Power Randomization in ALOHA Access Control for Wireless Networks¹

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Abstract

Randomization of transmission power in ALOHA access control for wireless networks is introduced and analyzed. It is shown that the power randomization, introduced appropriately, leads to a substantial throughput improvement and to elimination of the bistable behavior.

1 INTRODUCTION

Dynamic control of ALOHA for the purpose of increasing its throughput and eliminating the bistable behavior has been the topic of a number of publications, [1] - [5]. In parallel, for systems with capture capabilities, the improvement of ALOHA has been addressed on the basis of power control schemes, [6], [7]. In this paper, we consider the randomization of the transmission power as a tool for improving performance of ALOHA in an access channel of a wireless network with the base station having the capture capability. More precisely, we consider the access channel defined as follows:

(a) It is a collision channel with zero propagation delay; the network consists of $1 \ll M < \infty$ users communicating through this channel.

(b) Transmission of an access packet across a channel requires a unit interval of time; time is slotted with a

slot duration equal to 1.

(c) Every user has a single buffer capable of storing one access packet; a user generates an access packet with probability p_a during each time slot $[n, n + 1)$. Since only one packet can be successfully transmitted across the collision channel during a time slot, to avoid triviality, it is assumed that $Mp_a \leq 1$.

(d) If an access packet is generated during time slot $[n, n + 1)$ by a user having its buffer full, the newly generated packet is rejected; otherwise, it is stored in the buffer.

(e) If a positive acknowledgment is received by an active user at time $n + 1$, the access packet transmitted during time slot $[n, n + 1)$ is eliminated from its buffer.

(f) There is no performance degradation due to the near-far effect.

(g) Every user that has a packet attempts a transmission at the beginning of time slot $[n, n + 1)$ with probability p using the transmission power which is chosen randomly according to the uniform distribution in the domain $[P_{min}, P_{max}]$.

(h) The base station has the capture capability with the relative capture power, β , defined as follows:

$$\beta = \frac{P_{cap}}{P_{max} - P_{min}},$$

where $0 < P_{cap} \leq P_{max} - P_{min}$ and P_{cap} is the minimum power difference between two packets to ensure the capture effect.

Remark 1: Assumption (c) can be relaxed by assuming that each user has a buffer capable of storing N

¹S. M. Meerkov acknowledges the support of Lady Davis Professorship of the Technion, Israel, during the preparation of this work.

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packets. All the results obtained below remain qualitatively true, but the analysis becomes more complicated.

Remark 2: Assumption (f) is also introduced for simplicity. It is justified, however, by the existing standards. Indeed, the IS-95 digital cellular standard includes the power control to adjust for fading of the received signals. It is shown in [8],[9] that the users can adjust their transmitted power to compensate the near-far effect.

The goal of this paper is to investigate the effect of the power randomization on the performance of ALOHA in channels defined by assumptions (a) - (h). The remainder of this paper is structured as follows: In Section 2, dynamic equations of the network are derived. Their asymptotic approximation and analysis are presented in Sections 3 and 4, respectively. The conclusions are formulated in Section 5. A technical derivation is provided in the Appendix.

2 DYNAMIC EQUATIONS

Let $h(n)$ be the number of users having a packet in their buffers at time n . Due to the assumptions formulated above, the evolution of h can be characterized as follows:

$$h(n+1) = h(n) + \zeta(n, n+1) - \psi(n+1), \quad (1)$$

where ζ characterizes the arrival process and $\psi \in \{0, 1\}$ defines the packet transmission.

The sequence $\zeta(n, n+1)$ represents the process of packet arrival during $[n, n+1)$. The event $\zeta(n, n+1) = l, l = 0, 1, \dots, M - h(n)$ means that l packets have arrived into the buffers during $[n, n+1)$. Due to assumptions (c)-(d),

$$\begin{aligned} P[\zeta(n, n+1) = l | h(n)] \\ = \binom{M-h(n)}{l} p_a^l (1-p_a)^{M-h(n)-l}. \end{aligned} \quad (2)$$

The sequence $\psi(n+1), n = 0, 1, \dots$, represents the process of a packet transmission. The event $\psi(n+1) = 1$ means that a successful transmission has occurred during $[n, n+1)$; the event $\psi(n+1) = 0$ implies no successful transmission has occurred during $[n, n+1)$. Let $p_c(k)$ denote the probability that one user among k active users is captured by the base station due to the power difference. Then, due to assumption (h), $p_c(k)$ is (see the Appendix for the derivation):

$$\begin{aligned} p_c(k) &= k \int_{P_{cap}}^{P_{max}-P_{min}} \left(\frac{x - P_{cap}}{P_{max} - P_{min}} \right)^{k-1} dx \\ &= (1 - \beta)^k. \end{aligned} \quad (3)$$

Therefore, from assumption (g) and (3),

$$\begin{aligned} P[\psi(n+1) = 1 | h(n)] &= h(n)p(1-p)^{h(n)-1} \\ &+ \sum_{k=2}^{h(n)} \binom{h(n)}{k} p^k (1-p)^{h(n)-k} p_c(k). \end{aligned} \quad (4)$$

Thus, the performance of the network is described by the nonlinear, stochastic, finite difference equations (1)-(4). In general, analysis of these equations is difficult. However, the asymptotic technique of [10] can be employed in order to obtain a relatively simple analysis procedure.

3 ASYMPTOTIC APPROXIMATION

In terms of the normalized variable $x = h/M$, equation (1) can be rewritten as

$$x(n+1) = x(n) + \frac{1}{M} [\zeta(n, n+1) - \psi(n+1)], \quad (5)$$

where, due to assumption (a), $\frac{1}{M} = \epsilon \ll 1$. To simplify this equation, we use the asymptotic theory of [10]. This theory allows one to convert the stochastic finite difference equation (5) into a deterministic finite difference equation with the right hand side equal to the conditional expectation of the right hand side of (5), calculated using the probability mass functions (2) and (4). As it is shown in [10], if ϵ is sufficiently small (i.e., in the case under consideration, the number of users is sufficiently large), the solutions of the original equation (5) and its asymptotic approximation are ϵ -close to each other in probability (see [10] for details).

The methods of [10] are applicable if the conditional expectations of both random sequences in the right hand side of (5) have the average value and the variance of the order 1. Since $\psi(n+1)$ takes values on $\{0, 1\}$, it suffices to show that the average and the variance of $\zeta(n, n+1)$ are of the order 1. To accomplish this, using (2), we write:

$$\begin{aligned} E[\zeta(n, n+1) | x(n)] &= Mp_a(1-x(n)) \\ Var[\zeta(n, n+1) | x(n)] &= Mp_a(1-x(n))p_a, \end{aligned}$$

which mean that the required conditions are satisfied, since Mp_a is of the order 1.

Next, we construct the asymptotic approximation of (5) by taking the conditional expectations of both random sequences in its right hand side:

$$y(n+1) = y(n) + \epsilon[(1-y(n))Mp_a - F(y(n))], \quad (6)$$

where

$$\begin{aligned} F(y(n)) &= My(n)p(1-p)^{My(n)-1} \\ &= \sum_{k=2}^{My(n)} \binom{My(n)}{k} p^k (1-p)^{My(n)-k} p_c(k). \end{aligned} \quad (7)$$

Since, as it follows from [10], the solutions of (5) and (6) are ϵ -close to each other in probability, the simplified equation (6) constitutes the basis for the analysis presented below.

4 PERFORMANCE ANALYSIS

Steady States: Let y_s denote the steady state value of the averaged normalized occupancy of the buffers in the network. Then, from (6), we obtain

$$(1 - y_s)Mp_a = F(y_s). \quad (8)$$

The left hand side of (8) is referred to as a **load line** and the right hand side of (8) is referred to as a **transmission line**. Thus, every intersection of the load line and the transmission line defines a steady state of the network, y_s . The load line and the transmission lines of the network defined by (a)-(h) are illustrated in Figure 1. As it follows from the Jury stability test [11], if at the point of intersection the slope of the load line is smaller than the slope of the transmission line, the steady state is asymptotically stable. Thus, for example, the first intersection of the load line with the transmission line corresponding to $\beta = 0.3$ (see Figure 1) is asymptotically stable, the second is unstable, and the third is asymptotically stable again (bistable behavior). Note that for $\beta < 0.2$, the intersection is unique and asymptotically stable.

Figure 1: Load line and transmission lines.

Steady State Performance: As it is shown in [10], if (8) has a unique solution (i.e. the load line and the transmission line have only one intersection), the solutions of equations (5) and (6) are close to each other in probability on the infinite time interval. Therefore, the performance of the network for all t can be characterized by the properties of (6). This means that the network under consideration has the following performance characteristics:

- 1) The steady state throughput, TP , is

$$TP = F(y_s), \quad (9)$$

where y_s is the solution of (8) and $F(\cdot)$ is defined in (7). Thus, the steady state throughput is the ordinate of the intersection of the load line and the transmission line, and the steady state number of busy users is the abscissa of this intersection (see Figure 1).

- 2) The steady state time delay, TD , as it follows from Little's law, is

$$TD = \frac{My_s}{F(y_s)}. \quad (10)$$

- 3) The steady state probability of call rejection, PR , is

$$PR = y_s, \quad (11)$$

which is due to the fact that y_s represents the number of busy users.

If (8) has multiple solutions (i.e., the load and the transmission lines have more than one intersections), the global analysis of PR, TD, PR can be conducted using the large deviations approach to characterize the residence time of (5) in the domains of attraction of all asymptotically stable steady states of (6), using the theory of [12] (see also [5]).

Elimination of bistable behavior. As it follows from Figure 1, the relative capture power may lead to the stabilization of the unstable ALOHA, i.e., to the elimination of the bistability and to stabilization of the network in a unique steady state with a relatively high throughput. The following theorem quantifies this effect.

Theorem: Consider a network defined by (a)-(h). Then there exist M_0 and β such that the network, with $M \geq M_0$ satisfying the condition

$$(1 - y)Mp_a < F(y), \quad \forall y \in \Omega, \quad (12)$$

where $\Omega = \{y \in [\frac{1}{Mp}, 1] : \frac{\partial F(y)}{\partial y} \leq 0\}$, has a unique globally asymptotically stable steady state.

Proof: Since the transmission line has a maximum on the interval $[\frac{1}{Mp}, 1]$ and the load line is a monotonically decreasing function, the network has a unique steady state with a relatively high throughput if the load line has an intersection only with the monotonically increasing part of the transmission line, i.e., $(1 - y)Mp_a < F(y), \forall y \in \Omega_1$ where $\Omega_1 = \{y \in [\frac{1}{Mp}, 1] : \frac{\partial F(y)}{\partial y} \leq 0\}$.

Since $M \gg 1$, from (7), $F(y)$ can be approximated as follows: $F(y) \simeq e^{-\beta G} - e^{-G} + \beta G e^{-G}$ where $G = My p$. Then, if $p\beta \ll 1$, the relative capture power satisfying (12) is

$$\beta < \frac{Gp_a + p(1 - e^{-G} - Mp_a)}{p(G - Ge^{-G})}, \quad \forall G \in \Omega_2, \quad (13)$$

where $\Omega_2 = \{G \in [1, Mp] : 1 + \beta - \beta G - \beta e^{(1-\beta)G} \leq 0\}$. The boundary of the stability region with a relatively

high throughput in p_a versus β plane is shown in Figure 2. For a given p_a , this Figure allows one to select β which ensures high performance characteristics. The β selected can be implemented by the choice of P_{min} (or P_{max}) as indicated in assumption (h).

Figure 2: Stability and bistability areas in (β, p_a) -plane

5 CONCLUSIONS

Power randomization in ALOHA access channel of a wireless network leads to an improvement of the steady state throughput and to elimination of the bistable behavior. These effects are quantified by relationships (9) (steady state throughput), (10) (steady state time delay), and (11) (probability of call rejection). Figure 1 illustrates these performance measures. In addition, the stability area, as a function of the input calls intensify (p_a) and the relative capture power (β), is quantified by inequalities (12) and (13) and is illustrated in Figure 2. Thus, power randomization may be viewed as a potentially productive tool for improvement of performance in wireless networks.

Appendix: Derivation of Formula (3)

Let $\{P_1, P_2, \dots, P_l\}$ be a set of l real numbers where $P_i = P_{min} + i\Delta, i = 1, 2, \dots, l$, Δ is an arbitrarily small positive constant, and $P_l = P_{max}$. Assume that every user having a packet attempts a transmission with probability p with the randomly and equiprobably chosen power whose value belongs to the set $\{P_1, P_2, \dots, P_l\}$. Let l_c be a positive integer such that $P_{cap} = l_c \Delta$ and let $\hat{p}_c(k)$ denote the probability that one user among k active users is captured due to the power difference based on the above assumption. Then

$$\begin{aligned} \hat{p}_c(k) = & k \frac{1}{l} \left(\left(\frac{l-l_c}{l} \right)^{k-1} + \left(\frac{l-l_c-1}{l} \right)^{k-1} \right. \\ & \left. + \dots + \left(\frac{1}{l} \right)^{k-1} \right). \end{aligned}$$

Since $l = \frac{P_{max}-P_{min}}{\Delta}$, $l_c = \frac{P_{cap}}{\Delta}$, $\hat{p}_c(k)$ becomes

$$\begin{aligned} \hat{p}_c(k) = & k \frac{\Delta}{P_{max}-P_{min}} \left(\left(\frac{P_{max}-P_{min}-P_{cap}}{P_{max}-P_{min}} \right)^{k-1} \right. \\ & + \left(\frac{P_{max}-P_{min}-P_{cap}-\Delta}{P_{max}-P_{min}} \right)^{k-1} + \dots \\ & \left. + \left(\frac{\Delta}{P_{max}-P_{min}} \right)^{k-1} \right) \\ = & k \sum_{i=0}^{l-l_c-1} \left(\frac{P_{max}-P_{min}-P_{cap}-i\Delta}{P_{max}-P_{min}} \right)^{k-1} \\ & \frac{\Delta}{P_{max}-P_{min}}. \end{aligned}$$

As we let Δ approach 0, $\hat{p}_c(k)$, in the limit, equals $p_c(k)$. Therefore,

$$\begin{aligned} p_c(k) = & \lim_{\Delta \rightarrow 0} k \sum_{i=0}^{l-l_c-1} \left(\frac{P_{max}-P_{min}-P_{cap}-i\Delta}{P_{max}-P_{min}} \right)^{k-1} \\ & \frac{\Delta}{P_{max}-P_{min}} \\ = & k \int_{P_{cap}}^{P_{max}-P_{min}} \left(\frac{x-P_{cap}}{P_{max}-P_{min}} \right)^{k-1} \\ & \frac{dx}{P_{max}-P_{min}} \\ = & (1-\beta)^k. \end{aligned}$$

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