Upper-Bounds on Bit Error Rate of OCDMA Systems Using the Time Spreading/Wavelength Hopping Codes

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Rigorous upper-bounds on the bit error probabilities for optical CDMA systems with and without optical hard-limiter utilizing hybrid time-spreading/wavelength-hopping codes are analytically derived. The maximum auto-correlation and cross-correlation values of the codes are 1 and 2 respectively.

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Up to now various code families for an optical code division multiple access (OCDMA) system have been suggested from one-dimensional (1-D) codes [1] encoded in time-domain to the two-dimensional (2-D) codes with every pulses of a codeword encoded in time and wavelength [2] or time and space [3] domain. The equation to calculate bit error rate (BER) for the system using time spreading/wavelength hopping (TS-WH) 2-D codes had not been derived. This paper analyzes the TS-WH 2-D code system, suggesting the expression for estimating the upper bound of BER with is newly derived here. The system with an optical hard limiter is also analyzed. In OCDMA system with 2-D codes, we can also use optical hard-limiter (OHL) as well as the system with 1-D codes. The hard limiter is an effective component to mitigate the channel interference due to many other simultaneous users so that the bit error probability is improved and the number of simultaneous users is increased [1,4]. The analysis focuses on the exact influences due to multiple users' interferences because it can be assumed that thermal and shot noise in photo-detection process of the OCDMA system is negligible [4]. The result is applied to OCDMA system using Eqc-Prime code, which had not been analyzed the performance and the performance improvement by using OHL is proven.

Signature codes used in analysis are binary (0, 1) sequence with length *F* and each of Hamming weight *K*. Each pulse ("1" value) in the binary sequence has a wavelength among the available n_λ wavelengths. We assumed that for any codeword, autocorrelation side-lobe is no greater than one and for any two distinct code-words, the cross-correlation is no greater than two, because most of previously proposed codes [1~3] have the maximum autocorrelation value of one or the maximum cross-correlation of two. Chip synchronization and Of-Off Keying signaling are also assumed for calculating an upper-bound on BER [1]

The probability that one of the pulses (i.e. binary one) in a signature sequence lines up or hits a pulse in another sequence is K^2/F and the probability that two pulses of different sequences have the dame wavelength is $1/n_\lambda$. And interference is only occurred when both time slots and wavelengths in two sequences coincide respectively. Consequently, the probability that one chip of interference occurs is given by $P_1 = K2/Fn_\lambda$, which includes the probability, P_2, to hit two chips at the same time. The probability to have no interference is therefore given by $P_0=1-P_1-P_2$. We assume that data (0, 1) have equal probability 1/2. The probability to have the interference with magnitude I'_{CS} between two sequences is then given by sum of P_0, P_1, and P_2 as following:

$$P_{I'_{CS}}(I'_{CS}) = \left\{1-\frac{K^2}{2Fn_\lambda}\right\}\delta(I'_{CS}) + \left\{\frac{K^2}{2Fn_\lambda}-\frac{K^2(K-1)^2}{8F^2n_\lambda^2}\right\}\delta(I'_{CS}-1) + \left\{\frac{K^2(K-1)^2}{8F^2n_\lambda^2}\right\}\delta(I'_{CS}-2).$$

When there are *N* users transmitting simultaneously, the total interference I_1 at first user's receiver is the superposition of the interferences on the first user created by the remaining *N-1* users. Therefore, this process has the property of a multinomial probability. The expression for the ways reproducing total interference $(I_1=n)$ is $\sum_{i=0}^{\lceil n/2 \rceil} {}_{N-1}C_{n-i} \times {}_{n-i}C_{n-2i}$, where $\lceil x \rceil$ denotes the maximum integer value no more than x. Bit error occurs only when total interference is greater than the threshold of the receiver. The upper-bound on BER is then given by

$$\therefore BER \le \frac{1}{2}\sum_{n=Th}^{(N-1)}\sum_{i=0}^{\lceil n/2 \rceil}\frac{(N-1)!}{(n-2i)!i!(N-1-n+i)!}P_2^iP_1^{n-2i}P_0^{N-1-n+i} + \frac{1}{2}\sum_{n=N}^{2(N-1)}\sum_{i=(n-N+1)}^{n/2}\frac{(N-1)!}{(n-2i)!i!(N-1-n+i)!}P_2^iP_1^{n-2i}P_0^{N-1-n+i}$$

where $P_1 = \frac{K^2}{2Fn_\lambda} - \frac{K^2(K-1)^2}{8F^2n_\lambda^2}$, $P_2 = \frac{K^2(K-1)^2}{8F^2n_\lambda^2}$, $P_0 = 1-\frac{K^2}{2Fn_\lambda}$.

Let consider the OCDMA system with OHL positioned at the stage before the detector. In this case, bit error occurs only when interferences are on more chips than *Th*. The formula for the ways reproducing a pattern with pulses from *K* to the threshold level *Th* is equal to ${}_KC_{Th}$. Let *p* denote the probability that two chips in different code sequences coincide in both time slot and wavelength respectively, The value p becomes $K/2Fn_{\lambda}$. Let q be defined by $1-p$. Then, the probability that magnitude of interference generated by $N-1$ other users on any chip has a value more than one is given by $1-q^{N-1}$ [1]. Let assume the worst case that two chips of interference between two users always occurs. Since the probability of interference on any chip is given by $1-q$, the upper-bound on BER for TS-WH 2-D OCDMA with OHL is therefore derived as

$$BER \le \frac{1}{2}\frac{K!}{(K-Th)!Th!}\prod_{m=0}^{Th-1}(1-q^{(N-1-m/2)}) \quad \text{where} \quad q \equiv 1-\frac{K}{2Fn_{\lambda}}.$$

An exact upper-bound on the BER for the OCDMA systems using hybrid time-wavelength spreading codes in which maximum auto-correlation value has up to one and maximum cross-correlation is up to two was rigorously derived. These results were applied to the Eqc-Prime code [3], for the first time. The BER vs. the number of simultaneous user without OHL is represented in Fig. 2. This figure shows that users more than 20,000 can simultaneously communicate each other maintaining BER below 10^{-9} in an OCDMA system with 101 wavelengths and chip length of 20,301. The OCDMA systems with OHL were also analyzed and the resultant BER is compared with the system without OHL in Fig. 3. These results demonstrate that the OHL under the same condition can greatly reduce BER of the system. For an example, when 2400 users communicate simultaneously at 53 wavelengths and 5565 chip lengths, the OHL can reduce BER of the system from 5.2×10^{-8} to 4.9×10^{-23}.

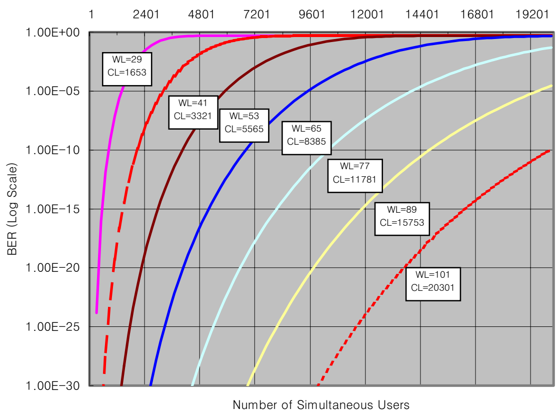

Fig. 2 BER vs. simultaneous users without OHL
- WL: the number of wavelengths
- CL: the chip length of code-word

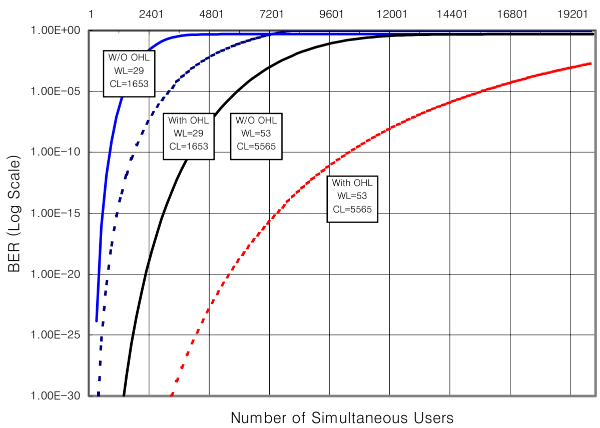

Fig. 3 Comparison of BER with and without OHL
- dashed line: OCDMA system with OHL
- solid line: OCDMA system without OHL
- W/O: without

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