Reduced Complexity Sequential Sequence Detection Using Modified Fano algorithm for V-BLAST Systems

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***Abstract*— Layered space-time architectures employed in BLAST system can provide remarkable capacity using multiple antennas in Rayleigh fading environments. In this paper, we propose a sub-optimal but computationally efficient Modified Fano Detection algorithm (MFD). This algorithm utilizes the QR decomposition of the channel matrix and the sequential detection scheme based on tree searching in order to find the optimal symbol sequence. For more reliable signal detection, the decoder makes intentional backward movement at the stage one step greater than the end of tree. Simulation results show that the performance of MFD is comparable to that of ML detector with reduced complexity. The computation time of MFD is about 45 times shorter than that of ML even at low SNR region.**

keywords-MIMO, V-BLAST, Sphere Decoding, Fano Algorithm

I. INTRODUCTION

In an independent Rayleigh scattering environment, multiple antenna systems provide enormous increase in spectral efficiency compared to single antenna systems [1]. To take advantage of the multiple antenna systems, the V-BLAST (*Vertical Bell Labs Layered Space-Time*) architecture was proposed in [2]. For the detection of the vertically layered codes at a receiver, a variety of detection algorithms, such as cancellation based zero-forcing (ZF) and minimum mean square error (MMSE) criterion with optimal ordering, ZF or MMSE Sorted QR Decomposition (ZF or MMSE SQRD) algorithm, and so on, were introduced [2]-[5]. Though these schemes provide a reasonable performance at a feasible complexity, these performances may not be sufficient to apply to systems with higher-order modulation like 16QAM and 64QAM.

The *joint*-Maximum Likelihood (JML) or *individual*-ML (IML) detector for Multi-Input Multi-Output (MIMO) systems [6] is known to be optimal with perfect Channel State Information (CSI) at the receiver. The performance of JML or IML is far superior to the aforementioned sub-optimal detection schemes. However, when a large number of antennas are used together with higher modulation constellations, ML detection is not feasible for real-time implementations. Therefore, we must seek for ML based approach with reduced complexity in order to apply to user terminals for the next generation systems.

Recently, sphere decoding (SD) has been getting attention by many researchers investigating MIMO detection algorithms. A variety of approaches regarding SD have been proposed in [8]-[11] to avoid the exponential complexity of ML detector after the concept of SD was introduced in [7].

In this paper, we propose a sub-optimal but computationally efficient modified Fano detection algorithm in V-BLAST system, which yields the performance close to that of ML with reduced complexity. The Fano algorithm, one of the various sequential decoding algorithms for decoding convolutional codes, was originally proposed by Wozencraft [12] and subsequently modified by Fano [13], [14]. Sequential decoding can be viewed as a trial-and-error technique for searching out the correct path in the code tree. It also achieves asymptotically the same error probability as ML decoding without searching all possible states. The important characteristic of the proposed MFD is an intentional backward movement at the stage one step greater than the end of tree.

This paper is organized as follows. In Section II, we describe the system model. Proposed V-BLAST detection scheme using modified fano algorithm is investigated in Section III. We present the simulation results regarding performance and computational complexity in Section IV and finally make conclusions in Section V.

II. SYSTEM DESCRIPTION

We consider a multiple antenna system[1] with m transmit and $n(\geq m)$ receive antennas, as depicted in Fig. 1. At the transmitter, input data stream $\{b(n)\}$ is demultiplexed into m data substreams. These substreams are mapped into $M-$PSK or $M-$QAM symbols $\mathbf{s}(n)$ and transmitted over the m antennas at the same time.

At the receiver, the baseband equivalent model of received signal vector at sampling time $\mathbf{y} = [y_1\, y_2\, \cdots\, y_n]^T$ is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{s} = [s_1\, s_2\, \cdots\, s_m]^T$ denotes the vector of transmitted symbols and $\mathbf{w} = [w_1\, w_2\, \cdots\, w_n]^T$ represents the zero-mean

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[1] Throughout this paper, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote matrix transpose, Hermitian transpose, and conjugate, respectively.

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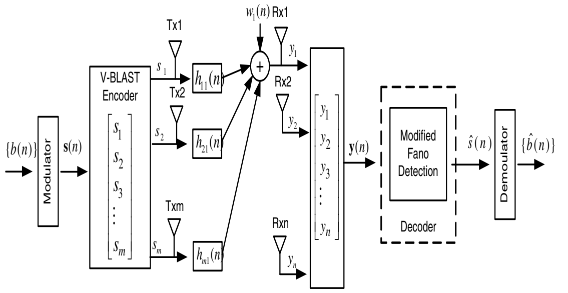

Fig. 1. System Model for V-BLAST system with (m, n) transceiver

complex additive white Gaussian noise of variance σ_w^2 while the average transmit power of each antenna is normalized to one. The $n \times m$ channel matrix $\mathbf{H}$ contains uncorrelated complex Gaussian fading gains with unit variance and changes independently from frame to frame. We assume that the independent fading gains are perfectly known by the receiver.

Using QR decomposition of the channel matrix $\mathbf{H}$, the received signal vector $\mathbf{y}$ in (1) is rewritten as

$$\mathbf{y} = \mathbf{QRs} + \mathbf{w}, \tag{2}$$

where the $n \times m$ matrix $\mathbf{Q}$ is an unitary matrix and the $m \times m$ matrix $\mathbf{R}$ is an upper triangular matrix. By multiplying the received signal $\mathbf{y}$ with $\mathbf{Q}^H$, the sufficient statistic for the transmit vector $\mathbf{s}$ is obtained as

$$\tilde{\mathbf{y}} = \mathbf{Rs} + \boldsymbol{\eta}, \tag{3}$$

where the statistical properties of the noise term $\boldsymbol{\eta} = \mathbf{Q}^H\mathbf{w}$ remain unchanged.

In order to detect the transmitted symbol sequence, ML detector is optimal but it requires an intensive computational efforts. Therefore, we propose sub-optimal detection algorithm for reaching near ML performance with reduced complexity in the following sections.

III. PROPOSED V-BLAST DETECTION ALGORITHM

This section develops the computationally efficient V-BLAST detector using the modified Fano algorithm. As mentioned previously, the Fano algorithm was originally introduced as a sequential decoding scheme for convolutional codes to avoid the exponential complexity of a Viterbi algorithm (VA) according to the constraint length. While the VA explores all possible paths to find the optimal symbol sequence, the Fano decoding algorithm only does the most promising paths. The decoder works by generating hypotheses about the transmitted codeword sequence and computes a metric between these hypotheses and the received signal. It goes forward or backward repeatedly in tree searching, so that it finds the most likely path. In the following subsections, we give a full detail of the proposed detector.

A. Fano-Like Metric Bias

In order to employ Fano algorithm in MIMO systems, it is necessary to adjust a metric on symbol based operation for the purpose of taking into account the lengths of the different paths being compared. This metric throughout this paper is called a Fano-like metric bias [8]. Note that from (1) the average value of the smallest path is

$$E\{\|\mathbf{y} - \mathbf{Hs}\|^2\} = E\{\|\mathbf{w}\|^2\} = n\sigma_w^2. \tag{4}$$

Therefore, the Fano-like metric bias for the tree level k can be defined as follows

$$F_k = F_{k-1} + \alpha\sigma_w^2 r_{k-1,k-1} \ (k = 2, 3, \cdots, m), \tag{5}$$

where $F_1 \triangleq 0$ and $0 \leqslant \alpha \leqslant 1$. The value of α is simply set to be 0.5 in our simulations, and also it can be determined by simulations

B. Modified Fano Algorithm Description

In a M-ary tree structure to apply the modified Fano algorithm to the V-BLAST system in Section II, there are m^k branches leaving each node in the level k $(k = 1, 2, \cdots, m)$. The decoding process of the modified version begins at the root node and terminates if one of two conditions is satisfied. The original Fano decoding process, on the other hand, terminates only when the decoder reaches the end of the tree (i.e. level m). One condition to improve the BER performance is that we introduce a scheme which increases the number of backward movement intentionally in the level m+1. We denote this parameter as N_{bmi}. The other is that we set the repeated number of the accumulated minimum metric at the level m. We also denote this parameter as N_{rnmm}. From the root node with a threshold $T = 10$ and the metric value $M = 0$, the decoder looks forward to the best of the m^k succeeding nodes at the tree level k. The best node is determined by the biased cumulative path metric which is based on the Euclidean distance.

Due to the upper triangular structure of $\mathbf{R}$ in (3), the k-th element of $\tilde{\mathbf{y}}$ is rewritten as

$$\tilde{y}_k = r_{k,k}y_k + \sum_{i=k+1}^{M} r_{k,i}y_i + \eta_k. \tag{6}$$

Using the Fano-like bias in (5), the l-th biased branch metric at the tree level k is computed as follows

$$BM_{lk} = |\tilde{y}_k - c_l|^2 + F_k \ (k = 2, \cdots, m, l = 1, \cdots, M), \tag{7}$$

where $|\cdot|$ denotes the Euclidean distance and c_l does an element of the signal constellation. Among all the branch metrics calculated at each level, the node having the minimum branch metric corresponds to the best one. We assume that all the branch metrics are zeros if the decoder gets to the level $m + 1$.

Let M_F and M_B represent the metrics of the forward and backward node being examined, respectively [14]. If $M_F \leq T$, the decoder move forward to the best of the m^k succeeding nodes. Furthermore, if this node is visited for the first time and one of two conditions is not satisfied, a threshold tightening is performed. On the other hand, if $M_F > T$, the decoder looks backward to its ancestor node at the tree level $k - 1$

and checks whether M_B is less than or equal to T. If it is greater than T, then T is increased by $\triangle$ and the look forward to the best node step is repeated. If $M_B \leq T$, the decoder moves back to the preceding node. For convenience, we call this node A. If this backward movement was from the worst of the m^k nodes succeeding node A, the decoder again looks back to the ancestor node. If not, the decoder looks forward to the next best of the m^k nodes succeeding node A and checks if $M_F \leq T$. Especially, in case that the decoder ever looks backward from the origin node, we assume that the metric value of the preceding node is ∞.

Through this decoding process, if the decoder reaches at the tree level k, it computes the following metric in [6],

$$-2\text{Re}\{\mathbf{s}^*\mathbf{H}^*\mathbf{y}\} + \text{Re}\{\mathbf{s}^*\mathbf{H}^*\mathbf{H}\mathbf{s}\}, \tag{8}$$

during the limited N_{bmi} (i.e., 200 in our case), the minimum value of these metrics would be repeated. Finally, among the candidate symbol sequences, the decoder finds the optimal symbol sequence which has the minimum metric calculated in (8).

The proposed modified Fano algorithm for the V-BLAST detector can be summarized in the following lists, where l_k indicates that the decoder is located at the tree level k, $LFBN = 0$ means that it looks forward to the best node, and $WN = 1$ represents that the backward movement is from the worst of the m^k nodes succeeding node A.

```
$T$ =10, $M$ =0, $k = 0$
$\triangle$= 3, $N_{\text{bmi}} = 0$
while $l_k < n_T + 1$
  k=k+1;
  if $l_k == n_T + 1$
    $N_{\text{bmi}} = N_{\text{bmi}} + 1$;
  end if
  if $N_{\text{bmi}} == 200$
    break;
  end if
  if $LFBN == 0$
    if $l_k <= n_T$
      compute branch metric ($BM$) from (7)
    else
      $BM = 0$;
      $T = -100$;
    end if
    if $MF <= T$
      move forward
      if $l_k == n_T$
        find optimal symbol sequence from (8)
        if $N_{\text{rnmm}} == 3$
          break;
        end if
      end if
      if first visit
        $T = MF$;
        $LFBN = 0$;
      end if
    else
      $WN = 1$;
      while $WN == 1$
        if $MB <= T$
          move backward
          $M = MB$;
          if from worst node
            $LFBN = 1$;
            $WN = 0$;
          else
            $WN = 1$;
          end if
        else
          $T = T + \triangle$;
          $LFBN = 0$;
        end if
      end while
    end if
end while
```

IV. SIMULATION RESULTS

Monte Carlo simulations are used to compare the proposed V-BLAST detector with ML and MMSE-OSIC (Ordered Successive Interference Cancellation) detectors in terms of the the average bit error rates (BER) performance and the corresponding computational complexity. The parameters in our simulations are shown in TABLE I.

TABLE I
SIMULATION PARAMETERS

Antenna configuration	4×4
Modulation	16QAM
Channel model	Rayleigh flat fading
Channel estimation	Perfect
N_{bmi}	200
N_{rnmm}	1, 2, 3

A. Performance Comparison

The average bit energy to noise power ratio (E_b/N_o) is the SNR at the receiver normalized by the number of bits per symbol, thus $E_b/N_o = m/(\log_2(M)\sigma_w^2)$ holds. Fig. 2 shows the result for the proposed V-BLAST detection algorithm and other schemes. The performance of proposed algorithm approaches the BER of ML detection as the repeated number of minimum metric (N_{rnmm}) increases and also outperforms MMSE-OSIC even in case of $N_{\text{rnmm}} = 1$.

B. Complexity Comparison

This system is simulated using MATLAB V6.5. The MATLAB command "etime" is used to compare the computation time of each algorithm. It returns the time in seconds that has elapsed between the start and the end point of the desired algorithm. These results would be presented as a measure of the computational complexity. TABLE II and Fig. 3 show the computation time for each approaches. Even though the SNR at the receiver is 0dB and the parameter N_{rnmm} is 3,

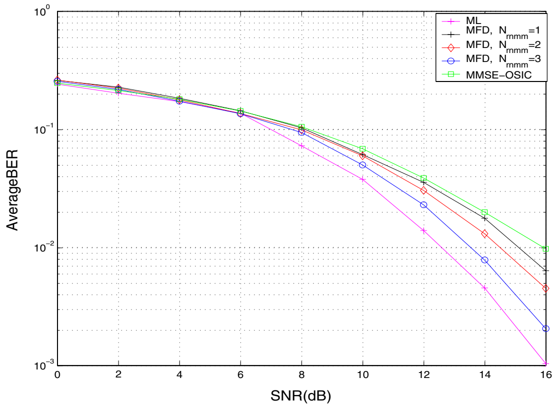

Fig. 2. Performance comparison between MFD and other schemes with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

the computation time of the proposed MFD for the V-BLAST system is about 45 times less than ML detector with achieving near ML performance. Furthermore, its computation time is about 302 times shorter at high SNR region.

TABLE II

COMPUTATION TIME OF VARIOUS SCHEMES [S]

Detection Scheme	SNR			
	0dB	4dB	12dB	16dB
ML	15.5448	16.2807	15.5550	15.9628
MFD $N_{\mathrm{rnmm}} = 1$	0.2840	0.0442	0.0245	0.0168
MFD $N_{\mathrm{rnmm}} = 2$	0.3200	0.0617	0.0442	0.0345
MFD $N_{\mathrm{rnmm}} = 3$	0.3478	0.0705	0.0601	0.0529
MMSE-OSIC	0.0016	0.0010	0.0010	0.0010

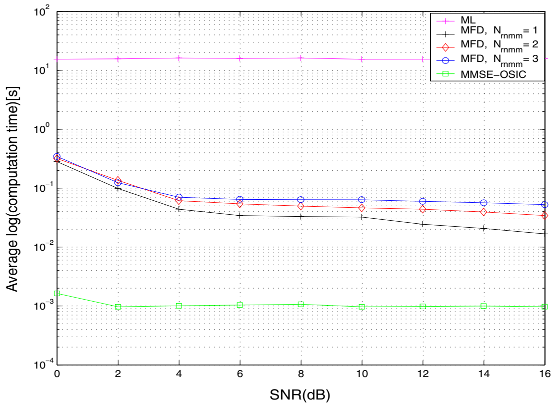

Fig. 3. Comparison of computation time between MFD and other schemes with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

As explained in Section III, the proposed decoder moves forward or backward repeatedly until it finds optimal symbol sequence. To show the tendency of computation efforts according to various N_{rnmm} values, the average number of decoding steps for the proposed V-BLAST detector is plotted in Fig. 4. From this result, we observe that the number of average decoding steps decrease dramatically as the SNR increase. We also present the average number of decoding steps at each tree level in Fig. 5 through Fig. 8.

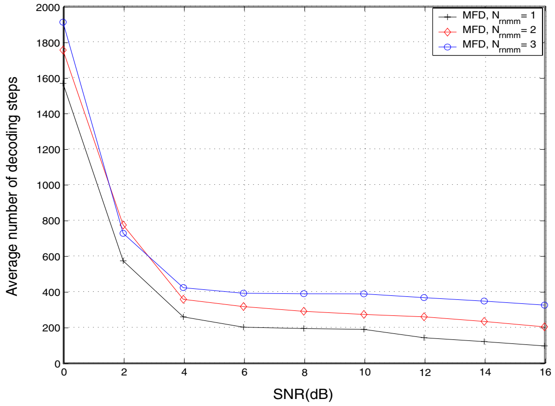

Fig. 4. The average number of decoding steps for MFD with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

C. Complexity and Performance Tradeoff

In this subsection, we also consider the proposed MFD together with ML and MMSE-OSIC detection schemes in terms of performance and complexity tradeoffs. As mentioned previously, for more reliable signal detection, we add two parameters, N_{bmi} and N_{rnmm}, in our approach. As N_{bmi} and N_{rnmm} increase, the BER performance of MFD is close to that of ML and outperforms MMSE-OSIC. However, the improvement is achieved at the cost of the increase of the computation time. Nevertheless, the MFD has much less computationally complexity compared to the ML detection.

V. CONCLUSIONS

We present a sub-optimal, reduced complexity MFD algorithm well-suited to MIMO systems with layered space-time architecture. The MFD is based on sequential sequence detection scheme using tree searching with backward movement on purpose. The results show that although there is a tradeoff between performance and complexity, the performance of MFD detection algorithm is improved significantly with much less computational complexity than that of ML detector. Hence, in many cases of interest, the proposed algorithm is a good candidate detection scheme for the V-BLAST system. We also expect that our proposed algorithm can be applied to a variety of future mobile terminals as well as other MIMO schemes in high traffic demanding environments.

VI. ACKNOWLEDGEMENT

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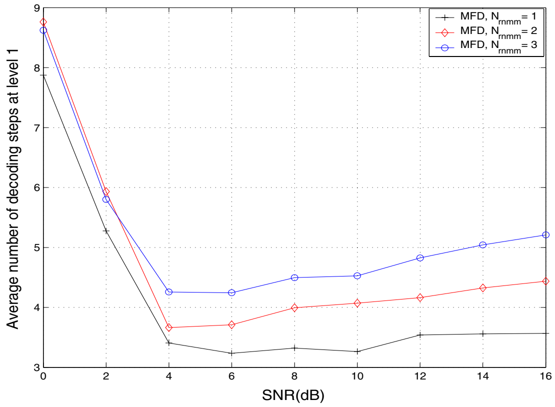

Fig. 5. The average number of decoding steps at level 1 for MFD with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

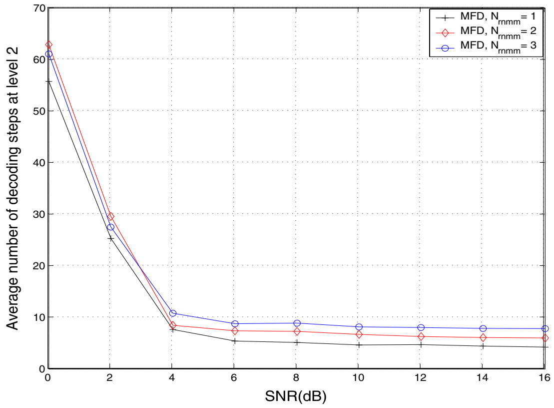

Fig. 6. The average number of decoding steps at level 2 for MFD with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

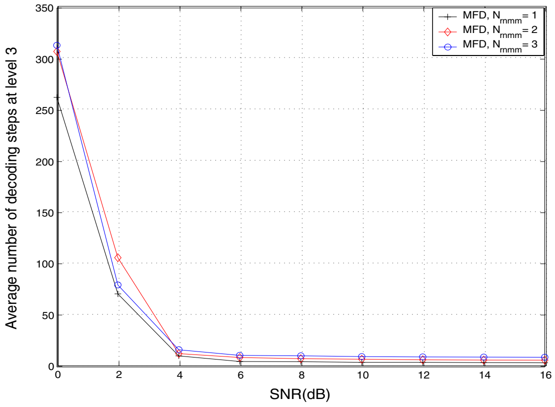

Fig. 7. The average number of decoding steps at level 3 for MFD with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz:

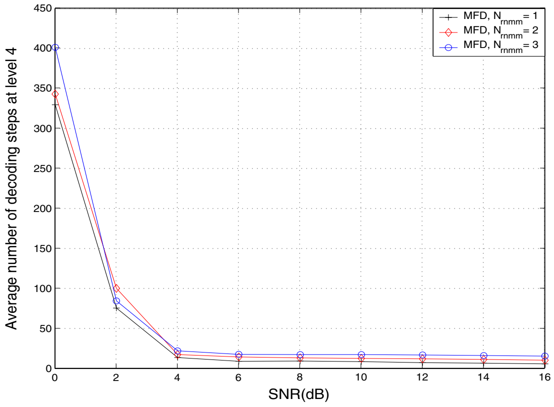

Fig. 8. The average number of decoding steps at level 4 for MFD with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16bps/Hz: