

The Performance of FHSS BFSK Systems with Limiter-Squarer Detectors in Impulsive Noise Environment

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Abstract - In this paper, the noncoherent reception performance of frequency hopping spread spectrum communication systems operated in channels with impulsive noise and selective fading is investigated. Binary frequency shift keying modulation and noncoherent demodulation are assumed and limiter-squarer detectors are used to detect signals. The bit error rates are obtained as functions of channel and system parameters, in the multi-hopping and multi-user synchronous multiple access case.

1. INTRODUCTION

The performance of FHSS communication systems in Gaussian noise multipath fading channels has been investigated by several authors [e.g., 1, 2]. In [1], the fast FHSS (FFHSS) communication system with binary FSK (BFSK) modulation was investigated. The channel model under consideration was a noisy multipath channel with very slow fading. In [2], assuming that the channel is jammed by intentional jammer whose jamming power resource is Gaussian noise, numerical results of error rates are obtained as a function of signal-to-jamming power ratio.

It is well-known that in some cases the Gaussian noise assumption cannot be entirely justified. For example, the non-Gaussian nature of atmospheric noise is clearly shown in [3]. In several studies the effects and implications of non-Gaussian impulsive man-made noise have also been discussed [4]. Models for impulsive noise can either be empirical or physically motivated for man-made noise. The non-Gaussian nature of certain noise has important implication for receiver design and evaluation of system performance. For instance, the ϵ -contaminated mixture noise model was used in the de-

tection of signals in non-Gaussian noise [5].

In this paper, when a signal is transmitted by the FFHSS-BFSK communication system through selective Rayleigh fading channels with impulsive noise, we will investigate the performance of the limiter-squarer (LS) detector. It is well-known in signal detection theory that the structure of a matched filter followed by an envelope detector is not optimum in non-Gaussian noise environment: the LS detector which is known as a robust detector is instead commonly used in impulsive noise environment.

2. SYSTEM MODEL

2.1 THE TRANSMITTER AND CHANNEL

The transmitted signal is given by [2]

$$S(t) = A \cos[2\pi(f_c + f_l + a_k \Delta)t + \theta_l],$$
$$\left[\frac{k+(l-1)}{L}\right]T \leq t < \left(\frac{k+l}{L}\right)T, 1 \leq l \leq L, \quad (1)$$

where $A = \sqrt{2E_b/T}$ is the amplitude, f_c is the carrier frequency, f_l is the l -th hopping frequency in the interval $[k + (l-1)/L]T \leq t < (k+l)/L]T$, a_k is a rectangular pulse of duration T with $Pr\{a_k = -1\} = Pr\{a_k = +1\} = 1/2$, Δ is one-half the spacing between two FSK tones and satisfies $\Delta = \frac{j}{2T}$ for some integer j [2], the phase angle θ_l is uniformly distributed between 0 and 2π , E_b is the energy per bit, and L is the number of chips per one data bit. For each time interval, the hopping frequency f_l takes on one value from the set $H = \{F_0, F_0 + C/T, F_0 + 2C/T, \dots, F_0 + (K-1)C/T\}$, where $F_0 \gg (K-1)C/T$, C is a positive integer, and K denotes the number of frequencies used in hopping or the maximum number of users.

The transmitted signal is assumed to be propagated

through a noisy multipath fading channel. The noise is modeled by the ϵ -contaminated mixture noise model [5], for which the pdf is

$$f(x) = (1 - \epsilon)f_N(x) + \epsilon f_T(x). \quad (2)$$

In (2) f_N is a Gaussian pdf with mean zero and variance σ_N^2 , and f_T is a zero mean pdf which has a variance σ_T^2 larger than σ_N^2 . In this paper, we will assume that f_T is also a Gaussian pdf in order to obtain explicit expressions of the pdfs of detector outputs.

Let the delay power spectrum of the channel be $\phi_c(\tau)$, which is defined as the autocorrelation function of the impulse response of the channel. Then the range of the values of τ over which $\phi_c(\tau)$ is essentially nonzero is called the multipath time delay spread or multipath spread of the channel and is denoted by τ_{max} . In this paper, we investigate the performance of the LS receiver in the selective fading channels, for which $0 < \tau_{max} \leq T$.

2.2 THE RECEIVED SIGNAL AND RECEIVER

The received signal can be written as, for $[k + (l - 1)/L]T \leq t < (k + l/L)T$,

$$R(t) = A \sum_{m=0}^{M-1} r_m \cos[2\pi(f_c + f_l + a_k \Delta)(t - \tau_m) + \psi_m] + N(t), \quad (3)$$

where the multipath strength r_m has the Rayleigh pdf

$$P_{r_m}(r) = \frac{2r}{b_m} \exp\left\{-\frac{r^2}{b_m}\right\}, \quad r \geq 0, \quad (4)$$

the path delay τ_m of the m th multipath signal is relative to the time reference $\tau_0 = 0$, and the random phase ψ_m is uniformly distributed over $[0, 2\pi]$. It is assumed that r_m , τ_m , and ψ_m , $m = 0, 1, \dots, M-1$, are statistically independent of each other. The white noise $N(t)$ is assumed to be zero-mean and statistically independent of the random variables resulting from the multipath phenomenon, r_m , τ_m , and ψ_m . It is also assumed that the random variables in $S(t)$ and $R(t)$ are independent of each other.

The receiver shown in Fig. 1 has two branches each with in-phase and quadrature subbranches. In no-noise situation, the upper and lower branch outputs are A^2 when the transmitted data bit is "+1" and "-1", respectively. Let the outputs of the upper and lower branches for $[n + (l - 1)/L]T \leq t < (n + l/L)T$ be $X(n, l)$ and $Y(n, l)$, respectively. Then, we have

$$X(n, l) = \begin{cases} X_i^2(n, l) + X_q^2(n, l), & \text{if } |X_i(n, l)| \leq a, |X_q(n, l)| \leq a; \\ X_i^2(n, l) + a^2, & \text{if } |X_i(n, l)| \leq a, |X_q(n, l)| > a; \\ a^2 + X_q^2(n, l), & \text{if } |X_i(n, l)| > a, |X_q(n, l)| \leq a; \\ 2a^2, & \text{if } |X_i(n, l)| > a, |X_q(n, l)| > a; \end{cases} \quad (5)$$

with a similar expression for $Y(n, l)$, where $X_i(n, l)$ and $Y_i(n, l)$ are the outputs of the in-phase branches, and the $X_q(n, l)$ and $Y_q(n, l)$ are the outputs of the quadrature branches. Denoting the dehopping signal by

$$D(t) = 2 \cos(2\pi f_l t + \varphi_l),$$

$$\left[n + \frac{l-1}{L}\right]T \leq t < \left(n + \frac{l}{L}\right)T, \quad (6)$$

where φ_l is uniformly distributed over $[0, 2\pi]$, $X_i(n, l)$ and $X_q(n, l)$ are given by

$$X_i(n, l) = \frac{2}{T} \int_{[n + \frac{l-1}{L}]T}^{[n + \frac{l}{L}]T} R(t)D(t) \cos[2\pi(f_c + \Delta)t] dt \quad (7)$$

and

$$X_q(n, l) = \frac{2}{T} \int_{[n + \frac{l-1}{L}]T}^{[n + \frac{l}{L}]T} R(t)D(t) \sin[2\pi(f_c + \Delta)t] dt, \quad (8)$$

with similar expressions for $Y_i(n, l)$ and $Y_q(n, l)$.

From (7), it is easy to see that $X_i(n, l)$ can be written as

$$X_i(n, l) = S(n, l) + I(n, l) + N(n, l). \quad (9)$$

Note that the desired main path component $S(n, l)$ is a zero-mean Gaussian random variable, since it is the product of a Rayleigh random variable and the cosine of a uniform random variable. Using the central limit theorem, the interference component $I(n, l)$ can be approximated as a zero-mean Gaussian random variable with variance α^2 , since it is the sum of independent random variables [2]. The noise component $N(n, l)$ in (9) is modeled as a random variable with pdf given by (2).

3. STATISTICAL CHARACTERISTICS OF DETECTOR OUTPUTS AND PERFORMANCE ANALYSIS

3.1 THE LS RECEIVER OUTPUT

The pdf of $X(n, l)$ can be shown to be

$$f_{X(n, l)}(x) = \frac{(1 - \epsilon)^2}{2\alpha_N^2} \exp\left\{-\frac{x}{2\alpha_N^2}\right\} + \frac{\epsilon^2}{2\alpha_T^2} \exp\left\{-\frac{x}{2\alpha_T^2}\right\} + \frac{\epsilon(1 - \epsilon)}{\alpha_N \alpha_T} \exp\left\{-\frac{x}{2\alpha_S^2}\right\} I_0\left(-\frac{x}{2\alpha_D^2}\right) \quad (10)$$

for $0 < x \leq a^2$, where α_N , α_T , α_S , and α_D depend on channel characteristics [7] and $I_0(\cdot)$ is the modified Bessel function,

$$f_{X(n, l)}(x) = \frac{(1 - \epsilon)^2}{2\alpha_N^2} \exp\left\{-\frac{x}{2\alpha_N^2}\right\}$$

$$\begin{aligned}
& \left[1 + \frac{4}{\pi} \left\{ \frac{\gamma_{NN}}{\sqrt{x-a^2}} - \theta_I(x) \right\} \right] \\
& + \frac{\epsilon^2}{2\alpha_T^2} \exp \left\{ -\frac{x}{2\alpha_T^2} \right\} \\
& \left[1 + \frac{4}{\pi} \left\{ \frac{\gamma_{TT}}{\sqrt{x-a^2}} - \theta_I(x) \right\} \right] \\
& + \frac{\epsilon(1-\epsilon)}{\alpha_N\alpha_T} \left[\exp \left\{ -\frac{x}{2\alpha_S^2} \right\} I_0 \left(-\frac{x}{2\alpha_D^2} \right) \right. \\
& + \frac{2}{\pi} \left\{ \frac{1}{\sqrt{x-a^2}} \left[\gamma_{NT} \exp \left\{ -\frac{x}{2\alpha_N^2} \right\} \right. \right. \\
& \left. \left. + \gamma_{TN} \exp \left\{ -\frac{x}{2\alpha_T^2} \right\} \right] \right. \\
& \left. - \exp \left\{ -\frac{x}{2\alpha_S^2} \right\} \right. \\
& \left. \int_0^{2\theta_I(x)} \cosh \left(-\frac{x \cos \theta}{2\alpha_D^2} \right) d\theta \right] \quad (11)
\end{aligned}$$

for $a^2 < x < 2a^2$,

$$\begin{aligned}
f_{X(n,l)}(x) &= \left[(1-\epsilon) \operatorname{erfc} \left(\frac{a}{\sqrt{2}\alpha_N} \right) \right] \\
&+ \epsilon \operatorname{erfc} \left(\frac{a}{\sqrt{2}\alpha_T} \right) \delta(x-2a^2), \quad (12)
\end{aligned}$$

for $x = 2a^2$, and $f_{X(n,l)}(x) = 0$ for $x \leq 0$ and $x > 2a^2$, where $\operatorname{erfc}(x)$ is the complementary error function, $\theta_I(x) = \arcsin \sqrt{1 - \frac{a^2}{x}}$, and $\gamma_{ij} = \sqrt{\frac{\pi\alpha_i^2}{2}} \exp \left\{ \frac{a^2}{2\alpha_i^2} \right\} \operatorname{erfc} \left(\frac{a}{\sqrt{2}\alpha_i} \right)$ with $i, j = N, T$.

3.2 PERFORMANCE ANALYSIS

The probability of one chip error is

$$\begin{aligned}
p_l &= \frac{1}{2} Pr \{ X(n,l) > Y(n,l) | a_n = -1 \} \\
&+ \frac{1}{2} Pr \{ X(n,l) < Y(n,l) | a_n = 1 \}. \quad (13)
\end{aligned}$$

Using the results in Section 3.1, we have

$$\begin{aligned}
p_l &= \frac{1}{2} \int_0^{2a^2} \int_y^{2a^2} v_X(x)v_Y(y) |_{a_n=-1} dx dy \\
&+ \frac{1}{2} \int_0^{2a^2} \int_x^{2a^2} v_X(x)v_Y(y) |_{a_n=+1} dy dx, \quad (14)
\end{aligned}$$

where $v_X(x)$ is the pdf (10)-(12) and v_Y is obtained from v_X by substituting a_n with $-a_n$. As we can see from (10)-(14), we cannot in general obtain a closed-form expression for the probability of bit error. We will thus use numerical calculation to compute the probability of bit error of the LS receiver.

For a hard decision scheme, the probability of bit error is

$$P_e = \sum_{i=\lfloor \frac{L+1}{2} \rfloor}^L \binom{L}{i} (1-p_l)^{L-i} p_l^i \quad (15)$$

in the binary symmetric channel (BSC). If we have U users and K frequency slots, then the probability of error is given by [6]

$$P_{e,K,U} = \left(1 - \left(1 - \frac{1}{K} \right)^{U-1} \right) P_e. \quad (16)$$

To show the probability of bit error in various cases, let us define the threshold to noise ratio (TNR) as $\text{TNR} = a/\sigma_N^2$, and the ratio of the noise variances as $\mu = \sigma_T^2/\sigma_N^2$. Fig. 2 shows the probability of bit error as a function of SNR for $\mu = 100.0$ with SNR defined by $\text{SNR} = 10 \log_{10} [E_b / (1-\epsilon)\sigma_N^2 + \epsilon\sigma_T^2]$ when $\epsilon = 0.1$: the values of TNR are 4.0, 8.0, and 20.0, and $T_0 = 0.5T$, where T_0 is the maximum value of time delay. In Fig. 3, we show the bit error probability for the multi-user and multi-hopping case for $T_0 = 0.1T$, $\epsilon = 0.1$, $\mu = 100.0$, $L = 2$, $K = 20$, and $U = 10$.

When the SNR is low, the performance of the LS receiver is better than that of the squarer (SQ) receiver under impulsive noise environment. For example, in Fig. 3, the LS receiver with $\text{TNR} = 20$ has approximately 3 dB SNR gain over the SQ receiver when $P_e = 10^{-2}$. The reason for this is that the limiter with proper value of TNR reduces the effects of impulsive (large-valued) noise.

When the SNR is high, on the other hand, the SQ receiver has better performance than the LS receiver if the value of TNR is small. This can be explained as follows. In the case of high SNR, the limiting property of the LS detector prevents the detector from fully exploiting the information of the large-valued transmitted signal.

4. CONCLUDING REMARKS

We investigated the performance of the FFHSS-BFSK communication system using LS detectors. The channel was modeled as a selective Rayleigh fading channel with ϵ -contaminated mixture noise.

The probability of bit error of the FFHSS-BFSK communication systems was obtained by numerical analysis in various cases of the ϵ -contaminated mixture noise, fading characteristics, multi-user, and multi-chip hopping.

The performance of the LS receiver was shown to be better than that of the SQ receiver if the TNR was chosen properly or if SNR was low. At high SNR, the SQ receiver had better performance than the LS receiver when the value of TNR was small.

ACKNOWLEDGEMENT

This research was supported by the Non-Directed Research Fund, Korea Research Foundation, in 1993, for which the authors would like to express their thanks.

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and selective Rayleigh fading channel, when $T_0 = 0.5T$, $\epsilon = 0.1$, and $\mu = 100.0$ for one-chip case. The bit error probabilities of LS (TNR = 4.0, 8.0, and 20.0) and SQ receivers are plotted.

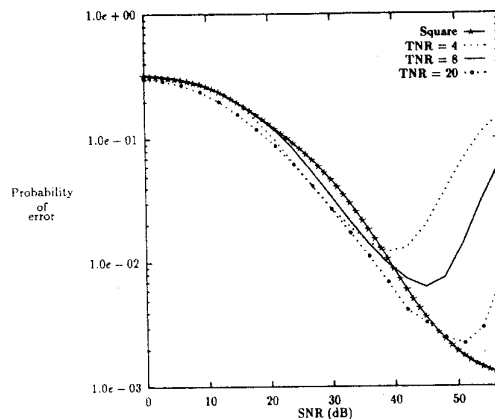


Fig. 3. The probability of bit error versus SNR in impulsive noise and selective Rayleigh fading channel, when $T_0 = 0.1T$, $\epsilon = 0.1$, $\mu = 100.0$, and the number of chips are 2. Asynchronous multiple access is assumed, the number of frequency slots is 20, and the number of users is 10. The bit error probabilities of LS (TNR = 4.0, 8.0, and 20.0) and SQ receivers are plotted.

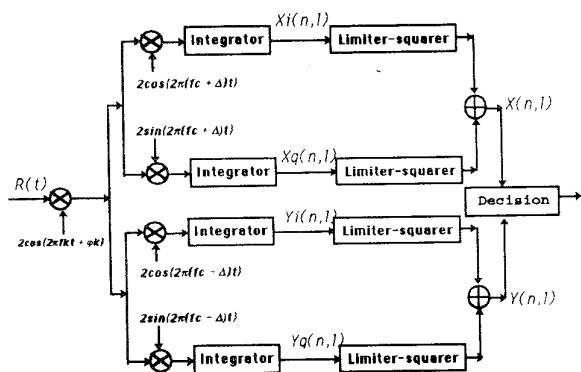


Fig. 1. A block diagram of the limiter-squarer receiver.

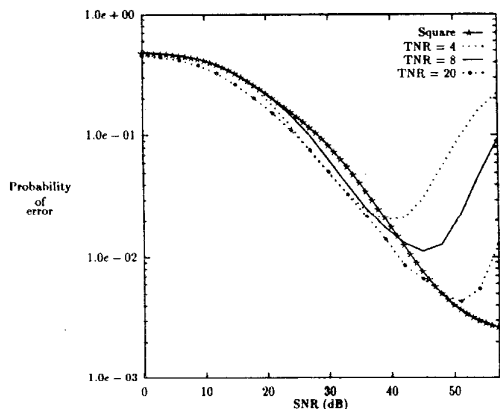


Fig. 2. The probability of bit error versus SNR in impulsive noise