

# Vector Watermarking Robust to Both Global and Local Geometrical Distortions

Dong-Hyuck Im, Hae-Yeoun Lee, Seung-Jin Ryu, and Heung-Kyu Lee

**Abstract**—A blind watermarking algorithm for vector graphic images is presented. The algorithm is resilient to both global and local geometrical distortions. The polygonal line is represented by the wavelet descriptor. An additive watermarking scheme is used to embed the watermark by slightly modifying the wavelet descriptor, and that causes invisible distortions to the coordinates of the vertices. The invariant properties of the wavelet descriptor ensure that the presented algorithm is resilient against both global and local geometrical distortions. Using vector graphic images from contour maps, we demonstrate that the presented algorithm outperforms the algorithm based on the Fourier descriptor.

**Index Terms**—Local geometrical distortion, robust watermarking, vector watermarking, wavelet descriptor.

## I. INTRODUCTION

VECTOR graphic images are widely used in digital maps, geographical information systems, cartoons, and 2-D graphics. The copyrights of these images need to be protected when they are used commercially. Digital watermarking provides such protection. While numerous watermarking schemes have been developed for raster graphic images, a limited number of schemes have been developed for vector graphic images, which use geometrical primitives such as points, lines, curves, and polygons.

It is an important challenge for most current watermarking algorithms to ensure that they are robust against geometrical distortions [1]. Global geometrical distortions do not remove the embedded watermark. However, they desynchronize its location and make automatic blind detection impossible. Local geometrical distortions are particularly difficult to resist because they desynchronize the location of the watermark and destroy watermarks without loss of perceptual image quality. Given that parts of vector graphic images are often modified in routine work, robustness against global and local geometrical distortions is essential for the successful watermarking of vector images.

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Watermarking schemes using curve reparameterization [2] and mesh spectral analysis [3] are proposed by Ohbuchi *et al.* Robustness is not considered in [2]. While the algorithm in [3] is robust against various attacks, it has high computing complexity and is not suitable for watermarking curves. Gou and Wu [4] proposes a robust watermarking scheme that uses B-spline control points. The scheme needs a preprocessing step before detecting the watermark, in order to align the test curve with the original one. Solachidis and Pitas [5] proposes a vector watermarking algorithm that uses the Fourier descriptor of polygonal lines. Although [5] is robust against global geometrical distortion, such as rotation, scaling, and translation (RST), it is weak against local geometrical distortion. Modifying the locations of a few vertices drastically decreases the performance of vector watermarking systems that use the Fourier descriptor.

In this letter, we present a blind vector watermarking algorithm that is resilient to both global and local geometrical distortions. The wavelet descriptor [6] is adopted to analyze the shape of polygonal lines, and an additive watermarking scheme is used to embed the watermark into the polygonal lines. The invariant properties of the wavelet descriptor against RST and local geometrical distortions are analyzed. Simulation results indicate that the presented algorithm is robust against both global and local geometrical distortions and outperforms Solachidis' algorithm [5].

The remainder of this letter is organized as follows. Section II presents and explains the proposed watermarking algorithm. Section III demonstrates the invariance of the presented scheme against both global and local geometrical distortions. Simulation results are shown in Section IV. Section V concludes.

## II. PRESENTED WATERMARKING ALGORITHM

A blind vector watermarking algorithm is presented. First, we explain the wavelet descriptor and how to solve the starting point-dependent problem of the wavelet descriptor for robust watermarking. Then, we describe the process by which watermarks are embedded and detected.

### A. Wavelet Descriptor and Synchronization

In shape recognition and retrieval, the wavelet descriptor is used widely, because it has many desirable properties, such as multiresolution representation, invariance, uniqueness, and stability [6]. The wavelet descriptor decomposes a curve into components of difference scales. The coarsest scale components carry the global approximation information, while the finer scale components contain information about local details.

Let us denote a clockwise-oriented closed-plane curve with parametric coordinates  $x(t)$  and  $y(t)$  by

$$\alpha(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, t(l) = \frac{l}{L}, 0 \leq l \leq L \quad (1)$$

where  $t$  is the normalized arc length,  $l$  is the arc length along the curve from a certain starting point  $t_0$ , and  $L$  is the total arc length. By applying the wavelet transform to the parameterized coordinates, we obtain

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_a^M(t) \\ y_a^M(t) \end{bmatrix} + \sum_{m=M-m_0}^M \begin{bmatrix} x_d^m(t) \\ y_d^m(t) \end{bmatrix} \quad (2)$$

where

$$x_a^M(t) = \sum_n a_n^M \phi_n^M(t), \quad y_a^M(t) = \sum_n c_n^M \phi_n^M(t) \quad (3)$$

are called the approximation coefficients at scale  $M$  and

$$x_d^m(t) = \sum_n r_n^m \psi_n^m(t), \quad y_d^m(t) = \sum_n d_n^m \psi_n^m(t) \quad (4)$$

are called the detailed signals at scale  $m$ , where  $m = M - m_0$  is the finest scale and  $m = M$  is the coarsest scale.  $\phi(t)$  and  $\psi(t)$  denote a scaling function and a mother wavelet function, respectively. Then, we can use the wavelet coefficients  $a_n^M, c_n^M, r_n^m$ , and  $d_n^m$  in (3) and (4) as the planar curve descriptor.

It is a well-known problem that the wavelet descriptor is not invariant to changes in starting point, which is critical for robust watermarking. To avoid this drawback, the same starting point should be used in the procedures for both watermark embedding and detection. Kith and Zahzah [7], [8] proposed several solutions using the following measures: furthest distance, maximum curvature, principal axis, and natural axis. We adopted the best measure, the furthest distance, to synchronize the starting point during watermark embedding and detection. When using this method, the starting point is fixed at the contour point such that its distance to the centroid is maximal. The point  $(x_{i_{\max}}, y_{i_{\max}})$  with  $i_{\max} = \arg \max_{1 \leq i \leq N} s(i)$  is chosen to the starting point.

Multiple points could be selected as the starting point. If ambiguity occurs, a combined method could be applied.

### B. Watermark Embedding

After the starting point has been determined, we apply the wavelet descriptor to the vertices of the polygonal line. Then, we embed the watermark into the magnitude of the detailed wavelet coefficients in order to use the invariant properties of the wavelet descriptor. A bi-polar ( $\pm 1$ ) random sequence from a pseudo-random generator is used as the watermark, which has zero mean value and unit variance. The watermark is multiplicatively embedded as follows:

$$|Z'(k)| = |Z(k)| + p|Z(k)|W(k) = |Z(k)|[1 + pW(k)] \quad (5)$$

where  $|Z(k)|$  is the magnitude of the original wavelet coefficients from the curve  $L$ ,  $W(k)$  is the bi-polar watermark,  $|Z'(k)|$  is the watermarked magnitude of the wavelet coefficients,  $k = 0, 1, \dots, N-1$ , and  $p$  is a factor that determines

the watermark's power. Given that the magnitude of the watermarked line's wavelet descriptor must be nonnegative, the multiplicative factor  $p$  must be less than 1. After the watermark has been embedded, an inverse wavelet transform of the watermarked wavelet coefficients produces the watermarked curve  $L'$ .

### C. Watermark Detection

The same starting point is chosen for both the embedding and detection of the watermark by using the furthest distance measure. Then, the wavelet descriptor is applied to the vertices of the polygonal line. We used the same method for detecting the watermark as Solachidis' algorithm [5]. Let  $|Z'(k)|$  be the magnitude of the detailed wavelet coefficients after applying the wavelet descriptor to the polygonal line  $L'$ , which is watermarked by  $W'$ . To determine whether the watermark  $W$  is in  $L'$  or not, the correlation  $c$  between  $|Z'(k)|$  and the watermark  $W$  is computed as follows:

$$c = \frac{\sum_{k=1}^N |Z'(k)|W(k)}{\sum_{k=1}^N |Z(k)|W(k) + p \sum_{k=1}^N |Z(k)|W(k)W'(k)}. \quad (6)$$

The mean value of correlation,  $\mu_c$ , can be computed with the following assumptions: both  $|Z'(k)|$  and  $W$  are independent and identically distributed random variables, and  $W$  has zero mean value and unit variance. Then  $\mu_c$  becomes

$$\mu_c = \begin{cases} \frac{1}{N} \sum_{k=1}^N |Z(k)|p, & \text{if } W = W' \\ 0, & \text{if } W \neq W' \\ 0, & \text{if no watermark is present.} \end{cases} \quad (7)$$

Since we assumed that  $W'$  has zero mean value, we can compute  $\mu_{|Z(k)|}$  and  $\mu_c$  without knowing the original wavelet coefficient  $Z(k)$  as follows:

$$\mu_{|Z'(k)|} = \overline{|Z(k)|} + p\overline{|Z(k)W'(k)|} = \overline{|Z(k)|} = \mu_{|Z(k)|}. \quad (8)$$

The normalized correlator  $c'$  can be computed by  $c/\mu_c$ , and the value is in the range  $[0, 1]$ . Instead of  $c$ , we use a normalized correlator  $c'$  in detection. The mean value of the normalized correlator  $c'$  equals 1 if  $W = W'$ . The detection rule is simple. If the normalized correlator is higher than the threshold, the watermark is present in the line. Otherwise, the watermark is not present. When determining the threshold, a false positive probability and a false negative probability are taken into account.

## III. ROBUSTNESS OF THE PRESENTED ALGORITHM

### A. Global Geometrical Distortion

In a  $\beta$  scaled polygonal line, the magnitude of the wavelet descriptor becomes  $\beta|Z(k)|$ . As far as the polar coordinates are concerned, we have

$$\begin{aligned} \tilde{\theta}_n^m &= \arctan\left(\frac{\beta d_n^m}{\beta r_n^m}\right) = \arctan\left(\frac{d_n^m}{r_n^m}\right) = \theta_n^m \\ \tilde{A}_n^m &= \sqrt{(\beta d_n^m)^2 + (\beta r_n^m)^2} = \beta \sqrt{(d_n^m)^2 + (r_n^m)^2} = \beta A_n^m \quad (9) \end{aligned}$$

where  $r_n^m, d_n^m$  are the wavelet coefficients of the detailed signals. However, the normalized correlator  $c'_s$  remains invariant because both the numerator  $c_s$  and the denominator are multiplied by  $\beta$  as follows:

$$c_s = \sum_{k=1}^N |Z(k)| W(k)\beta + p |Z(k)| W^2(k)\beta \quad (10)$$

$$c'_s = \frac{\sum_{k=1}^N |Z(k)| W(k)\beta + p |Z(k)| W^2(k)\beta}{\sum_{k=1}^N p |Z(k)| \beta} = \frac{c}{\mu_c} = c' \quad (11)$$

where  $Z(k)$  is an original polygonal line, and  $W(k)$  is the watermark. Therefore, the presented watermarking method is invariant against scaling attacks.

The displacement of a curve affects only the approximation coefficients. After translation, we have

$$\tilde{\theta}_n^m = \arctan\left(\frac{d_n^m}{r_n^m}\right) = \theta_n^m \quad \tilde{A}_n^m = \sqrt{(d_n^m)^2 + (r_n^m)^2} = A_n^m \quad (12)$$

for the wavelet coefficients  $r_n^m, d_n^m$  of the detailed signals. Given that our watermark embedding scheme only uses the detailed coefficients that are invariant under translation, the presented method is robust to translation attacks.

By rotating a curve by a counterclockwise angle  $\phi$  with the centroid as the pivot point, we have

$$\tilde{\theta}_n^m = \theta_n^m + \phi, \quad \tilde{A}_n^m = A_n^m \quad (13)$$

for the wavelet coefficients of the detailed signal. The same relationship also holds for the polar coordinate representation of the approximation coefficients. As shown in (13), rotation does not affect the magnitude of the line's wavelet descriptor. Therefore, the presented method is invariant against rotation attacks.

### B. Local Geometrical Distortion

The wavelet descriptor has the stability property, which means that small differences in the shapes of curves correspond to small differences in their representations, and vice versa [9]. We consider a class of square-integrable functions  $f \in L^2([0, 1])$  and their corresponding wavelet frame representations [10], [11]. The frame is a concept that is more general than the basis. By choosing  $\psi(x)$  such that functions

$$\{\varphi_0^0\} \cup \{\psi_n^m(x)\}_{m \in -N; n \in \mathbb{Z}_m} \quad (14)$$

constitute a frame in  $L^2([0, 1])$ , we have

$$A \|f\|^2 \leq |\langle f, \tilde{\varphi}_0^0 \rangle|^2 + \sum_{m \in -N} \sum_{n \in \mathbb{Z}_m} |\langle f, \tilde{\psi}_n^m \rangle|^2 \leq B \|f\|^2 \quad (15)$$

for  $f \in L^2([0, 1])$  and  $0 < A < B < \infty$ . The 2-norm of the wavelet representation is defined as follows:

$$\|W(f)\| \equiv \left( |\langle f, \tilde{\varphi}_0^0 \rangle|^2 + \sum_{m \in -N} \sum_{n \in \mathbb{Z}_m} \left( |\langle f, \tilde{\psi}_n^m \rangle|^2 \right) \right)^{1/2}. \quad (16)$$

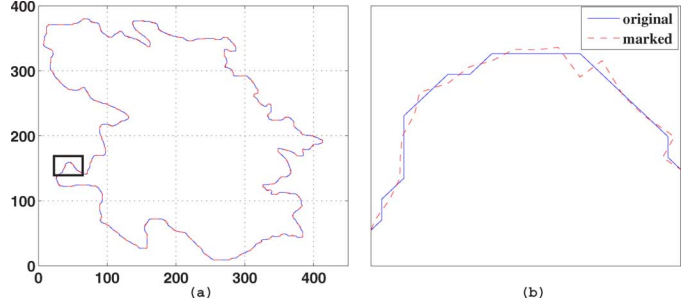


Fig. 1. Vector graphic image with (a) a watermarked polygonal line overlays the original line and (b) zoomed-in view of (a).

We can see that if two representations are close, the curves that they represent should be close as well [6]. Thus, a small change in the shape of a curve will not cause a large change in its wavelet representation, which means that the wavelet representation is stable with respect to local geometrical distortion. From the viewpoint of watermarking, detection performance is affected little by local geometrical distortion. In contrast, the basis functions of the Fourier descriptor are sinusoids that are periodic and global (not sufficiently localized in space). As a result, the entire shape can be changed by a small perturbation of one coefficient, and the whole coefficients can be changed by only a small changes in shapes. Due to the lack of stability, watermarking systems that use the Fourier descriptor have difficulties in detecting watermarks in locally distorted images.

## IV. SIMULATION RESULTS

We tested the presented algorithm and Solachidis' algorithm [5] on 100 polygonal lines from several contour maps. Global and local geometrical distortions were applied to the watermarked vector graphic images. We set the watermark embedding strength  $p$  at 0.8 through experiments. In Fig. 1, the watermarked curve overlays the original curve, using a dotted line and solid line, respectively. A portion of Fig. 1(a) is enlarged in Fig. 1(b). The Hausdorff distance [12] can quantify the difference between the original and watermarked curve, and the distance is 2.36.

### A. Global Geometrical Distortion

RST attacks were applied to the watermarked vector graphic images with the presented algorithm. We tested with several keys. The empirical distribution of the normalized correlator of 1000 watermark detections with an erroneous key (left side) and 1000 watermark detections with the correct key (right side) is shown in Fig. 2(a). The empirical distribution after translating the polygonal line by  $-100$  pixels on the x-axis and 200 on the y-axis is shown in Fig. 2(b). The empirical distributions after scaling  $0.5 \times$  and rotating 30 degrees are shown in Fig. 2(c) and (d), respectively.

Against RST attacks, the empirical distributions represent well-separated probability-distribution-functions (PDFs), which means that the watermarking method has few detection errors. It is evident that the presented watermarking algorithm is robust against global geometrical distortion.

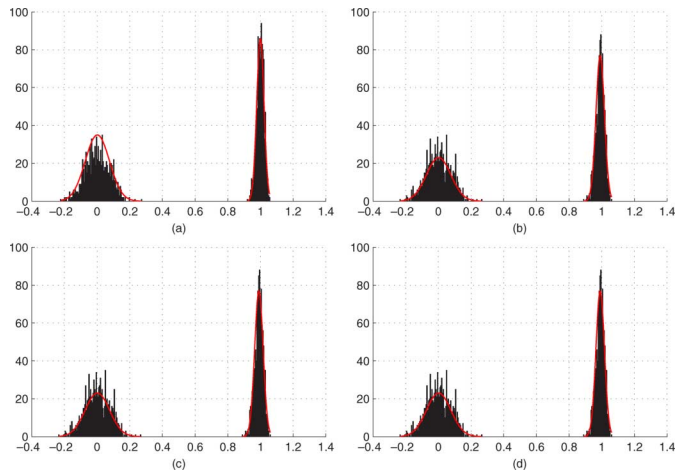


Fig. 2. Empirical probability distribution of the normalized correlator on 1000 watermarked polygonal lines with an erroneous key (left) and the correct key (right) against: (a) no attack, (b) translation, (c) scaling, and (d) rotation.

TABLE I  
AVERAGE OF NORMALIZED CORRELATION FOR 100 WATERMARKED  
VECTOR IMAGES AGAINST GEOMETRIC DISTORTIONS

	Presented algorithm	Solachidis' algorithm [5]
No attack	0.917	0.943
Rotation 30 deg.	0.917	0.943
Scaling 0.5 $\times$	0.917	0.943
Translation	0.917	0.943
Local distortion	0.802	0.243

Table I summarizes the average normalized correlation value of 100 watermarked vector images using the presented algorithm and that of [5] against global geometrical distortion. The presented algorithm performed as well as that of [5].

### B. Local Geometrical Distortion

We modified the local part of the watermarked polygonal lines as shown in Fig. 3. About 10% of the adjacent vertices around (150, 50) in Fig. 3(a) are locally modified by moving  $-20$  pixels on the x-axis and  $+20$  on the y-axis. The modified part is indicated by the arrow in Fig. 3(b). The watermarked polygonal line with the Solachidis' algorithm was modified in the same way.

The empirical distributions from the presented algorithm and Solachidis' algorithm are shown in Fig. 3(c) and (d), respectively. The correlation PDF of the presented algorithm was separated well. The average normalized correlation values of the correct and erroneous parts were 0.8306 and 0.0016, respectively. However, the detection performance of the Solachidis' algorithm was not good as that of the presented watermarking algorithm. The average normalized correlation values of the correct and erroneous parts were 0.1638 and 0.0016, respectively, and the correlation PDF is not separated well. These correlation distributions support the assertion that the presented watermarking algorithm is more efficient than Solachidis' algorithm against local geometrical distortion. Table I summarizes the average normalized correlation value of 100 watermarked vector images against local geometrical distortion. The presented algorithm outperformed Solachidis' algorithm.

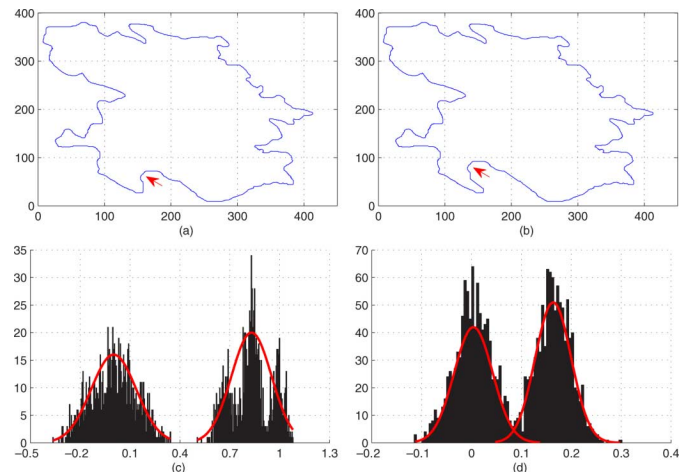


Fig. 3. (a) Watermarked line and (b) its locally distorted line. Empirical probability distribution of the normalized correlator output on 1000 polygonal lines with an erroneous key (left) and the correct key (right) against local distortions: (c) the presented algorithm and (d) Solachidis' algorithm [5].

## V. CONCLUSION

We have presented a blind watermarking algorithm for vector graphic images by using the wavelet descriptor. The algorithm is robust against both global and local geometrical distortions and outperforms Solachidis' algorithm, which uses the Fourier descriptor [5], especially against local geometrical distortion.

However, like Solachidis' algorithm, the presented algorithm is still not robust to malicious attacks such as polygonal line cropping, vertex insertion, and vertex deletion. Our future work includes improving robustness against these smart and deliberate attacks.

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