BENDING HISTORY IN FINITE ELEMENT INVERSE ANALYSIS OF DEEP DRAWING PROCESSES

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ABSTRACT: This paper introduces a new approach to take account of bending history in finite element inverse analysis of a sheet metal forming process. A modified membrane element was adopted for finite element inverse analysis so that bending–unbending energy was additionally imposed on the total plastic energy. Bending–unbending regions were predicted from the geometry of the final shape and tools. The proposed method was applied to deep drawing processes of a cylindrical cup. The blank shape and the distribution of the thickness strain were compared with those obtained from the incremental finite element analysis to evaluate the effect of the bending history. The proposed method reduced the difference between the results of the inverse analysis and those of the incremental analysis. The analysis was also carried out changing the thickness of the initial blank to investigate the effect of bending deformation. The results showed that the difference was remarkably reduced as the thickness of the initial blank increased. This indicates that the suggested method is useful to obtain more accurate results especially when bending effects are significant.

KEYWORDS: Finite Element Inverse Analysis, Bending Effect, Deep Drawing Process, Modified Membrane Element.

1 INTRODUCTION

Deep drawing processes are widely used in automotive industries. There are several process parameters, such as the shape of the tools, which affect the formability of deep drawn products. However, process parameters are so strongly coupled that the design of a deep drawing process requires repetitive numerical analyses with tremendous computing time. To reduce computation time and cost, an efficient method is needed in the preliminary design stage. Finite element inverse analysis was developed by Chung and Richmond [1,2], Guo et al. [3,4], Lee and Huh [5], and Kim and Huh [6,7] to reduce the cost and time of the analysis of a deep drawing process and calculate an initial blank shape within a small amount of time in the preliminary design stage. In inverse analysis, initial shapes and final thickness or strain distributions are calculated by comparing the desired final shape and initial shape based on the deformation theory. Many researches on inverse analysis only considered membrane deformation since it is dominant in many sheet metal forming processes. However, bending effects are significant in deep drawing processes and the error may increase when disregarding bending effects [5,6,8]. Bending effects have been considered in inverse analysis by Guo and Batoz [3], and Lee and Cao [9] using shell elements, but these methods could not properly consider the bending history. A blank undergoes bending and unbending during the deep drawing process as it passes over the die radius. However, the bending–unbending effects are neglected in conventional inverse analysis because there is no information on the deformation path. This paper introduces a new approach to consider the bending history in the inverse analysis of a deep drawing process. Inverse analyses of a cylindrical deep drawing process verified the validity of the proposed method.

2 METHODS

2.1 FORMULATION OF FINITE ELEMENT INVERSE ANALYSIS

Formulation of finite element inverse analysis has been developed for 4-node quadrilateral elements. The membrane deformation energy was obtained using the method suggested by Lee and Huh [5]. The deformation tensor was calculated from the initial and final coordinates. The logarithm strain tensor was obtained after calculating the Green strain tensor.
tensor and the principal stretch \( \lambda_1, \lambda_2 \) by solving the eigenvalue problem. The constitutive equation for a normal anisotropy material was expressed as Eq. (1) based on the Hencky’s deformation theory. The stress tensor was calculated using Eq. (1) and the effective strain–stress curve expressed in Eq. (2) equation. The membrane deformation energy was obtained from Eq. (3).

\[
\sigma = K (\varepsilon_n + \varepsilon)^T
\]

\[
\delta W^m = \int_a \sigma : \varepsilon \, d\Omega
\]

A modified membrane element proposed by Huh et al. [10] was used to take the plastic bending energy into account. The curvature \( \kappa \) was assumed as the difference of the kink angle between the two neighboring elements (Eq. (4)).

\[
\kappa = \frac{1}{L} \left| w_i' - w_i \right|
\]

where \( L \) is the distance between the centers of two neighboring elements and \( w_i', w_i \) are derivatives of normal displacements in the direction normal to the boundary. The plastic bending energy was expressed as follows [10]:

\[
\delta W^p = \int_a M_1 \delta \kappa_1 \, d\Gamma + \int_a M_2 \delta \kappa_2 \, d\Gamma
\]

where \( M_1 \) and \( M_2 \) are the bending moments corresponding to \( \delta \kappa_1 \) and \( \delta \kappa_2 \) respectively, which were defined in Eq. (4).

Finite element discretization was carried out for quadrilateral elements, and bending energy was derived as Eq. (6):

\[
W^p(X) = \sum_{i=1}^w \int_{e_i} CB_i B_i U \, d\Gamma
\]

where \( B_i \) is expressed as Eq.(7), \( n_i^p \) is the unit normal vector of the \( i^{th} \) element, and \( n_i^p \) is the unit tangential vector of the \( i^{th} \) element. \( H \) is the matrix of the shape functions. \( C \) is the bending stiffness coefficient and is expressed as Eq. (8).

\[
B_i = \frac{1}{L_i} \left( n_i^p \frac{\partial H}{\partial n_i^p} - n_i^{p2} \frac{\partial H}{\partial n_i^{p2}} \right)
\]

\[
C = \frac{r^3 \sigma (1 + r)(2 + r)}{18 \Delta \varepsilon (1 + 2r)}
\]

Total plastic deformation energy became:

\[
W_p = W_p^m + W_p^p
\]

where \( W_p^m \) is the membrane deformation energy, and \( W_p^p \) is the bending deformation energy.

The principle of virtual work can be expressed as the minimum of the plastic potential energy [11]. The plastic potential energy was defined as the difference between the plastic deformation energy and equivalent external work [5] as follows: where, \( W_p \) is the total plastic deformation energy, \( W_f \) is the work done by the blank holding force, and \( W_s \) is the work done by the friction force.

\[
\min \psi(X) = W_p(X) - W_f(x) - W_s(x)
\]

The equivalent external work done by the friction force and the BHF was calculated as Lee and Huh proposed [5]. The initial coordinates which minimize the plastic potential energy were obtained from the Newton–Raphson method.

\[
\frac{\partial R(X)}{\partial X} \delta X = -[R(X)]_{\alpha}(\delta X)
\]

The Convergence condition as in Eq. (13) was used.

\[
\sum_{i=1}^{\delta X} \delta X_i \leq \delta_i
\]

2.2 CONSIDERATION OF THE BENDING HISTORY IN FINITE ELEMENT INVERSE ANALYSIS

A blank undergoes bending and unbending during a deep drawing process as it passes over the die radius. A new approach was introduced to take the bending history into consideration in inverse analysis. Bending–unbending history was considered using the following assumption of the bending deformation path during a deep drawing procedure (Figure 1):

1. Elements which are located on the die or punch radii at the final state do not undergo bending–unbending.

2. Bending deformation is negligible for elements located on the bottom of the cup or flange region.
3. Elements located on the wall at the final state undergo bending–unbending. The blank is assumed to contact fully with the die as it passed over the die radius. The bending energy corresponding to the bending deformation of each element was calculated from Eq. (5). The curvature and kink angle of wall-elements was obtained from the assumption of full contact between the blank and the die. The unbending energy of wall-elements was assumed to be the same as the bending energy.

2.3 NUMERICAL ANALYSIS OF A CYLINDRICAL CUP DRAWING PROCESS

The presented algorithm was implemented in a finite element code and applied to a cylindrical cup drawing process. The tool geometry for deep drawing of a cylindrical cup is shown in Figure 2.

The finite element model of the cylindrical cup consisted of 288 elements and 319 nodes (Figure 3).

A quarter model was used considering the symmetry of the model. The material properties and analysis conditions are shown in Table 1. Final thickness distributions were obtained with and without considering the bending history. The thickness and strain distribution along the line marked on Figure 3 were compared with the result obtained from the incremental finite element analysis with 3325 elements and 3424 nodes. The inverse analyses were carried out with the variation of the thickness of the initial blank from 0.5 to 1.5 to investigate the effect of bending deformation.

3 RESULTS

Final thickness distribution obtained from the finite element inverse analysis is shown in Figure 4. The thickness of the bottom of the cup calculated from the inverse analysis with considering bending history was thinner than that from the inverse analysis without considering bending history.

Table 1: Material properties and analysis condition

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lankford value</td>
<td>r = 1.82</td>
</tr>
<tr>
<td>Initial thickness</td>
<td>t_0 = 1.0mm</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>μ = 0.1</td>
</tr>
<tr>
<td>blank holding force</td>
<td>F_b = 40kN</td>
</tr>
</tbody>
</table>

Figure 1: Assumption of bending deformation path during a deep drawing process

Figure 2: Geometric description of tooling for deep drawing of a cylindrical cup

Figure 3: Finite element mesh geometry of a cylindrical cup: cup height=40mm (288 elements and 319 nodes)

Figure 4: Thickness distribution of the blank at the final state in a cylindrical cup drawing; (a) without considering bending history, (b) considering bending history
The results from the inverse analysis were compared with the result from the incremental analysis in Figure 5. The difference between the result from the inverse analysis and that from the incremental analysis was reduced when bending history was considered. The final thickness was calculated with the variation of the initial thickness from 0.5mm to 1.5mm by 0.5mm. The results are shown in Figure 6. More accurate final thickness distribution was obtained when the bending history was considered. The difference between the final thickness distribution obtained from the inverse analysis and that from the incremental analysis increases as the initial thickness becomes thicker. The thickness distribution obtained from the inverse analysis with considering the bending history was more accurate than that from the inverse analysis without considering the bending history.

4 DISCUSSION

Only membrane deformation has been considered in many previous researches on finite element inverse analyses, since bending deformation is negligible in most sheet metal forming processes [5,6,8]. However, bending deformation becomes significant during deep drawing processes. There have been several studies that consider bending deformation during deep drawing processes using shell elements [3,4,9,12], but the previous researches did not consider the bending history. Elements located on the wall at the final state undergo bending and unbending during a deep drawing process. The bending deformation of the elements which do not have the curvature at the final stage was not considered in previous studies [3,4,9,12]. The proposed method can consider the bending history in inverse analysis by assuming the deformation path and adding the bending–unbending energy corresponding to the bending deformation. The total plastic deformation energy is assumed to be the sum of the plastic membrane deformation energy and the plastic bending energy, since membrane and bending deformation are not strongly coupled [10]. Observation of the deformation shape during the deep drawing process obtained from the incremental analysis verifies the assumption of the deformation path which was stated in Chapter 2.2. The unbending energy of the elements located on
the wall region at the final state was assumed to be the same as the bending energy because it is impossible to know the deformation path from the inverse analysis. The final thickness of the bottom of the cup and the punch radius became thinner and closer to that from the incremental analysis when the bending history was considered. The bending energy, corresponding to the amount of the bending– unbending energy of the wall region, was added to the total deformation energy to consider the bending history. This means that elements on the wall region became stiffer, and it makes the elements on the wall region difficult to flow during the deep drawing process. Therefore, the thickness of the bottom of the cup and the punch radius became thinner and that of the wall region became thicker when the bending history was considered. The thickness of the elements located on the punch radius is much smaller in case of the incremental analysis than in case of the inverse analysis. Since inverse analysis simplifies the deformation path and contact condition, errors should exist between the results from inverse analysis and incremental analysis. The other reason of discrepancy between inverse analysis and incremental analysis is due to the different finite element mesh system. The error reduced when bending history was considered. The bending–unbending effect becomes significant as the initial thickness increases. The amount of added bending energy increases as the initial blank thickness increases. Thus, the difference between the thickness distributions obtained from the inverse analysis with and without considering the bending history increases as the initial blank thickness increases.

5 CONCLUSION

A new method was introduced to take account of the bending history during the finite element inverse analysis of the deep drawing process. A modified membrane element was adopted, and bending–unbending regions were predicted from the geometry of the final shape. The bending energy, corresponding to the amount of bending–unbending energy of the wall region, was added to the total deformation energy. The proposed method was applied to the inverse analysis of cylindrical cup drawing processes, and the final thickness and thickness strain from the inverse analysis were compared to those from the incremental analysis. The algorithm increased the accuracy of the inverse analysis due to the consideration of the bending history. The accuracy was remarkably improved as the thickness of the initial blank increased. Finite element inverse analysis with the method suggested is useful to obtain more accurate thickness distribution especially when bending effects are significant.

6 REFERENCES