Analyzing the Critical Path for the Well-Formed Workflow Schema^{*}

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Abstract

Though the concept of the critical path in the workflow is important because it can be utilized in many issues in workflow systems, the critical path in the context of the workflow has not been much addressed in the past. In this paper we propose a method to systematically determine the critical path in a workflow schema built by the workflow control constructs described in our workflow model.

1. Introduction

Nowadays, the concept of a workflow attracts a great interest in an objective of automating and computerizing business processes in whole or part. A business process is abstracted to a workflow which is composed of “a collection of activities” interconnected by various workflow control flows [1]. The critical path in a workflow schema is defined as “the longest execution path in a workflow schema” which directly affects the total workflow completion time. The activities in the critical path are called **critical activities**. We expect that the concept of the critical path and the critical activity can be effectively utilized in many workflow issues, for example, workflow resource/time management.

In Section 2, we describe the workflow control constructs and model considered in the paper. Section 3 presents our proposed method to determine the critical path in a workflow. Finally, we summary the paper in Section 4.

2. Workflow Control Constructs and Model

A workflow schema, which is a description of a workflow, is a network of nodes and directed edges. It begins from the start node and ends with the final node. Each node denotes an activity, and each directed edge denotes a transition between two nodes, i.e., a branch of execution. Figure

1 shows graphical expressions of various workflow control constructs with which we can fairly describe current complicated business logic. Note that for a split control construct α , there is a set of join control construct C_α that can be combined with α whose resulting workflow schema can produce a semantically meaningful flow of control. For example, $C_{AND-split} = \{AND-join, Selective-AND-join, Preference-OR-join\}$.

Definition 1 Any subnetwork in a workflow schema that satisfies one of the following conditions is called a **non-sequential control block**(, simply **control block**). 1) A subnetwork that starts from a split control construct and ends with a matching join control construct. 2) A subnetwork that consists of all the nodes and edges between the source and the destination of the LOOP.

Definition 2 Non-sequential control blocks and their nested control blocks are called **well-formed control blocks**. A workflow schema composed of well-formed control blocks and simple nodes is called a **well-formed workflow schema**.

When we assume that workflow requests arrive by a Poisson process and the service time in each activity is exponential, a well-formed workflow schema can be modeled

Figure 1. Workflow Control Constructs

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Figure 2. Workflow Queuing Network

as a M/M/1 queuing network such as telephone networks or computer communication networks, which allows us to analyze some important workflow properties. In a M/M/1 workflow queuing network, each activity is an independent M/M/1 queuing system [2]. We can, therefore, specify the arrival and departure rate in each activity as in Figure 2 from the information of the initial request rate, the service rate in each activity, and the branch probabilities in OR-split, Preference-OR-split, and LOOP control constructs.

1. All the simple nodes belong to the critical path.
2. For (each top-level control block) {
 3. WHILE (the top-level control block is a nested control block) {
 4. IF (its innermost control block is a LOOP atomic control block)
 - 4.1 Transform the LOOP into a sequence control construct.
 5. ELSE IF (its innermost control block is an atomic control block consisting of a split and a join control construct)
 - 5.1 Select a path with the longest average execution time in the block.
 - 5.2 Transform the path into an activity.
 - /* Here, the top-level control block is an atomic control block */
 6. Determine the longest execution path in the control block which is the sub-critical path of the top-level control block.
7. Combine all the sub-critical paths with simple nodes. The resulting path becomes the critical path of a workflow schema.

Figure 3. CPWS Algorithm

3. Finding the Critical Path

The critical path in a workflow schema is the longest execution path from the start node to the final node. From the aforementioned workflow queuing network, we propose a method called the Critical Path in a Workflow Schema (CPWS) that determines the critical path in a workflow schema. As many workflow instances for the same workflow schema can be concurrently executed, some workflow instances should wait at the queue of an activity during other workflow instances that arrives ahead are served. This means that the *average execution time* of a workflow instance in an activity is the *average service time* in the activity plus the *average waiting time* at the queue of the activity.

It is clear that since a workflow schema is a sequence of simple nodes and/or top-level control blocks, all the simple nodes and the longest execution path in a top-level control block are to be part of the critical path of a workflow schema. Therefore, CPWS basically finds out a sub-critical path with the longest average execution time in each top-level control block and then combines these sub-critical

Figure 4. An Overall Example of the CPWS

paths to form the critical path as explained in Figure 3. Finding a subcritical path in each control block, which corresponds to the step 4, step 5, and step 6 in the CPWS algorithm, can be easily performed using the queuing theory.

Figure 4 show the overall example of our CPWS method. Let the service rate to the workflow schema be 10 and the branch probabilities p_1 , p_2 , and p_3 be 0.4, 0.5, and 0.6, respectively. Thus, we can specify the arrival and departure rate at each activity as in Figure 4-(a), which is based on Section 2. And, let the service rate μ_i of activity i be $[\mu_1, \mu_2, \dots, \mu_{15}] = [13, 15, 7, 8, 20, 18, 20, 14, 16, 20, 10, 15, 15, 6, 7, 25]$. As a result, we can determine the critical path, $\{1, 2, 4, 5, 8, 9, 14, 15, 16\}$, in the workflow schema by combining these sub-critical paths.

4. Conclusion

We understand that the concept of the critical path may be very useful because it can be utilized in many workflow issues, for example, workflow resource management, workflow time management, etc. In this paper we first specify various workflow control constructs which can be enough to support the context of current complex business processes. And then, we propose a method to determine the critical path in a well-formed workflow schema.

References

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