

# Hybrid Impedance/Time-Delay Control from Free Space to Constrained Motion

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**Abstract:** In this paper, hybrid impedance/time-delay control is used for a robot to successfully achieve contact tasks without changing a control algorithm or controller gains throughout all three modes: free space, contact transition and constrained motion. In order to absorb impact forces and stabilize the system upon collision with a stiff environment, a nonlinear bang-bang impact controller is developed. The proposed controller uses hybrid impedance/time-delay control in free space and this control input alternates with zero when no environment force is sensed due to loss of contact. The discontinuous on-off control action depending on the state of contact quickly dissipates the impact energy during impact transient. After impact transient, the hybrid impedance/time-delay control algorithm is employed. Hybrid impedance/time-delay control achieves optimal responsiveness and has good disturbance rejection properties since disturbances are attenuated by a direct estimation technique using time delay. Therefore, this bang-bang control method provides stable interaction between the robot with severe nonlinear joint friction and a stiff environment and achieves rapid response. The proposed controller requires specific knowledge of only one system parameter, the inertia, for its implementation. It is shown via experiments that a robot can successfully work with only one control algorithm from free space to constrained motion under the nonlinear bang-bang impact controller. The proposed controller can be best applied for robots working in unstructured environments.

## 1. Introduction

As technology develops, robots are expected to perform more sophisticated and diverse tasks. In most cases except surveillance, we want robots to manipulate the world/environment they encounter for our benefit, not just maneuver around in it avoiding obstacles. In general, such manipulations involve active interaction with an unknown or changing environment whether intentional or accidental, and this requires making and breaking contact with objects frequently. When robots move from free-space to constrained motion, impact generally occurs and the ability to achieve stable and smooth contact transition becomes crucial for successful manipulation. In contrast to its importance, however, relatively few research works have addressed the area of impact control [1-13]. Most of these approaches use switching of

controllers upon impact, which are not suitable for robots to achieve contact tasks or desired dynamics in unstructured environments in a seamless fashion. Thus, in this paper, we propose an interaction controller that makes a robot establish stable contact and achieves the desired state/dynamics in an unstructured environment without changing a control algorithm or gains. It is called a nonlinear bang-bang impact force control (NBBIC) using a hybrid impedance/time-delay control algorithm (also called natural admittance/time-delay control (NAC/TDC)) and is an improved version of our earlier work [14-16].

The hybrid impedance/time-delay controller is chosen because it compensates for disturbances effectively, thus absorbing impact energy quickly. Consider a task which involves establishing a desired dynamic interaction with a stiff environment. Because hitting a stiff wall with a nonzero approach velocity can excite high frequency vibration, a reckless strategy of pushing the wall back with a high force to settle the vibration would not be a good choice. In order to establish stable contact and reach the desired state, instead, a robot must exert control action in such a manner to absorb the interaction forces while achieving the desired dynamic response. Reaction forces are developed as a result of contact, but the attempt to apply increased forces to compensate for the reaction force can destabilize the system. In our controller, therefore, we use a mitigated control action to absorb the environment force by on-off control action instead of continually pushing the wall during contact transient. The off state of control input helps reduce further impact vibrations by letting the impact oscillations subside naturally.

Under NBBIC, a robot can successfully achieve desired interaction dynamics with one control algorithm without changing controller gains in all three modes: free space, transition and constrained motion. The NBBIC best applies for robots working in unstructured environments. If a robot accidentally hits a stiff environment with high approach velocity while following a desired trajectory or maneuvering around in free space, it can quickly alleviate impact oscillation via the bang-bang action of NAC/TDC. After impact transient, it can continue to perform contact tasks, such as emulation of desired dynamics, without changing its controller gains if they are chosen properly. For example, while a rover robot moves quickly to explore the Mars surface, it may encounter a hard object. Upon collision, the

NBBIC can quickly absorb impact energy without damaging the robot itself or the object. Then, the robot can perform contact tasks with the object, such as shaking hands or drilling.

The organization of this paper is as follows: Section 2 describes the integration of hybrid impedance/time-delay control with the proposed bang-bang impact control. Section 3 verifies the performances of NAC/TDC and NBBIC via experiments. Section 4 discusses conclusions and suggestions for future work.

## 2. Control Design

In this section, a brief description of natural admittance control and time delay control is presented along with their control laws. Additionally, this section develops a hybrid natural admittance/time delay (NAC/TDC) control.

### 2.1 Natural Admittance Control

Under natural admittance control (NAC), the target dynamics to be achieved are explicitly chosen to be the natural system dynamics. Colgate and Newman show separately that the maximum target admittance which does not violate the passivity constraint is:

$$Y = \frac{1}{M_s s} \quad (1)$$

which is the high-frequency asymptote of the driving-point admittance  $Y_i$  where  $M_s$  is the end-point mass [17,18]. Thus, the achieved admittance approaches but should not exceed that of a pure inertia equal in magnitude to the end-point mass.

While Eq. (1) describes the responsiveness of maximum target admittance, such behavior is not practical in performing contact tasks which require dynamic interaction with the environment. Pure mass itself cannot produce useful interactive tasks since there is no restoring force which can be generated by stiffness or damping. To achieve better interactive behavior and shape the low-frequency response, emulation of desired stiffness,  $K_{des}$ , and damping,  $B_{des}$ , need be included.

To satisfy the passivity condition of Eq. (1), the target dynamics are chosen to be:

$$Y_{des} = \frac{1}{(K_{des}/s) + B_{des} + M_s s} < \frac{1}{M_s s} \quad (2)$$

The simplest form of natural admittance control that achieves the target dynamics of Eq. (2) can be described as follows:

$$F = G_v(v_{cmd} - v) + \left(\frac{1}{s}K_{des} + B_{des}\right)(v_{des} - v) \quad (3)$$

where  $v_{cmd} = \{F_s + (K_{des}/s + B_{des})(v_{des} - v)\} / (M_s s)$  and  $v_{des}$  and  $v$  represent the desired velocity and the velocity at the motor port,

respectively. In Eq. (3), the first term corrects deviations of the actual response from the modeled response,  $v_{cmd}$ . The second term is the feedforward term which imposes desired dynamics implicitly. While a robot tries to achieve desired target dynamics of Eq. (2) the desired dynamics generate a virtual force, composed of virtual spring force and virtual damping force, on the end effector. Therefore, this virtual force should be also accounted for in the force feedback loop through  $v_{cmd}$ . In reality, however, this target admittance cannot be achieved due to undesirable dynamic effects such as friction. To mask these effects, the sensed environment force  $F_s$  is fed back as a velocity command.

Thus, natural admittance control achieves the maximum passive responsiveness to sensed forces and has good disturbance rejection properties. Natural admittance control tries to approach the driving-point admittance of pure endpoint inertia at high frequencies, while it emulates the desired stiffness  $K_{des}$  and damping  $B_{des}$  at low-frequencies to achieve better interactive behavior.

However, like many other interaction controllers, NAC does not achieve the desired performance due to inherent nonlinear dynamics, modeling uncertainties and digital sampling. This problem can be solved by using a feedforward compensator, which does not affect stability but does affect the transient response and the equilibrium state. Newman includes a feedforward term,  $F_f$ , to account for Coulomb friction and other disturbances [19]. However, this process is complicated and system dependent.

Better compensation may be achieved by a simpler estimation technique that evaluates a function representing the effect of uncertainties. Youcef-Toumi proposed a time delay control (TDC) for such a purpose [20]. Hsia also used a similar approach [21]. The technique uses a recent past observation of the system's response and its control input to directly estimate the unknown dynamics and unexpected disturbances at any given instant through time delay. The controller updates its observation every sampling period and uses this information to counteract the unknown dynamics and disturbances simultaneously. Then the desired dynamics are inserted into the plant.

In the next section, a hybrid impedance/time-delay control algorithm will be developed to enhance natural admittance control by combining it with time delay control.

### 2.2. Design of Hybrid Impedance/Time-Delay Control

Consider a single-input single-output single DOF robotic system that can be described by the nonlinear dynamic equation:

$$\ddot{x}(t) = f(x, \dot{x}, t) + h(x, \dot{x}, t) + b(x, \dot{x}, t)u(t) + d(t) \quad (4)$$

where  $x$ ,  $\dot{x}$  and  $\ddot{x}$  are states,  $u(t)$  is a control input,  $b(x, \dot{x}, t)$ , is a control distribution term,  $d(t)$  represents unknown disturbances, and  $f(x, \dot{x}, t)$  and  $h(x, \dot{x}, t)$  represent known and unknown nonlinear dynamics of the system, respectively. The variable  $t$  represents time. The term  $h(x, \dot{x}, t)$  includes actuator saturation and stiction as well as Coulomb friction and nonlinear spring characteristics in the transmission. The system output is the variable  $x$  and the reference model for  $x$  is defined by

$$\ddot{x}_r(t) = c_r x_r(t) + a_r \dot{x}_r(t) + b_r r(t) \quad (5)$$

where  $x_r(t)$  and  $\dot{x}_r(t)$  are model states,  $r(t)$  is a reference input and  $c_r$ ,  $a_r$  and  $b_r$  are known constants.

If we set  $b$  equal to a constant for the case where the control distribution term is assumed known as in [22], and assume that no information is available on the nonlinear dynamics, that is  $f$  is zero, the TDC law can be derived as follows [16,23,24]:

$$u(t) = u(t-L) + (1/b) [-\ddot{x}(t-L) - K_p(x_r(t) - x(t)) - K_v(\dot{x}_r(t) - \dot{x}(t)) + c_r x(t) + a_r \dot{x}(t) + b_r r(t)]. \quad (6)$$

In this particular law, the delayed control action term and the delayed acceleration term attempt to compensate for the nonlinear dynamics and disturbances, while the desired reference model is followed by adjusting the error dynamics.

Now, let us examine the structure of the NAC law in the time domain,

$$u(t) = K_{des}(x_{des}(t) - x(t)) + B_{des}(\dot{x}_{des}(t) - \dot{x}(t)) + G_v(\dot{x}_{cmd}(t) - \dot{x}(t)) \quad (7)$$

where

$$\dot{x}_{cmd}(t) = \int \frac{F_s(t) + K_{des}(x_{des}(t) - x(t)) + B_{des}(\dot{x}_{des}(t) - \dot{x}(t))}{M_s} dt. \quad (8)$$

Notice that the term,  $\dot{x}_{cmd}(t)$ , can be regarded as a reference input,  $r(t)$ , in the time delay control law. The NAC stiffness and damping terms are essentially the position and velocity error terms in TDC, respectively. Therefore, if we set  $a_r = b_r = G_v$  and  $r(t) = \dot{x}_{cmd}(t)$ , we can nest the NAC loop inside the TDC loop.

Thus, the hybrid impedance/time-delay control law becomes:

$$u(t) = u(t-L) + (1/b)[- \ddot{x}(t-L) + G_v(\dot{x}_{cmd}(t) - \dot{x}(t)) + K_{des}(x_{des}(t) - x(t)) + B_{des}(\dot{x}_{des}(t) - \dot{x}(t)) + c_r x(t)] \quad (9)$$

where  $\dot{x}_{cmd}(t)$  follows (8) and  $c_r x(t)$  enhances system stability.

Hybrid NAC/TDC achieves optimal responsiveness and provides good trajectory following since the nonlinear effects and disturbances are attenuated by a direct estimation technique. In order to implement the control law only one system parameter, the mass, needs to be estimated.

### 2.3 Nonlinear Bang-Bang Impact Control

Based on the discussion in the previous section, a nonlinear bang-bang impact controller is proposed. During free-space motion, NAC/TDC is used. During impact transient when contact is broken due to bouncing, no control input is applied; and when contact is made, NAC/TDC is used. This bang-bang control approach repeats until steady state is attained. NAC/TDC is used again after contact is established. NAC/TDC also brings a robot back to contact with an environment when it stops in free space due to the zero control input during contact transient. The

resulting control strategy for a single input system is described as follows.

1) In Unconstrained Motion: NAC/TDC

$$\text{If } F_s > F_{impact}, \text{ then NAC/TDC.} \quad (10)$$

2) In Contact Transition: Bang-Bang Control

If the manipulator and the environment are in contact,

$$\text{i.e., } F_s > F_{sw}, \text{ then NAC/TDC.} \quad (11a)$$

If the manipulator and the environment are out of contact, i.e.,  $F_s < F_{sw}$ , then

$$u(t) = 0. \quad (11b)$$

$$\text{If } F_s < F_{sw} \text{ and } |v| < v_{threshold}, \text{ then NAC/TDC.} \quad (11c)$$

3) After Impact Transient: NAC/TDC

$$\text{If } F_s > F_{sw}, \text{ then NAC/TDC.} \quad (12)$$

$F_{impact}$ ,  $F_{sw}$ , and  $v_{threshold}$  are threshold values to detect impact, switching and zero velocity, respectively, and are dependent on the sensitivity of the torque and position sensors.

However, the zero control input of (11b) alone cannot achieve the desired dynamics if a robot stops while it is out of contact with the environment after bouncing off from the environment. In most cases, the nonlinear spring characteristics of robot transmissions and link can bring the robot back to the environment after impact due to its high restoring force. If the restoring spring force cannot overcome friction and inertia of the robot, however, it may stop in free space after it bounces off from the environment. Eqn. (11c) prevents this from occurring.

## 3. Experiments

### 3.1 Experimental Setup

An experimental setup was constructed as shown in Fig. 1. The aluminum robot arm is attached to a Himmelstein model MCRT 28002T(5-2), 500lb-in (56.5 Nm) range, non-contact rotating torque transducer via a Tran-torque coupling. The torque meter is used to measure the contact force at the tip of the aluminum link when it interacts with an environment. The other shaft of the torque meter is coupled to the HD Systems harmonic drive CSF-20-1000-2A-GR (gear ratio of 100:1) via a Thomas miniature flexible disc coupling, which provides relatively high torsional stiffness along the shaft axis and compliance along all five remaining degrees of freedom. The harmonic drive is attached to a Maxon Precision Motors DC motor 148867, which is connected to a Hewlett Packard HEDM-5500 two channel incremental optical encoder with a resolution of 1,000 counts-per-revolution with quadrature output (0.0016 rad/pulse).

For data acquisition and control, a motion control interface card, Precision MicroDynamics model MFIO-4A Dual, is inserted into a PC Pentium III (500Mhz). This I/O card has four digital-to-analog converters (DAC), four analog-to-digital converters (ADC) and four encoder interface channels used for input/output between the computer and external devices.

For controller implementation, control algorithms were written in C++ and implemented on a real time OS, QNX RTP, at a sampling rate of 1 kHz. The control command generated by the computer is sent to a PWM motor amplifier, Maxon Precision Motors ADS 50/10, through the DAC. The sensed motor

position is imported into the computer through the encoder interface channel. The torque meter is connected to a Himmelstein model 701 universal strain gauge amplifier and the measured torque transducer analog input is processed via the I/O board ADC.

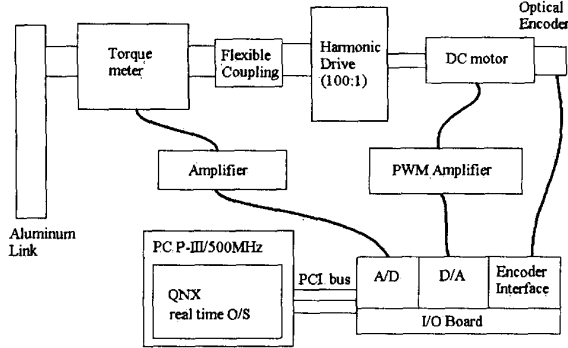


Fig. 1. Schematic of experimental setup

The total inertia of the arm was estimated as  $19.5\text{ kg}$  through a series of experiments and the total static friction force was measured to be  $1.96\text{ Nm}$  by torque meter. An aluminum plate and a Styrofoam block were used to make hard and soft environments, respectively.

### 3.2 Effect of Sampling Time

Theoretically, the smaller the sampling time, the better the performance of NAC/TDC of Eqn. (9). In reality, however, there is a limit on how small we can make the sampling time. We observe that a sampling time  $T$ , smaller than a certain value, can deteriorate a control system with limited sensor resolution by increasing numerical differentiation errors in the process of estimating velocity/acceleration from position/velocity sensor signals.

In order to examine such a phenomenon for velocity, let's define that  $\tilde{y}_k$  and  $\tilde{y}_{k-1}$  are sensed values of exact positions,  $y_k$  and  $y_{k-1}$ , at time  $kT$  and  $(k-1)T$ , respectively. Then the estimated velocity at time  $kT$ ,  $\tilde{v}_k$ , can be obtained by following backward numerical differentiation:

$$\tilde{v}_k = \frac{\tilde{y}_k - \tilde{y}_{k-1}}{T} = \frac{y_k + \varepsilon_k - y_{k-1} - \varepsilon_{k-1}}{T} = \frac{y_k - y_{k-1}}{T} + \frac{\varepsilon_k - \varepsilon_{k-1}}{T}, \quad (13)$$

where  $\varepsilon_k$  and  $\varepsilon_{k-1}$  represent errors in  $\tilde{y}_k$  and  $\tilde{y}_{k-1}$ , respectively, due to limited sensor resolution. If we use exact positions  $y_k$  and  $y_{k-1}$  instead of  $\tilde{y}_k$  and  $\tilde{y}_{k-1}$ , the numerical differentiation error is represented by the following equation [25]:

$$v_k = \frac{y_k - y_{k-1}}{T} + \frac{T}{2} \left. \frac{d^2 y}{dt^2} \right|_{Max} \quad (14)$$

for  $(k-1)T \leq t \leq kT$ . Therefore, from equations (13) and (14), we can obtain

$$|e_k|_{Max} = |v_k - \tilde{v}_k|_{Max} = \frac{T}{2} \left. \frac{d^2 y}{dt^2} \right|_{Max} + \frac{|\varepsilon_k - \varepsilon_{k-1}|_{Max}}{T}. \quad (15)$$

Fig. 2 plots equation (15), the maximum error vs. sampling time with our sensor resolution of  $1.57 \times 10^{-5}$  radians. This figure shows that a sampling time smaller than  $10\text{ ms}$  can increase the numerical differentiation error. The experimental results of Fig. 3 show that the velocities obtained at sampling time  $0.83\text{ kHz}$  include large errors.

However, our experiments show that the best performance can be achieved with a sampling time of  $1\text{ ms}$  and the performance deteriorated below and beyond that value. This is because the control system performance is affected not only by the numerical differentiation error but also by other factors such as the refresh rate of control input.

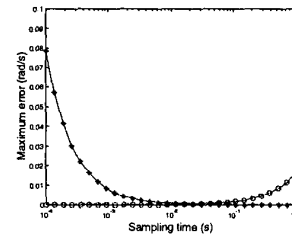
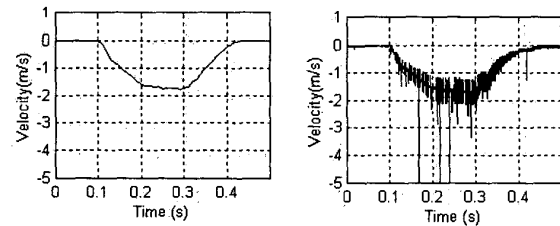


Fig. 2. Numerical differentiation error vs. sampling time: marks 'o' and '\*' represent  $\left. \frac{d^2 y}{dt^2} \right|_{Max} T/2$  and  $|\varepsilon_k - \varepsilon_{k-1}|_{Max} / T$ , respectively, and solid line is their summation.



(a) For sampling time  $10\text{ ms}$  (b) For sampling time  $0.83\text{ ms}$   
Fig. 3. Velocities obtained using numerical differentiation in experiments

### 3.3 Experiments for NAC/TDC

In this section, the performance of NAC and the proposed NAC/TDC are compared. For experiments, the one-link robot is commanded to come into and break out of contact with an environment by setting a desired position,  $x_{des}$ , to be a sinusoidal function with amplitude of  $2.6\text{ mm}$  and frequency of  $0.2\text{ Hz}$ . For the inertia parameter, the measured value of  $M_i = 19.5\text{ kg}$  is used. The controller gains are set as follows:  $G_v = 1180\text{ N/s/m}$ ,  $K_{des} = 5.9 \times 10^4\text{ N/m}$ ,  $B_{des} = 2900\text{ N/s/m}$ ,  $1/b = 5.9\text{ kg}$ . They are the maximum stable gains for contact with a hard environment, which is simulated by an aluminum plate in our experiments. Throughout these experiments,  $c_r$  is set to zero since its value did not make much difference in system performance.

The results for a contact task with a hard surface in Fig. 4 demonstrate that the performances of NAC/TDC are almost identical to that of NAC in performing a contact task. Graphs (c)

and (d) for contact force show that the robot arm was in and out of contact with the surface.

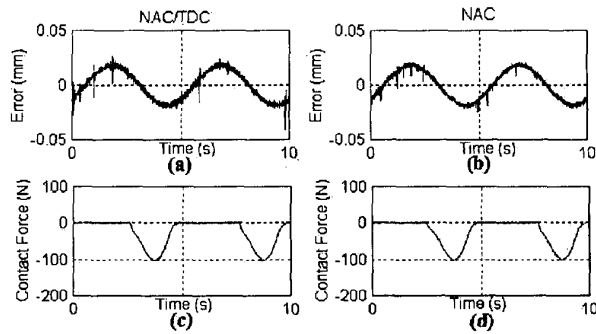


Fig. 4. Experimental results for hard surface

The experimental results demonstrate that the acceleration term and the effect of time delay in NAC/TDC do not deteriorate its control performance. It justifies using NAC/TDC in free space so that it can achieve stable contact tasks upon contact with a stiff environment with high approach velocity.

### 3.4 Experiments for Impact Control

In this section, the performance of the proposed bang-bang impact controller is presented. The experiment of impact control has three stages of motion: free space motion, impact and constrained motion. The control performances are compared when the robot hits the environment with an impact velocity of  $1.75 \text{ m/sec}$ . The control parameters are adjusted to produce the best performance and still achieve stable force control. For the bang-bang control, the control parameters are adjusted to be  $G_v = 1180 \text{ N/s/m}$ ,  $K_{des} = 5.9 \times 10^4 \text{ N/m}$ ,  $B_{des} = 2900 \text{ N/s/m}$  and  $1/b = 5.9 \text{ kg}$  for NAC/TDC.

Fig. 5 shows system responses when the link contacted the hard environment of an aluminum plate. For the same impact velocity of  $1.75 \text{ m/s}$ , all controllers experience an impact force of approximately  $160 \text{ N}$ . In Fig. 5 (a), we can see that the bang-bang control scheme effectively shortened the settling time down to 0.17 seconds. Under NAC and NAC/TDC without anti integral-windup, contact forces increased suddenly due to integral wind-up [24]. Fig. 5 (b) shows experimental results for NAC/TDC with anti integral-windup. This result displays a settling time of 0.22 seconds, longer than that of nonlinear bang-bang control. NAC is observed to exhibit similar performance with anti integral-windup. The steady state contact force is approximately  $10 \text{ N}$  for nonlinear bang-bang control and  $130 \text{ N}$  for NAC/TDC with anti-windup scheme. This is due to the fact that the controller gains for each controller are set to exhibit the best stable performance.

It was also observed that the proposed bang-bang impact control is better than or comparable to other existing impact control techniques for our system [25].

The NAC/TDC can be best applied for robots working in unstructured environments. If a robot accidentally hits a stiff environment with high approach velocity while following a desired trajectory in free space, it can quickly subside impact

oscillation via bang-bang action of NAC/TDC. After impact transient, it can continue to perform contact tasks, such as emulation of desired dynamics, without changing its controller gains. Thus, a robot can successfully work with only one control algorithm in three modes: free space, transition and constrained motion.

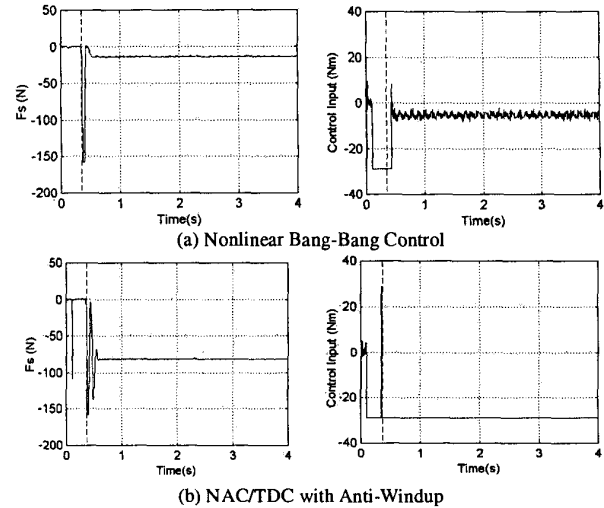


Fig. 5. Experimental results for impact control (Dashed line indicates impact time.)

An experiment was performed to demonstrate this advantage. First, the robot arm was commanded to travel down toward an aluminum plate on the table and hit the plate with impact velocity  $1.75 \text{ m/s}$ . Then, one end of the plate was moved up and down off the table while the robot link kept contact with the plate. For NAC/TDC, the control parameters are adjusted to be  $G_v = 5900 \text{ N/s/m}$ ,  $K_{des} = 5.9 \times 10^4 \text{ N/m}$ ,  $B_{des} = 2900 \text{ N/s/m}$  and  $1/b = 5.9 \text{ kg}$ , and remained the same throughout the experiment. As shown in the system responses of Fig. 6, the robot quickly absorbs impact energy upon hitting a stiff surface and successfully emulates desired dynamics.

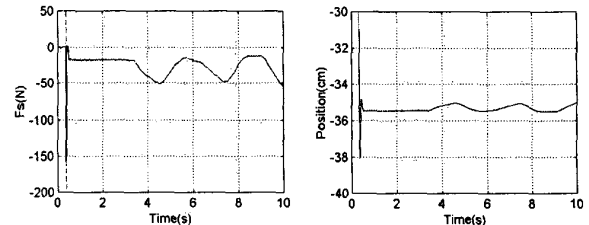


Fig. 6. Emulation of desired dynamics after impact (Dashed line indicates the impact time.)

## 4. Conclusions

In this paper, a hybrid impedance/time-delay control and a nonlinear bang-bang impact controller are developed and their performances are demonstrated via experiments. Experimental results show that the proposed nonlinear bang-bang impact control

is very effective in contact transient after contact with a stiff environment. It is shown via experiments that under NBBIC, a robot can successfully interact with an environment without changing a control algorithm and controller gains throughout all three modes: free space, transition and constrained motion. The hybrid impedance/time-delay controller is very effective for robots with severe nonlinearities such as friction and joint flexibility because unknown dynamics and disturbances are attenuated by a direct estimation technique using time delay.

The proposed controller, NBBIC, can be used for mobile robots and cooperative robot tasks as well as for robot interactions in unstructured environments such as space exploration. Future work includes experimental verification of the proposed control method for multi-degree of freedom robots as well as a more formal representation of stability analysis.

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