A Fast Computational Method for Minimum Square Error Transform

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ABSTRACT

In this paper, we describe a basic minimum square error transform for point pattern matching and propose a fast computational method for minimum square error transform. The computational analysis revealed that the proposed method is faster than that of Groen et al. [3].

INTRODUCTION

Image matching is important in many applications, including registration, navigation, change detection, and stereo-mapping. The brute-force approach to image matching involves correlation, performed either in the space or spatial frequency domain; in either case, the cost of matching grows with the image area [1]. A more economical approach is to extract a discrete set of feature points from the two images that are to be matched, and match the resulting two point patterns. The cost of this process grows with the number of feature points rather than with the image area. In recent years, many researchers [1,2,3,4,5] have directed considerable attention to the problems of point pattern matching. In this paper, we describe the basic minimum square error (MSE) transform for point pattern matching and propose a fast computational method for MSE transform of Ref. [3].

We can say that two point patterns are equivalent when there exists an appropriate functional relation between the two coordinate systems

$$x = f(r,s) \quad (1a)$$
$$y = g(r,s) \quad (1b)$$

which is satisfied by all point pairs of the two point patterns. In practical situations, however, two point patterns are seldom equivalent in the strict sense stated above. Only an approximate relation between the two point patterns exists, and it can be determined by finding a transform which minimizes some error criterion. The approximate relations f and g between the two points when there are n points in each point pattern are determined in a minimum-squares sense; i.e., by minimizing the residual error

$$E = \Sigma\, w_i \{[x_i - f(r_i, s_i)]^2 + [y_i - g(r_i, s_i)]^2\} \quad (2)$$

where w_i is a coefficient giving the possibility to weight the error with the likehood of the match ($w_i > 0$, $\Sigma w_i = 1$). When equal weights are applied, $w_i = 1 / n$.

EXISTING MSE TRANSFORM

Groen et al. [3] restricted f and g to be transforms which are defined by

$$f(r_i, s_i) = s \cos\theta\, (r_i - r_0) + s \sin\theta\, (s_i - s_0) \quad (3a)$$
$$g(r_i, s_i) = -s \sin\theta\, (r_i - r_0) + s \cos\theta\, (s_i - s_0) \quad (3b)$$

where r_0, s_0 is the translation, θ is the rotation, and s is the scale.

They defined:

$c_x = \Sigma_i\, w_i\, x_i$
$c_y = \Sigma_i\, w_i\, y_i$
$c_r = \Sigma_i\, w_i\, r_i$
$c_s = \Sigma_i\, w_i\, s_i$

$c_{xx} = \Sigma_i w_i x_i x_i$
$c_{yy} = \Sigma_i w_i y_i y_i$
$c_{rr} = \Sigma_i w_i r_i r_i$
$c_{ss} = \Sigma_i w_i s_i s_i$

$c_{xr} = \Sigma_i w_i x_i r_i$
$c_{xs} = \Sigma_i w_i x_i s_i$
$c_{yr} = \Sigma_i w_i y_i r_i$
$c_{ys} = \Sigma_i w_i y_i s_i$

By setting the partial derivatives of E to r_0, s_0, θ and s to zero, they obtained

$\partial E/\partial r_0 = 0$:
$$r_0 = c_r - c_x / s \cos\theta + c_y / s \sin\theta$$

$\partial E/\partial s_0 = 0$:
$$s_0 = c_s - c_y / s \cos\theta + c_x / s \sin\theta$$

$\partial E/\partial s = 0$:
$$s(c_{rr} - 2c_r r_0 + r_0^2 + c_{ss} - 2c_s c_0 + s_0^2) = (c_{xr} + c_{ys} - c_x r_0 - c_y s_0) \cos\theta + (c_{xs} - c_{yr} - c_x s_0 + c_y r_0) \sin\theta$$

$\partial E/\partial\theta = 0$:
$$\sin\theta\, (c_{ys} + c_{xr} - s_0 c_y - r_0 c_x) = \cos\theta\, (c_{xs} - c_{yr} - s_0 c_x + r_0 c_y)$$

From these equations the transform parameters can be calculated :

rotation:
$$\tan\theta = (c_{xs} - c_{yr} - c_x c_s + c_y c_r) / (c_{xr} + c_{ys} - c_x c_r - c_y c_s)$$

scale:
$$s = \{(c_{xr} + c_{ys} - c_x c_r - c_y c_s)\cos\theta + (c_{xs} - c_{yr} + c_x c_s + c_y c_r)\sin\theta\} / (c_{rr} + c_{ss} - c_r c_r - c_s c_s)$$

translation:
$$r_0 = c_r - c_x / s \cos\theta + c_y / s \sin\theta$$
$$s_0 = c_s - c_y / s \cos\theta + c_x / s \sin\theta$$

The transform error E equals :

$$E = c_{xx} + c_{yy} - c_x c_x - c_y c_y - s^2 (c_{rr} + c_{ss} - c_r c_r - c_s c_s)$$

FAST COMPUTATIONAL METHOD FOR MSE TRANSFORM

If we define a_0, a_1, b_0, and b_1 appropriately, Eq. (3a) and Eq. (3b) can be rewritten as

$$f(r_i, s_i) = a_0 + a_1 r_i - b_1 s_i \qquad (3a')$$
$$g(r_i, s_i) = b_0 + b_1 r_i + a_1 s_i \qquad (3b')$$

By substituting f and g defined in Eq. (3a') and Eq. (3b') into Eq. (2), we get the residual error

$$E = \Sigma w_i \{[x_i - (a_0 + a_1 r_i - b_1 s_i)]^2 + [y_i - (b_0 + b_1 r_i + a_1 s_i)]^2\} \qquad (4)$$

In order to minimize E, we find the partial derivatives of E with respect to a_k and b_k, and set them equal to zero,

$$\partial E / \partial a_k = 0 \qquad (5a)$$
$$\partial E / \partial b_k = 0 \qquad (5b)$$

for $k = 0, 1$. Thus from Eq. (5a) we obtain

$$\Sigma [x_i - (a_0 + a_1 r_i - b_1 s_i)] = 0 \qquad (6a)$$
$$\Sigma \{r_i [x_i - (a_0 + a_1 r_i)] + s_i [y_i - (b_0 + a_1 s_i)]\} = 0 \qquad (6b)$$

Similarly, we get from Eq. (5b)

$$\Sigma [y_i - (b_0 + b_1 r_i + a_1 s_i)] = 0 \qquad (6c)$$
$$\Sigma \{s_i [x_i - (a_0 - b_1 s_i)] + r_i [y_i - (b_0 + b_1 r_i)]\} = 0 \qquad (6d)$$

Then Eq.s (6a), (6b), (6c) and (6d) can be put into the following classical linear system of equations

$$\begin{bmatrix} a_0 \\ a_1 \\ b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} n & \sum r_i & 0 & -\sum s_i \\ \sum r_i & \sum (r_i^2 + s_i^2) & \sum s_i & 0 \\ 0 & \sum s_i & n & \sum r_i \\ -\sum s_i & 0 & \sum r_i & \sum (r_i^2 + s_i^2) \end{bmatrix}^{-1} \begin{bmatrix} \sum x_i \\ \sum (y_i s_i + x_i r_i) \\ \sum y_i \\ \sum (y_i r_i - x_i s_i) \end{bmatrix} \qquad (7)$$

or, more compactly

$$A = M^{-1} X \qquad (8)$$

where A, M, and X are defined appropriately.

The transform error E equals:

$$E = \Sigma x_i^2 + \Sigma y_i^2 - A^t X \tag{9}$$

Parameters for translation (r_0, s_0), rotation (θ), and scale (s) can be determined as follows:

$$\theta = \tan^{-1}(-b_1, a_1) \tag{10a}$$
$$s = a_1 / \cos\theta \tag{10b}$$
$$r_0 = (b_0 \sin\theta - a_0 \cos\theta) / s \tag{10c}$$
$$s_0 = -(a_0 \sin\theta - b_0 \cos\theta) / s \tag{10d}$$

Note that this MSE transform is much more efficient than the MSE transform proposed by Groen et al. [3]. The matrix M in this transform depends only on the points extracted from the model point pattern. Therefore, M^{-1} needs to be evaluated only once for each model point patterns. (A similar type of saving in computation time can be made in the case where a single observed point pattern is compared to many model point patterns.) The computational analysis of this fact will be given in next section.

COMPUTATIONAL ANALYSIS AND CONCLUSION

Consider the MSE transform between n pairs of corresponding points. Then, the MSE transform of Groen et al. [3] needs:

$C_x, ..., C_{ys}$	: 8n multiplications and 12n additions
θ	: 1 multiplication, 6 additions and 1 $\tan^{-1}$
s	: 9 multiplications, 10 additions and 2 sin/cos
E	: 1 multiplication, 7 additions

Therefore, in total, 8n+11 multiplications, 12n+23 additions and 3 trigonometric functions are needed for n pairs of corresponding points.

Our MSE transform needs:

X	: 4n multiplications and 6n additions
$M^{-1}X$	: 16 multiplications and 12 additions
$\Sigma x_i^2 + \Sigma y_i^2$	: 2n multiplications and 2n+1 additions
AX	: 16 multiplications and 12 additions

Therefore, in total, 6n+32 multiplications, 8n+25 additions and no trigonometric functions are needed for n pairs of corresponding points.

The speedup factor of our transform to that of Groen et al. [3] is as follows:

speedup factor for additions $\approx$ (12n +23) / (8n+25)
speedup factor for multiplications $\approx$ (8n+11) / (6n+32)

Currently, the proposed method is being used successfully in other research projects [6, 7]. The computational analysis revealed that the proposed method is faster than that of Groen et al. [3].

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