{1}}$.
$a_{2}$ (length unit) be the length from $v_{i_{1}}$ to $v_{i_{2}}$
$a_{n}$ (length unit) be the length from $v_{i_{n}}$ to the destination node.

In the route of another algorithm, let:
$b_{1}$ (length unit) be the length from the source node to $v_{j_{1}}$
$b_{2}$ (length unit) be the length from $v_{j_{1}}$ to $v_{j_{2}}$
$b_{m}$ (length unit) be the length from $v_{j_{m}}$ to the destination node.
Because the lengths from the source node to the destination node in two algorithms are the same, we have:

$$
\begin{equation*}
a_{1}+a_{2}+\ldots+a_{n}=b_{1}+b_{2}+\ldots+b_{m} \tag{1}
\end{equation*}
$$

Due to the characteristics of selecting longest segment, the segment from source to node $v_{i_{1}}$ must be longer than the one from the source node to node $v_{j_{1}}$. We have: $a_{1} \geq b_{1}$
If $\mathrm{a}_{2} \geq \mathrm{b}_{2}$, obviously we have: $a_{1}+a_{2} \geq b_{1}+b_{2}$
Let $A$ is the length between node $v_{j_{2}}$ and $v_{i_{2}}$. In case $a_{2} \leq b_{2}$ we will show that $i_{2} \geq j_{2}$, then we still have $a_{1}+a_{2} \geq b_{1}+b_{2}$ because $a_{1}+a_{2}=b_{1}+b_{2}+A$.

Indeed, if $i_{2} \leq j_{2}$, then the length from $i_{1}$ to $j_{2}$ must be longer than that one form $i_{1}$ to $i_{2}$ because it includes the length form $i_{1}$ to $i_{2}$. Therefore, the segment starting from $i_{1}$ and ending at $i_{2}$ is shorter than the segment starting from $i_{1}$ and ending at $j_{2}$. This contradicts with the selection of LS algorithm. So, we have $i_{2} \geq j_{2}$. That means we always have $a_{1}+a_{2} \geq b_{1}+b_{2}$.

Similarly: $a_{1}+a_{2}+a_{3} \geq b_{1}+b_{2}+b_{3}$ and so on...
Finally:

$$
\begin{equation*}
a_{1}+a_{2}+\ldots+a_{m} \geq b_{1}+b_{2}+\ldots+b_{m} \tag{2}
\end{equation*}
$$

On the other hand, due to $a_{1}+a_{2}+\ldots+a_{m}+\ldots+a_{n}>a_{1}+a_{2}+\ldots+a_{m}$, hence, we have:

$$
\begin{equation*}
a_{1}+a_{2}+\ldots+a_{m}+\ldots+a_{n}>a_{1}+a_{2}+\ldots+a_{m} \geq b_{1}+b_{2}+\ldots+b_{m} \tag{3}
\end{equation*}
$$

We see that (3) conflicts with (1). This means the assumption that another wavelength assignment algorithm employs less number of converters than the LS algorithm is not correct. Therefore, the usage of converters in our algorithm is optimized.

Theorem 4: The algorithm complexity is approximate $O(w)$, where $w$ is the capacity of each link.
Proof: Algorithm complexity is the number of steps to complete that algorithm.
First, we point out that the number of steps to complete LS algorithm is no longer than $O(w k N)$, where $w$ is the capacity of each link, $k$ is the number of nodes which have wavelength converters on the path from the source node to the destination node and $N$ in the number of immediate nodes between the source node and the destination node. This is made clear in Appendix section.

Because converter is one of network resources, hence, the number of nodes which have converter(s) in the network must be limited. As a results, the number of nodes having converter on the path for given connection must be limited. That means $k$ has upper bound. Furthermore, the number of nodes in the network also must be limited. Therefore, $N$ also has upper bound. Then $O(w k N)$ becomes $O(w)$. In addition, from simulation, the time to run LS algorithm is approximate with time of FF algorithm, which is $O(w)$. We can infer that, the complexity of LS algorithm is approximate $O(w)$ if above assumption is satisfied.

Theorem 5: LS algorithm requires only local link state information.
Proof: The LS algorithm only requires information on the route from the source node to the destination node for each given connection. Hence, it is obviously that it only needs local link state information. For achieving the local link state with the least overhead incurred, the solution has been recently proposed [11].

## 4. SIMULATION RESULTS

This work is implemented by using C programming language in the National Foundation Networks (NFSNET) [10]. The network includes 14 nodes and 21 links as shown in figure 4 . Each link is assumed to carry 16 wavelengths (one of the channel spacing standards specified by ITUT-G.962). We use share-per-node converter architecture [3] and full conversion configuration [10]. Among 14 nodes, only 6 nodes each has 1 converter, because when the number of nodes having converters increases, the benefits of wavelength converters tend to be saturated. These 6 nodes are: $1,5,6,8$, 11 , and 13 because they have more traffic than other nodes.

In this work, traffic model is assumed to follow Poisson process and holding time is assumed to follow exponential distribution with unit mean. Furthermore, we follow a uniform distribution in choosing source nodes and destination nodes. The traffic characteristic is unidirectional. The traffic from node A to node B is not the same as that one from node B to node A . In addition, we do not consider the delays of propagation and processing while implementing simulation. For simplicity, we use fixed routing algorithm. There exits only one fixed path for each possible connection. Major of the routes are Shortest-Path routes while other are not. The reason is that we want to enhance the use of converter by placing some nodes having converter in those routes.

Because among wavelength assignment algorithms, FF has almost the best blocking performance, hence, we only need to compare the blocking probability of FF algorithm with that of our algorithm. The following figure shows the blocking probability of the LS algorithm compared with the FF algorithm


Fig 4. The NSFNET with 14 nodes and 21 links


Fig 5. Comparison of blocking probability between FF algorithm and LS algorithm

## 5. CONCLUSIONS

In this paper, a dynamic wavelength assignment algorithm for all-optical WDM networks called Longest Segment (LS) algorithm is proposed. By connecting a minimum number of consecutive segments, LS algorithm establishes an optical path with the least wavelength conversions, leaving more converters available for future requests. Simulation result shows that blocking performance in our algorithm is much better than FF algorithm. In addition, the LS algorithm employs the minimum usage of converters, which no algorithms achieved previously. The low complexity is also one advantage of our algorithm. In conclusion, the LS algorithm provides much better blocking probability and uses much less wavelength conversion while the computational complexity is similar when compared with FF algorithm.

## 6. APENDIX

This section proves that the number of step to complete LS algorithm is $O(w k N)$

Consider the link $e_{1}$, the number of available wavelengths is no larger than $w$. In the worst case, the number of free wavelengths is $w$.
Assume that with $\lambda_{l}$, we have segment number 1 ending at node $v_{i_{1}}$.
With $\lambda_{2}$, we have segment number 2 ending at node $v_{i_{2}}$.

With $\lambda_{w}$, we have segment number $w$ ending at node $v_{i_{w}}$.
$v_{i_{1}}, v_{i_{2}}, \ldots, v_{i_{w}}$ may not exist if the respective wavelength is not available.
Let $x_{1}=\max \left\{i_{1}, i_{2}, \ldots, i_{w}\right\}$ then the selected segment starting form the source node will end at node $v_{x_{1}}$ and the number of steps for selecting the first segment is no larger than: $w x_{1}$.

Consider the second segment, starting from node $v_{x_{1}}$
Assume that with $\lambda_{l}$, we have segment ending at node $v_{x_{1}+j_{1}}$.
With $\lambda_{2}$, we have segment number 2 ending at node $v_{x_{1}}+j_{2}$.

With $\lambda_{w}$, we have segment number $w$ ending at node $v_{x_{1}+j_{w}}$.
$v_{x_{1}+j_{1}}, v_{x_{1}+j_{2}}, \ldots, v_{x_{1}+j_{w}}$ may not exist if the respective wavelength is not available.
Let $x_{2}=\max \left\{x_{1}+j_{1}, x_{1}+j_{2}, \ldots, x_{1}+j_{w}\right\}$ then the second selected segment starting form node $v_{x_{1}}$ will end at node $v_{x_{2}}$ and the number of steps for selecting the second segment is no larger than $w x_{2}$
Because there are $N$ intermediate nodes from source node to the destination node, we have: $x_{1}+x_{2} \leq N$
Consider the third segment
Assume that with $\lambda_{1}$, we have segment ending at node $v_{x_{2}+t_{1}}$.
With $\lambda_{2}$, we have segment 2 ending at node $v_{x_{2}+t_{2}}$.

With $\lambda_{w}$, we have segment $w$ ending at node $v_{x_{2}}+t_{w}$.
$v_{x_{2}+t_{1}}, v_{x_{2}+t_{2}}, \ldots, v_{x_{2}+t_{w}}$ may not exist if the respective wavelength is not available.
Let $x_{3}=\max \left\{x_{2}+t_{1}, x_{2}+t_{2}, \ldots, x_{2}+t_{w}\right\}$ then the third selected segment starting form node $v_{x_{2}}$ will end at node
$v_{x_{3}}$ and the number of steps for selecting the second segment is no larger than $w \cdot x_{3}$
Similarly, we have: $x_{1}+x_{2}+x_{3} \leq N$.
Suppose we have $k$ segments, then the number of steps is less than or equal to: $w\left(x_{1}+x_{2}+\ldots+x_{3}\right)$
Let: $S=x_{1}+x_{2}+\ldots+x_{k}$. We have
$x_{1} \leq N$
$x_{2} \leq N-x_{1}$
$x_{3} \leq N-x_{1}-x_{2}$
$\cdots$
$x_{k} \leq N-x_{1}-x_{2}-\ldots-x_{k-1}$
$\Rightarrow S=x_{1}+x_{2}+\ldots+x_{k} \leq(k-1) N-\left[\left(S-x_{k}\right)+\left(S-x_{k}-x_{k-1}\right)+\ldots+\left(S-x_{k}-x_{k-1}-\ldots-x_{2}\right)\right]$
$\Rightarrow S \leq(k-1) N-(k-1) S+\left[x_{2}+2 x_{3}+\ldots+(k-1) x_{k}\right]$ or
$\Rightarrow k S \leq(k-1) N+\left[x_{2}+2 x_{3}+\ldots+(k-1) x_{k}\right]$

Because $x_{i} \leq N$ with $1 \leq j \leq k$ we have:
$k S<(k-1) N+[1+2+\ldots+(k-1)] n$
$\Leftrightarrow k S<(k-1) N+\frac{k(k-1)}{2} N$
$\Leftrightarrow k S<N(k-1)\left(1+\frac{k}{2}\right)$
$\Leftrightarrow S<N \frac{(k-1)}{k}\left(1+\frac{k}{2}\right)$
$\Rightarrow S<N\left(1+\frac{k}{2}\right)$
$\Leftrightarrow w S<w N\left(1+\frac{k}{2}\right)$
As a result, the number of steps to complete the wavelength assignment in LS algorithm is no larger than $w \cdot S$.
The number of steps $\leq \mathrm{O}\left(w N\left(1+\frac{k}{2}\right)\right)=\mathrm{O}(w N k)$

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