Analysis of Incremental Relaying Protocol with RCPC in Cooperative Diversity Systems

†Poramate Tarasak, ‡Hlaing Minn, †Yong Hoon Lee
†Department of Electrical Engineering, Korea Advanced Institute of Science and Technology,
Daejeon 305-701, Korea, email: ptarasak@stein.kaist.ac.kr, yohlee@ee.kaist.ac.kr
‡Department of Electrical Engineering, University of Texas at Dallas
Richardson, TX 75083-0688, U.S.A., email: hlaing.minn@utdallas.edu

Abstract— **The incremental relaying protocol in this paper applies the idea of hybrid-ARQ with rate-compatible puncture convolutional code (RCPC) on the cooperative setting. It is shown that the incremental relaying protocol clearly achieves full diversity in the symmetric user case while still has significant performance gain over no-cooperation system in the asymmetric user case. Simulation results of bit-error rate (BER), throughput and percentage of cooperation of the protocol have been confirmed by analytical results which yield further insight to the operation of the protocol.**

I. INTRODUCTION

Relaying protocol is a crucial component to the success of cooperative diversity [1], [2]. Amplify-and-forward relaying and selection decoded-and-forward relaying proposed in [2] are proved to achieve full diversity. Incremental relaying protocol introduced in [2] also achieves full diversity. It exploits feedback information and relaying occurs only when necessary. The relay mobile will forward the relay signal only when it is instructed to do so from the destination. It is shown that this protocol outperforms any other protocols when transmission rate is taken into account [2]. However, most research works have considered the performance simulation and analysis of amplify-and-forward and decode-and-forward protocols [4]–[7]. Little attention is found on the incremental relaying protocol. During the review process of this paper, incremental relaying protocol in a relay network is recently presented in [10] where the problem was approached from an information theoretic perspective.

In this paper, we analyze the incremental relaying protocol with RCPC in a two-user cooperative diversity system. This protocol is a variant of the original one in [2] and can be considered as an extension of hybrid-ARQ with RCPC to the cooperative diversity system. The protocol exploits feedback information from the destination and also incorporates selection relaying protocol based on cyclic redundancy check (CRC). Note that the application of RCPC and CRC on cooperative systems has already been presented in [4], [5]. However, the incremental relaying protocol in this paper generalize existing protocols by allowing more than one relay transmission and exploiting the feedback information from the destination. It is shown that our incremental relaying protocol has several attractive features, e.g.

- A large gain is observed even when the interuser channel SNR is very low.
- Throughput performance is robust to the asymmetric user channels.
- The BER performance is always improved with the higher interuser SNR even when the SNR between user and the destination is low.
- Only a small fraction of resource is needed for relaying when the channels are symmetric and are at high SNRs.

II. SYSTEM MODEL

We consider two cooperative users, denoted as User 1 and User 2, transmitting their messages to the same destination. Each user employs a single transmit antenna and so does the destination. Each message has been appended with CRC bits before entering the convolutional code encoder. The coded bits are punctured according to the puncturing pattern. The output from the puncturing, which is RCPC, is BPSK modulated and is transmitted. We assume that both users share the same frequency and that the user mobile cannot transmit and receive signal at the same time. Each user accesses the channel via time-division multiple access frame. Each frame accommodates a transmission from User 1 or User 2 and doesn't need to have the same size depending on the puncturing pattern being used. The order of transmission at any frame is managed by the relaying protocol via the feedback information from the destination to be described in the next section. The interuser channels and the channels between the users and the destination are independent of each other. All channels experience frequency flat fading and are quasi-static , i.e. they are fixed during one round of incremental relaying protocol and change independently in the next round. (There is no time diversity in relaying or retransmission.) We also assume perfect channel state information and perfect synchronization.

The channel gains are modeled as independent zero-mean complex Gaussian random variables while the additive noise is modeled as independent circularly symmetric zero-mean complex Gaussian random variables. In the interuser channels, the channel gains are assumed to be reciprocal and the noise statistics at User 1 and User 2 are similar. We define $\rho, \gamma_{u1}, \gamma_{u2}$

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to represent the SNRs, which incorporate the effects of path loss and noise, in the interuser channel, the channel between User 1 and the destination, and the channel between User 2 and the destination, respectively. Therefore, $\rho, \gamma_{u1}, \gamma_{u2}$ are exponential random variables.

III. INCREMENTAL RELAYING WITH RCPC

The incremental relaying protocol in this paper applies the idea of hybrid-ARQ to the cooperative setting as well as the idea of selection relaying protocol in [2]. Conventional hybrid-ARQ exploits RCPC by transmitting the punctured bits as incremental redundancy in the retransmission. In contrast, the incremental relaying protocol transmits the incremental redundancy bits via the partner's mobile whenever the partner has a correct message. Next, we briefly describe hybrid-ARQ with RCPC in [8].

A rate of RCPC is $P/(P+l_k)$ where P is the puncturing period corresponding to P trellis intervals and each interval corresponds to one information bit. l_k is the number of additional coded bits in each puncturing period where $k = 1, 2, \ldots, k_{\max}$ denotes the transmission attempt corresponding to each code rate and puncturing pattern. At the first transmission, $N(P + l_1)/P$ coded bits are transmitted where N is the number of message bits in a frame. The destination decodes the message by a Viterbi decoder. If uncorrectable errors are found, the destination will inform the user that retransmission is required. Every time a retransmission is needed, the user increases the value of k and transmits $N(l_k - l_{k-1})/P, k = 2, 3, \ldots, k_{\max}$ punctured bits, that have not been transmitted. Retransmission repeats until the destination receives the message correctly or until $k_{\max}$ (the lowest code rate) is reached.

The situation becomes more complicated in the incremental relaying protocol. The protocol transmits the messages belonging to User 1 and User 2 alternatively in each frame until both messages are received correctly or the lowest code rate is reached. To transmit the message belonging to either user, it is not important who is transmitting. However, the partner will be responsible to transmit the relay message whenever he/she has received correct message and retransmission is needed. If the partner does not have the correct message, the user is responsible to transmit his/her own retransmission signals. In this manner, the protocol encourages cooperation while the message is still in error at the destination and avoids error propagation from the relay transmission.

The incremental relaying protocol operates as follows.

1) Set $k_1 = 1$. User 1 transmits his/her own message which is composed of $N(P + l_{k_1})/P$ bits. Both the destination and User 2 try to decode the message. At the destination, if uncorrectable errors are detected, a relay transmission is requested to User 2. Otherwise, the frame is successfully received and the destination does nothing.
2) Set $k_2 = 1$. In the next frame, User 2 transmits his/her own message which is composed of $N(P + l_{k_2})/P$ bits. Both the destination and User 1 try to decode the message. At the destination, if uncorrectable errors are detected, a relay transmission is requested to User 1. Otherwise, the frame is successfully received and the destination does nothing.
3) If both User 1 and User 2's messages are successfully received at the destination, both users will not receive the relay transmission requests. After some waiting time, both will know that their messages have been received successfully. If k_1 or k_2 reaches maximum value and the message is still in error, the destination will declare an error on this frame and accept decoded information in error.
4) Set $k_1 = k_1 + 1$. If a relay transmission is requested to User 2 and User 2 successfully decodes User 1's message, User 2 performs re-encoding (if have not done) User 1's message and transmits the $N(l_{k_1} - l_{k_1-1})/P$ punctured bits that have not been transmitted (by either User 2 or User 1). If a relay transmission is requested to User 2 but User 2 fails to decode User 1's message, User 2 does nothing. After a specified period of time without transmission, the destination will know that User 2 cannot relay the message. The destination will therefore inform User 1 to transmit the $N(l_{k_1} - l_{k_1-1})/P$ punctured bits that have not been transmitted. Now the destination tries to decode the message with additionally received signals. If uncorrectable error is still found, a relay transmission will be requested to User 2. At the same time, User 2 also tries to decode User 1's message with his/her additional received signals.
5) The same process as Step 4) occurs with the role of User 1 and User 2 switched (as well as the parameter k_1 changed to k_2). Then, go to Step 3).

Note that Step 3), 4) equivalently incorporate selection relaying protocol based on CRC to decide whether relay transmission by the partner is allowed. In this paper, we neglect the waiting time required at the destination and at the users in the performance evaluation. Other approach might be possible in practice, for example, another dedicated channel is used for this purpose. We also assume that feedback channel is perfect and always available to the users and that the rate loss due to feedback information and CRC is not taken into account.

For the protocol to be analytically tractable, we choose a Hagenauer's rate-$1/3$ RCPC as a mother code [8] as well as two puncturing patterns corresponding to the rates 2/3 and 1/2 for both users. Therefore, the maximum number of transmission attempts is $k_{max} = 3$.

IV. PERFORMANCE ANALYSIS

Without loss of generality, the performance of User 1 will be considered. First, let us focus on the conditional pairwise error probability (PEP) of RCPC. We define the following PEP's:

- $P_d^1(\gamma_{u1}) = Q\left(\sqrt{2d_1\gamma_{u1}}\right)$
- $P_d^2(\gamma_{u1}) = Q\left(\sqrt{2(d_1 + d_2)\gamma_{u1}}\right)$
- $P_d^2(\gamma_{u1}, \gamma_{u2}, 2) = Q\left(\sqrt{2(d_1\gamma_{u1} + d_2\gamma_{u2})}\right)$

- $P_d^3(\gamma_{u1}) = Q\left(\sqrt{2(d_1+d_2+d_3)\gamma_{u1}}\right)$
- $P_d^3(\gamma_{u1},\gamma_{u2},2) = Q\left(\sqrt{2(d_1\gamma_{u1}+(d_2+d_3)\gamma_{u2})}\right)$
- $P_d^3(\gamma_{u1},\gamma_{u2},3) = Q\left(\sqrt{2((d_1+d_2)\gamma_{u1}+d_3\gamma_{u2})}\right)$.

$P_d^k(.)$ is a PEP associated with the kth transmission and codeword of weight d. d_k is the weight of the codeword associated with the kth transmission. γ_{u1} can be interchanged with γ_{u2} as appropriate and can be replaced by ρ when the PEP is computed at the User 2 instead of the destination.

Next let us consider the frame error rate (FER) of RCPC which is bounded by [9]

$$P_F^k(.) \leq 1-(1-P_E^k(.))^B, \tag{1}$$

where B is the number of trellis intervals and $P_E^k(.)$ is the conditional error event probability (EEP). The union bound of conditional EEP is given by [8]

$$P_E^k(.) \leq \frac{1}{P}\sum_{d=d_{\text{free}}}^{\infty} a_d P_d^k(.), \tag{2}$$

where a_d is a number of paths at distance d and d_{free} is a free distance. The union bound of conditional BEP is given by [8]

$$P_b^k(.) \leq \frac{1}{P}\sum_{d=d_{\text{free}}}^{\infty} c_d P_d^k(.), \tag{3}$$

where c_d is information error weight of all the weight-d codewords. In the evaluation of (2) and (3), we include only those codewords whose distances are given in Table I in [8]. The arguments and superscripts of $P_F^k(.), P_E^k(.), P_b^k(.)$ follow those of PEP's depending on the kth transmission. In the following analysis, we assume that for given SNR values, all decoding attempts at the destination and User 2 are independent.

A. Bit Error Probability (BEP) Analysis

For an information frame, bit errors occur only after the third (re)transmission has been reached. This requires the destination to fail to decode the first and second transmissions. Three events can occur at User 2.

- User 2 fails to decode User 1's first and second transmissions. No relay transmission from User 2 is allowed. The probability of this event is $P(\#1) = P_F^1(\gamma_{u1}) \cdot P_F^1(\rho) \cdot P_F^2(\gamma_{u1}) \cdot P_F^2(\rho)$.
- User 2 fails to decode User 1's first transmission but decodes User 1's second retransmission successfully. Relay transmission occurs in the third transmission. The probability of this event is $P(\#2) = P_F^1(\gamma_{u1}) \cdot P_F^1(\rho) \cdot P_F^2(\gamma_{u1}) \cdot (1-P_F^2(\rho))$.
- User 2 decodes User 1's first transmission successfully. Relay transmission occurs in the second and third transmissions. The probability of this event is $P(\#3) = P_F^1(\gamma_{u1}) \cdot (1-P_F^1(\rho)) \cdot P_F^2(\gamma_{u1},\gamma_{u2},2)$.

Now the conditional BEP can be found by weighting (3) with the probabilities of the above three events as follows:

$$\begin{aligned} P_b(\gamma_{u1},\gamma_{u2},\rho) &= P_b^3(\gamma_{u1}) \cdot P(\#1) + P_b^3(\gamma_{u1},\gamma_{u2},2) \cdot \\ & \quad P(\#2) + P_b^3(\gamma_{u1},\gamma_{u2},3) \cdot P(\#3). \end{aligned} \tag{4}$$

The unconditional BEP can be found by averaging (4) with the statistics of $\gamma_{u1}, \gamma_{u2}, \rho$. One key difference of the above analysis and those in [3], [5] is the fact that here the averaging over the statistics of SNRs has to be done only after (4) is written since the BEP in (3) is not independent on the events at User 2 as discussed above.

B. Throughput Analysis

We adopt the normalized throughput defined in [8] as

$$R_{av} = \frac{P}{P+l_{av}}, \tag{5}$$

where l_{av} is the average number of additional transmitted bits per P information bits. The averaging is performed over all possible code rates. With the chosen RCPC scheme, we have

$$l_{av} = 4 \cdot P_{l1} + 8 \cdot P_{l2} + 16 \cdot P_{l3}, \tag{6}$$

where

$$\begin{aligned} P_{l1} &= E\left[1-P_F^1(\gamma_{u1})\right] \\ P_{l2} &= E\left[P_F^1(\gamma_{u1}) \cdot \left[(1-P_F^2(\gamma_{u1})) \cdot P_F^1(\rho) + \right.\right. \\ & \quad \left.\left.(1-P_F^2(\gamma_{u1},\gamma_{u2},2)) \cdot (1-P_F^1(\rho))\right]\right] \\ P_{l3} &= 1-P_{l1}-P_{l2}. \end{aligned}$$

The expectation is taken over the SNR statistics. Substituting the above probabilities into (6), we can obtain the (normalized) throughput from (5).

C. Percentage of Cooperation Analysis

The percentage of cooperation is defined as the ratio between channel utilization in relaying the partner's message and the total channel resource given to the user. If the percentage of cooperation is high, it implies that the user spends much of his resource to forward the relay signals.

Let us consider channel utilization of User 1 during P trellis intervals. 12 bits have to be sent for the first transmission. 4 incremental bits are possibly sent from either user for the second transmission. 8 incremental bits are possibly sent from either user for the third transmission. Therefore, the percentage of cooperation can be computed as

$$\%\text{coop} = \frac{4 \cdot Q_{u2}^2 + 8 \cdot Q_{u2}^3}{12 + 4 \cdot (Q_{u1}^2 + Q_{u2}^2) + 8 \cdot (Q_{u1}^3 + Q_{u2}^3)} \tag{7}$$

where

$$\begin{aligned} Q_{u1}^2 &= E\left[P_F^1(\gamma_{u1}) \cdot P_F^1(\rho)\right] \\ Q_{u1}^3 &= E\left[P_F^1(\gamma_{u1}) \cdot P_F^1(\rho) \cdot P_F^2(\gamma_{u1}) \cdot P_F^2(\rho)\right] \\ Q_{u2}^2 &= E\left[P_F^1(\gamma_{u2}) \cdot (1-P_F^1(\rho))\right] \\ Q_{u2}^3 &= E\left[P_F^1(\gamma_{u2}) \cdot \left[(1-P_F^1(\rho)) \cdot P_F^2(\gamma_{u2},\gamma_{u1},2)\right.\right. \\ & \quad \left.\left.+P_F^1(\rho) \cdot P_F^2(\gamma_{u2}) \cdot (1-P_F^2(\rho))\right]\right], \end{aligned}$$

and the expectation is taken over the SNR statistics.

Note that the BEP, throughput and percentage of cooperation in this section have no closed-form solution. Therefore, we resort to the numerical integration in order to obtain the analytical results. In addition, the limit-before-average approach [9] is applied in the integration similar to [3].

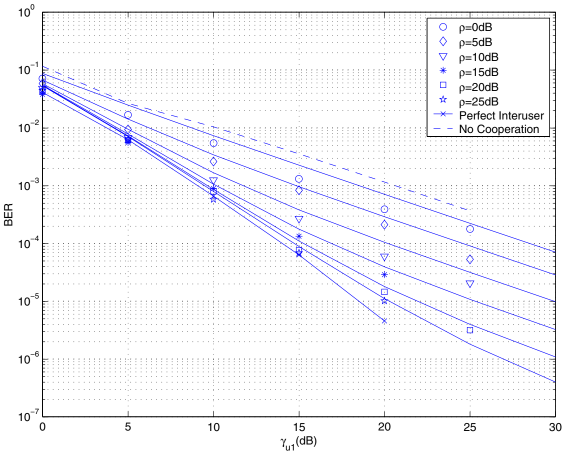

Fig. 1. BER of the incremental relaying protocol, $\gamma_{u1} = \gamma_{u2}$

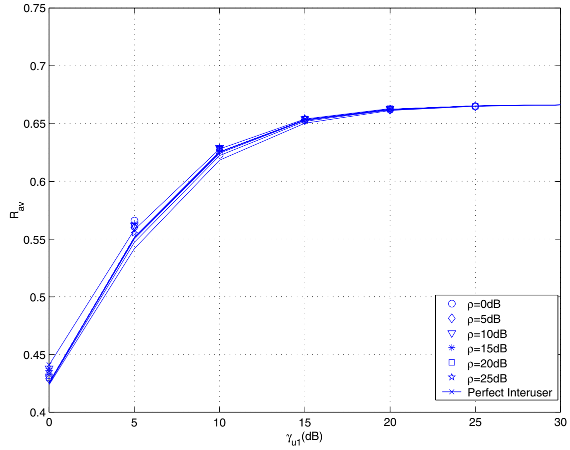

Fig. 2. Throughput of the incremental relaying protocol, $\gamma_{u1} = \gamma_{u2}$

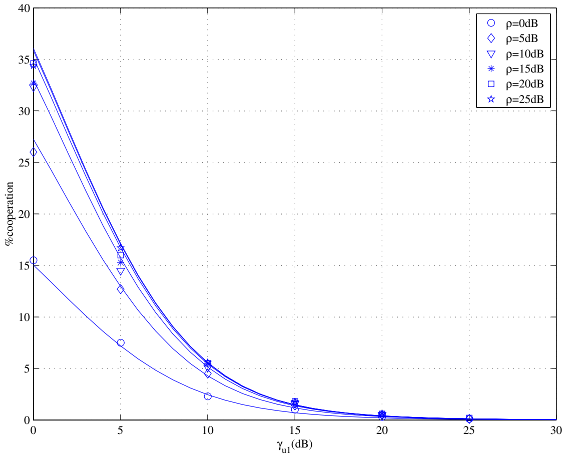

Fig. 3. % of cooperation of the incremental relaying protocol, $\gamma_{u1} = \gamma_{u2}$

V. RESULTS AND DISCUSSION

The information frame length is 128 bits which yields a 384-bit unpunctured coded frame. The analytical results of the cooperative schemes are shown as solid lines (without markers) while the simulation results are shown as markers.

A. *Symmetric User SNRs*

Due to symmetry, User 1 and User 2 have the same average performance. Fig. 1 shows the BER at various ρ's. Included in Fig. 1 are the performance of the same three-rate RCPC code in an incremental redundancy approach is applied in a no-cooperation system and the performance when the users have perfect knowledge of each other's message (perfect interuser). The perfect interuser case serves as a lower bound on the BER performance of the relaying protocol.

From the simulation results in Fig. 1, the slope of the perfect interuser indicates diversity of level two. As ρ increases, the performance curves bend toward the perfect interuser case. Compared with the no-cooperation, all cooperation cases have significant performance gains. Even at $\rho = 0$dB, the incremental relaying protocol achieves about 3-dB gain over the no-cooperation case. At $\rho = 25$dB, the performance closely approaches the perfect interuser case. Thus, the protocol enjoys full diversity gain at high ρ. The analytical results for $\rho = 0$ to 25dB are shown from the top solid line to the bottom one. It is observed that the analytical results agree with the simulation results well especially at high ρ and γ.

Fig. 2 shows throughputs of the protocol at various ρ's. For all ρ's, the throughput is a monotonic increasing function of γ_{u1}. At high γ_{u1}, the throughput reaches 2/3, which is the maximum possible value of this RCPC scheme. Note that the existing protocols [4], [5] would have their throughput fixed at 1/3 with this RCPC scheme since relay transmission is always done. The effects of different ρ's seem to be smaller as γ_{u1} increases.

Fig. 3 shows the percentage of cooperation versus SNR. Unlike other existing relaying protocols, the incremental relaying protocol has varying percentage of cooperation. As γ_{u1} increases, the percentage of cooperation is less while the BER performance still achieves diversity of level two for high ρ. This means that at high γ_{u1}, only a very small percentage of resource is needed to achieve diversity. This small percentage of cooperation should *not* be interpreted as a no-cooperation system since here the users are willing to cooperate at all time but do so only when they are requested. Therefore, only a very small fraction of resource of each individual user is used for relaying signals. This results in saving lots of (re)transmissions and gaining in throughput at high γ_{u1} as shown in Fig. 2. Again, the analytical results match very well with the simulation results.

B. *Asymmetric User SNRs*

In the asymmetric case, γ_{u2} is fixed at 10 dB. The analytical results of User 1 are shown as solid lines while those of User 2 are shown as dash-dot lines. The simulation results of User 1 are shown as empty markers while those of User 2 are shown as filled markers. As shown in Fig. 4, the BER curves of both User 1 and User 2 show performance gain over no-cooperation system in all cases. As ρ increases, further improvement

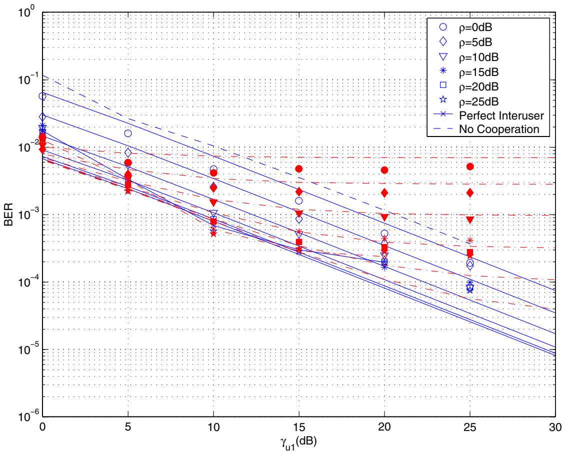

Fig. 4. BER of the incremental relaying protocol, γ_{u2} = 10dB

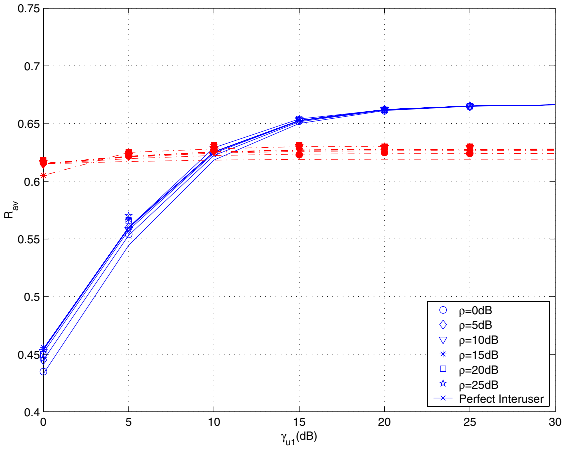

Fig. 5. Throughput of the incremental relaying protocol, γ_{u2} = 10dB

of User 1 over no-cooperation is observed although at high γ_{u1} the performance seems to be limited by the lower value of γ_{u2}. In contrast, the performance of User 2 is improved remarkably as ρ is larger. The performance of User 2 seems to be unaffected by γ_{u1} at high value of γ_{u1}. The analytical BER curves of User 1 at high γ_{u1} are lower than simulation results. This is attributed to the inaccuracy of union bounds at low γ_{u2} value. Nevertheless, the analytical and simulation results show similar behaviours.

The User 1's throughput in Fig. 5 has similar performance as that of the symmetric case. This means that the throughput performance of the incremental relaying protocol is robust to the asymmetric user SNRs. The throughput of each user does not change with the partner destination SNR.

The percentage of cooperation of the asymmetric case is shown in Fig. 6. Unlike the symmetric case, the User 1's percentage of cooperation does not converge to a very small value at high γ_{u1}. This is because User 1 still has to transmit the relaying message for User 2 whose γ_{u2} is lower. The analytical results match well and illustrate the same behaviour with the simulation results in most cases.

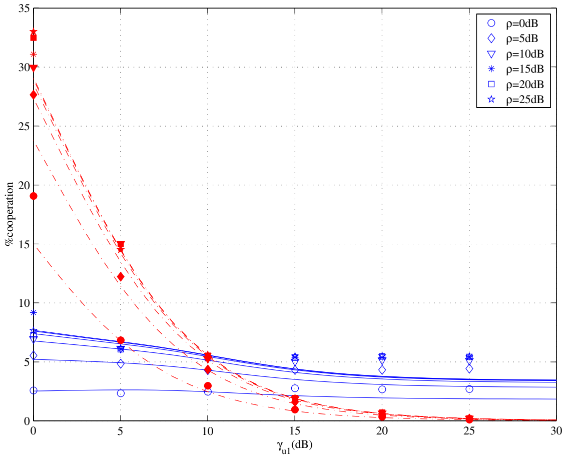

Fig. 6. % of cooperation of the incremental relaying protocol, γ_{u2} = 10dB

VI. CONCLUSIONS

An incremental relaying protocol with RCPC is shown to achieve full diversity and significant performance gain over no-cooperation system especially for the symmetric case. The protocol also has other attractive features such as throughput robustness to asymmetric user channels and large performance gain even when the interuser SNR is very low. The study on percentage of cooperation indicates that a very small fraction of resource is needed for relaying at high SNR in the symmetric case. The protocol is beneficial especially when time diversity is not available and can be easily extended to more than two cooperative users.

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