An Assignment Method for Part-Machine Cell Formation Problem In the Presence of Multiple Process Routes*

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Abstract

In this paper we consider the part-machine cell formation decision of the generalized Group Technology(GT) problem in which multiple process routes can be generated for each part. The existing p-median model and similarity coefficient algorithm can solve only small-sized or well-structured cases. We suggest an assignment method for the cell formation problem. This method uses an assignment model which is a simple linear programming. Numerical examples show that our assignment method provides good separable cells formation even for large-sized and ill-structured problems.
KEY WORDS: Group technology, cell formation, p-median model, assignment model.

1 INTRODUCTION

Group Technology(GT) is a manufacturing philosophy or concept that identifies and exploits the similarities of product design and manufacturing processes. One application of GT is a cellular manufacturing(CM). The first and most important step in CM is the cell formation, which groups parts with similar design features or processing requirements into families so as to take advantage of their similarities and forms machine cells by assigning machines dedicated to process only the parts belonging to each family.

The objective of cell formation decision is to create mutually disjoint machine cells independent of other cells. Cells are designed so that they can operate independently with minimum interaction between cells and the result of these separable cell formation is the simplified material flows that allow much easier control than job shop.

Many researchers have addressed the cell formation problem and proposed numerous methods for clustering machines and parts. Wemmerlov *et al.*[11] and Chu[1] provide extensive reviews of the cell formation methods. Most of the algorithms or methods developed so far usually assume that each part has only one fixed process route which indicates the sequence of facilities used to process each part. We will call the cell formation problem for such case the simple GT problem.

This assumption, however, may not be realistic in the CM environment since each operation of a particular part may be performed on alternative machines. In a functional layout which is adopted by job shop production, this is not a consideration because the parts can be routed to any available machines which are concentrated together on the functionally similar work center. On the other hand, in a GT layout which is adopted by CM, each manufacturing cell usually consists of functionally dissimilar machines and the existence of functionally similar machines assigned to different manufacturing cells makes multiple process routings for a part possible (Nagi *et al.*[6]). Hence, a methodology is needed to form machine-part cells under such flexible routings. This paper considers the cell formation problem in the presence of such multiple process routes for each part. We will call the cell formation problem including multiple process routes for a part the generalized GT problem.

Kusiak[4] first proposed the cell formation problem under multiple process routes. He suggested a p-median model as a part family formation method. However, the p-median model may require some prohibitive computation time to solve since it usually has too many 0-1 integer variables even for an intermediate-sized problem(Wei and Gaither[10]). Furthermore, to obtain the proper cell solution using the p-median model, one may have to solve a sequence of integer programming problems because the proper value of p is, actually, not known until the most proper cell solution is found (Srinivasan *et al.*[8,9]). Also, the model tries to group

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only parts into families and does not present any subsequent procedure to allocate machines to the corresponding families(Chu[2]). Kusiak and Cho[5] suggested another algorithm using similarity coefficients, but this solves only small-sized problems with completely independent cells.

Recently, Srinivasan *et al.*[8] presented an assignment model to solve the simple GT problem. Motivated by their work, we propose a method which uses an assignment model to overcome the limitations of the p-median model. The assignment model is a simple linear programming. The model is looking for a possible seed part family which has maximum similarity between those production routes of the parts in the family. Then our method expands this seed family while minimizing the number of exceptional elements which prevent from obtaining the completely independent cells. Numerical examples show that our method works much better than the p-median model.

The organization of this paper is as follows. In Section 2, we provide an assignment model. Section 3 shows the whole algorithm. In Section 4, test results of several numerical examples are presented. The last section summarizes the conclusion of this paper.

2 ASSIGNMENT MODEL

A basic tool frequently used to represent the GT cell formation problem is a machine-part incidence matrix $[a_{ij}]$, which consists of 0,1 entries where $a_{ij} = 1$ indicates that machine i is used to process part j and $a_{ij} = 0$, otherwise.

Cell formation solutions often contain exceptional elements that create interaction between two manufacturing cells. Exceptional elements can be considered to be bottleneck machines that are required to process parts belonging to two or more families or bottleneck parts that require to be processed on machines assigned to two or more cells. So as to create independent cells minimizing the number of exceptional elements, maximizing the sum of the similarity coefficients between parts or machines has been widely used.

The assignment model proposed in this section makes use of a square matrix of similarity coefficients between routes. A similarity coefficient is an indicator showing the degree of dependence between each pair of routes or machines. Let n be the number of part types, m be the number of machines, F_j be the set of process routes for part j and $r = \sum_{j=1}^{n} |F_j|$ be the total number of process routes. A similarity coefficient s_{kl} between routes k and l is defined as

$$s_{kl} = \begin{cases} -\infty, & \text{if } k, l \in F_j \text{ for some } j, \\ 0, & \text{if } k = l, \\ \sum_{i=1}^{m} \delta(a_{ik}, a_{il}), & \text{otherwise,} \end{cases}$$

where

$$\delta(a_{ik}, a_{il}) = \begin{cases} 1, & \text{if } a_{ik} = a_{il}, \\ 0, & \text{otherwise.} \end{cases}$$

Then our problem can be stated as follows:

$$\text{Maximize} \quad \sum_{k=1}^{r} \sum_{l=1}^{r} s_{kl} x_{kl} \tag{1}$$

$$\text{subject to} \quad \sum_{k \in F_j} \sum_{l=1}^{r} x_{kl} = 1, \; j = 1, \cdots, n \tag{2}$$

$$\sum_{l \in F_j} \sum_{k=1}^{r} x_{kl} = 1, \; j = 1, \cdots, n \tag{3}$$

$$x_{kl} \geq 0, \; k = 1, \cdots, r; l = 1, \cdots, r. \tag{4}$$

The decision variable $x_{kl} = 1$ if the parts produced by process routes k and l are considered as candidates to form a portion of a part family. We use the same objective as the p-median model. Constraint (2) implies that for each part j one process route $\bar{k} \in F_j$ is selected for the row index of the decision variable x_{kl} and that $x_{\bar{k}l} = 1$ for exactly one l and 0 for others. Similar interpretation can be applied to the constraint (3) except that the roles of row and column are interchanged. However, we allow that the process routes selected for row index and column index for a part can be different. In this case, both of the two process routes are considered as candidates in our cell formation procedure.

Let $\bar{x}$ be an optimal solution of the assignment problem and $R = \{k, l | \bar{x}_{kl} = 1\}$ be the set of routes selected for row or column indices. Then the set R can be partitioned into several disjoint and exhaustive subsets of the following three types:

Type I (closed loop): A subset $R_1 = \{k_1, \cdots, k_p\}$ of R such that $\bar{x}_{k_1,k_2} = \bar{x}_{k_2,k_3} = \cdots = \bar{x}_{k_{p-1},k_p} = \bar{x}_{k_p,k_1} = 1$.
Type II (open sequence with identical parts at the two ends): A subset $R_2 = \{k_1, \cdots, k_p\}$ of R such that
- i) $\bar{x}_{k_1,k_2} = \bar{x}_{k_2,k_3} = \cdots = \bar{x}_{k_{p-1},k_p} = 1$,
- ii) k_1 and k_p are two different process routes for an identical part.

Type III (open sequence with different parts at the two ends): A subset $R_3 = \{k_1, \cdots, k_p\}$ of R such that
- i) $\bar{x}_{k_1,k_2} = \bar{x}_{k_2,k_3} = \cdots = \bar{x}_{k_{p-1},k_p} = 1$,
- ii) k_1 and k_p are process routes for different parts.

In a closed loop, each part produced by the routes in the loop has identical row and column process routes. In a type II or type III open sequence, each part produced by the interior routes(routes except both ends) in the sequence has identical row and column process routes.

The set of parts produced by the routes in a closed loop or Type III open sequence provides a candidate for a seed part family. However, the set of parts from Type II open sequence can not be a candidate for a seed part family because the two process routes at the two ends of the sequence are two different routes for an identical part and this fact violates the requirement that only one route should be selected for a part.

From each of the Type I closed loops and Type III open sequences embedded in the optimal solution, we generate a candidate seed family. In all examples tested, we were always able to find a Type I or Type III subset. However, it could be possible that the optimal solution consists of only Type II open sequences. In this cases, we suggest to generate a candidate seed family from a Type II open sequence by choosing any one route from the two routes at the two ends of the sequence.

This assignment model has a totally unimodular constraint matrix and we can exclude the integer restriction. This saves considerable computational efforts even when the model is applied to large size problems. Furthermore, the number of decision variables of this model is the same as that of the p-median model but the number of constraints is $2n$ which is much smaller compared with $r^2 + n + 1$, the number of constraints of the p-median model.

3 Algorithm

Our algorithm is an iterative procedure. Let us start with an initial incidence matrix. From an optimal solution of the assignment model, we construct a part family. Remove all parts in the part family from the initial incidence matrix and obtain a reduced incidence matrix. Solve the new assignment problem and construct another part family. We repeat this procedure until all parts are removed.

At each iteration, we need to construct a part family from an optimal solution of the assignment problem. We start with the candidate seed families generated from Type I closed loop or Type III open sequence. For each candidate seed family, construct a machines cell which is the set of machines used for the production of the parts in the candidate seed family. Since there could be some other process routes which can be processed completely by the machines in this machine cell, we expand the candidate seed family by adding other parts into the family which have such a process route. We apply this procedure to each of the candidate seed families generated from the assignment model. Among the several expanded candidate part families, we choose a larger one which has minimum interaction with existing machine cells.

This procedure may not generate completely independent cells. As a result, some machines may be used by two or more different part families. If there are such bottleneck machines, then reassign these machines to the cell giving the smallest number of exceptional elements.

ALGORITHM

Step 0. (Initialization): Set the level number representing the iteration as $L = 1$.

Step 1. (Transformation): From the current incidence matrix, obtain a similarity coefficient matrix.

Step 2. (Solving assignment problem): Solve the assignment problem with the similarity coefficient matrix.

Step 3. (Identifying candidate seed families): Identify candidate seed families from the solution. A Type I closed loop or a Type III open sequence provides a candidate seed family. If there is no Type I closed loop or Type III open sequence, then generate candidate seed family from a Type II open sequence by choosing any one route from the two routes at the two ends of the open sequence.

Step 4. (Merging parts): For each candidate seed family, form a corresponding machine cell and merge other parts to the part family which have a process route completely processed within the machine cell.

Step 5. (Selecting a part family and a machine cell): Among the expanded candidate part families, choose the candidate family with the fewest number of exceptional elements with existing machine cells generated in previous iterations. If ties occur, choose the one with the largest number of parts.

Step 6. (Revising incidence matrix): Obtain a new reduced incidence matrix by deleting the parts in the selected part family from the current incidence matrix. If no parts are remaining, go to step 7. Otherwise, let $L = L + 1$ and go to step 1.

Step 7. (Reassigning machines and stopping): If bottleneck machines exist, reassign each of the machines to the cell giving the smallest number of exceptional elements. Stop.

4 NUMERICAL EXAMPLES

Only a few examples of incidence matrix have been given in the literature for the generalized GT problem. In order to test the effectiveness of our algorithm, we select three – small, medium, and large-sized – data sets for the generalized GT problem seen in the literature. As a primary measure of effectiveness of the cell formation methods, we evaluate the number of exceptional elements. This measure has been widely used as a critical criterion of effectiveness of the cell formation methods (Chu[1,2]).

Figure 1-(a) is the original incidence matrix given by Kusiak[4] and Figure 1-(b) shows the result by our algorithm. In this example with completely independent cells, we obtain the same result as the *p*-median model.

Insert **Figure 1-(a)** here
Insert **Figure 1-(b)** here

Figure 2-(a) is the incidence matrix given by Sankaran and Kasilingam[7] and Figure 2-(b) shows the result by our algorithm. In this example, we obtained a solution with two cells and 2 exceptional elements. On the contrary, the solution of the *p*-median model gives no visible cell formation as shown in Figure 2-(c).

Insert **Figure 2-(a)** here
Insert **Figure 2-(b)** here
Insert **Figure 2-(c)** here

Figure 3-(a) is the incidence matrix given by Nagi *et al.*[6] which is the largest example that has ever been published. Figure 3-(b) is the result by our algorithm that shows only 1 exceptional element. Figure 3-(c) is the result by the *p*-median model that gives 3 exceptional elements.

Insert **Figure 3-(a)** here
Insert **Figure 3-(b)** here
Insert **Figure 3-(c)** here

As shown in Figures, our model gives better results than the *p*-median model in terms of the number of exceptional elements. Since our algorithm solves an assignment problem at each iteration of the algorithm, at most L simple linear programming problems have been solved. The computational burden of this algorithm is very small. For example, our algorithm takes about 7 seconds in CDC CYBER 960 computer to solve Nagi *et al.*'s example but the p-median model requires about 147 seconds with a fixed value of $p = 5$. If we solve the p-median model for $p = 1, 2, \cdots, L$ and choose the best solution as Kusiak recommends, the p-median model will consume far more computation time(Srinivasan *et al.*[8]).

5 CONCLUSION

In this paper, an assignment model is presented to solve the generalized GT cell formation problem in which more than one process route can be planned for each part. Even for the Nagi *et al.*'s example which is known to be an ill-structured and large-sized problem, our model gives a good separable cell formation. Reducing the number of decision variables by defining the similarity coefficient not between routes but between machines could be an interesting future research area.

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Figure 1-(a). Kusiak's original incidence matrix

Machines \ Parts	1a	1b	1c	2a	2b	3a	3b	4a	4b	5a	5b
1			1		1	1		1	1		1
2		1	1	1			1				
3	1			1	1				1	1	
4	1	1				1	1	1		1	

Figure 1-(b). Result by our model

Machines \ Parts	2b	4b	5b	1b	3b
1	1	1	1		
3	1	1			
2				1	1
4				1	1

Figure 2-(a). Sankaran and Kasilingam's incidence matrix

Machines \ Parts	1a	1b	2a	2b	3a	4a	4b	5a	5b	5c	5d	6a	6b	7a	8a	8b	9a	9b	10a	10b
1					1			1	1					1		1				
2	1	1	1	1		1	1					1			1	1	1	1		
3		1						1	1	1	1			1					1	
4			1	1	1	1	1	1		1	1	1	1		1	1	1		1	1
5	1		1			1			1		1				1			1		1
6	1	1	1	1			1						1		1	1	1	1		

Figure 2-(b). Result by our model

Machines \ Parts	1a	2a	4a	6a	8a	9b	10b	3a	5a	7a
2	1	1	1	1	1	1				
4		1	1	1	1		1	1	1	
5	1	1	1		1	1	1			
6	1	1			1	1				
1								1	1	1
3									1	1

Figure 2-(c). Result by p-median model

Parts

Machines	1b	2b	4b	6b	8a	9a	3a	5a	7a	10a
2	1	1	1		1	1				
3	1							1	1	1
6	1	1	1	1	1	1				
4		1	1	1	1	1	1	1		1
5					1					
1							1	1	1	

Figure 3-(a). Nagi *et al.*'s incidence matrix

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Parts
                                            1111111111111111111111112
          11223344555667777788999999001112223334445556667890
          abababababcababcdefababcdeffababcabcabcabcabcabcaaaa

M   1       111111111
a   2                11       1111111111
c   3                                   111111111
h   4                                                      111
i   5                11111111        11
n   6     1 1 1 1    1    111 1 1 1 11
e   7      1 1 1 1    1111   1 1 1 1  1
s   8                                   111111111
    9     11  11  111
   10                                            1111111111
   11                                   111   111
   12     11111111111
   13                                                      11
   14                                            111   1111
   15                                                      111
   16                  11H111111111111111
   17                                            111111   1
   18              1    1  1     11    1  1  1  1  1  1
   19             1    1  1     11      1  1  1  1  1  1
   20               1    1  1     11    1  1  1  1  1  1
```

Figure 3-(b). Result by our model

Machines \ Parts	6	7	8	9	10	14	15	16	17	11	12	13	18	19	20	1	2	3	4	5
	b	a	a	a	b	a	a	a	a	c	c	c	a	a	a	a	a	a	a	a
2	1		1	1	1															
5	1	1			1															
7	1	1	1	1	1															
16		1	1	1	1															
19		1		1																1
10						1	1	1	1											
14						1		1	1											
17						1	1		1											
18						1	1	1												
3										1	1	1								
8										1	1	1								
11										1		1								
20										1	1	1								
4													1	1	1					
13													1	1						
15													1	1	1					
6																1	1	1	1	
9																1		1		1
12																1	1	1	1	1
1																	1	1	1	1

Figure 3-(c). Result by p-median model

Machines \ Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	b	b	b	b	c	a	d	b	b	a	a	a	a	b	b	b	a	a	a	a
7	1	1	1	1																
9	1		1		1															
12	1	1	1	1	1															
1		1	1	1	1															
20					1															
2						1		1	1	1										
5						1	1			1										
6						1	1	1	1	1										
16							1	1	1	1										
19							1		1					1	1	1				
3											1	1	1							
8											1	1	1							
11											1		1							
18											1	1	1							
10														1	1	1	1			
14														1		1	1			
17														1	1		1			
4																		1	1	1
13																		1	1	
15																		1	1	1