Multicriteria Fuzzy Control and Its Application to DC Servomotor Position Control

Kwang-Chun Kim and Jong-Hwan Kim*

Abstract— **In this paper, we present a multicriteria fuzzy controller(MFC) based on fuzzy measures and fuzzy integrals. The basic idea underlying this approach is based on analyzing the source of attributes of the system output responses and applying the fuzzy measure and integral theory to the existing fuzzy controllers. By this scheme, we can tune the three attributes including rise time, overshoot, and settling time of the output responses. We demonstrate that MFC have superior control performance compared to conventional fuzzy controller by computer simulations as well as via experiments performed on a DC servomotor position control. Moreover, our scheme can be easily implemented in practice simply by adding weight terms to an existing fuzzy rules and assigning the weight to each rule by employing the fuzzy measure and integral.**

KeyWords—**multicriteria fuzzy controller(MFC), fuzzy measure, fuzzy integral, DC servomotor position control.**

I. Introduction

Recently, Grabisch[1] presented a survey paper on fuzzy measures and integrals, where he mentioned that some application papers using fuzzy measures and integrals could be found for the prediction of wood strength[2], the evaluation of printed color images[3], the design of speakers[4], and the human reliability analysis[5], etc. Since Sugeno's thesis on fuzzy integrals[6], some theoretical work has been done to show the usefulness of fuzzy measures and integrals by Murofushi and Sugeno[7], and Grabisch *et al.*[8; 9]. However, in spite of their theoretical work, fuzzy measure and integral theory has not been widely used in an application field, at least until recently.

A multicriteria fuzzy controller(MFC) using fuzzy measures and fuzzy integrals was first for control by Kim [10]. The idea underlying that scheme is based on analyzing the source of attributes of the system output responses and applying the fuzzy measure and integral theory to the conventional fuzzy controllers. As already well known, since Zadeh's paper[11], fuzzy logic-based controllers have received considerable interest in recent years. These fuzzy controllers are based on human rules. In the rules, three attributes are defined such as *rise time*, *overshoot*, and *settling time* to be evaluated to get desired output responses. By assigning three partial evaluations to each rule and expressing grade of importance by a fuzzy measure, three partial evaluations can be aggregated using a fuzzy integral. Through the scheme, the three attributes of the output responses can be tuned under the same given fuzzy rules. By computer simulations, it is shown that the control scheme has an excellent tuning performance, compared to the existing fuzzy controllers without multicriteria function. And also we demonstrate the performance of our scheme via experiments performed on a DC servomotor position control. Another advantage of our scheme is that it can be easily implemented in practice simply by adding weight terms to an existing fuzzy rules and assigning the weight to each rule by employing the fuzzy measure and integral.

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II. Basic Concepts

As we will deal only with finite spaces in the subsequent application, we restrict original definitions to a finite space $X = \{x_1, \cdots, x_l\}$. Let us define fuzzy measures on the power set of X, denoted $P(X)$.

Definition 1. A fuzzy measure g defined on $(X, P(X))$ is a set function $g : P(X) \rightarrow [0,1]$ verifying the following axioms :

(1) boundary condition
$g(\phi) = 0,\ g(X) = 1$

(2) monotonicity
If $A, B \in P(X)$ and $A \subseteq B$, then $g(A) \le g(B)$

Fuzzy measures include belief measures, plausibility measures, probability measures, possibility and necessity measures, etc. Another widely used fuzzy measure introduced by Sugeno is the λ-fuzzy measure, which satisfies the following axiom:

$$g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B)$$

for every $A, B \in P(X)$ and $A \cap B = \phi$ where λ is a real number in $(-1, +\infty)$.

Now let us define Choquet fuzzy integral.

Definition 2. (Choquet Integral) Let h be a mapping from X to [0,1]. The Choquet fuzzy integral of h over a subset of $X \in P(X)$ with respect to a fuzzy measure g is defined:

$$\int_X h \circ g = \sum_{i=1}^{l} (h(x_i) - h(x_{i-1})) g(H_i)$$

where it is assumed without loss of generality that $0 \le h(x_1) \le \cdots \le h(x_l) \le 1$, $H_i = \{x_k | k = i, \cdots, l\}$, and $h(x_0) = 0$.

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III. Multicriteria Fuzzy Controller(MFC)

A. Fuzzy Reasoning with Weight

We first describe a fuzzy reasoning for the following fuzzy rules with weights.

$$\begin{array}{lll} R_1 & : \text{If } E_1^{(1)} \text{ and } \cdots \text{ and } E_m^{(1)}, & \text{then } U^{(1)} \text{ with } w^{(1)} \\ R_2 & : \text{If } E_1^{(2)} \text{ and } \cdots \text{ and } E_m^{(2)}, & \text{then } U^{(2)} \text{ with } w^{(2)} \\ & \vdots & \\ R_n & : \text{If } E_1^{(n)} \text{ and } \cdots \text{ and } E_m^{(n)}, & \text{then } U^{(n)} \text{ with } w^{(n)} \\ \text{Fact} & : \text{If } e_1^* \text{ and } \cdots \text{ and } e_m^* & \\ \hline \text{Cons} & : & \text{then } u^* \end{array}$$

where $E_i^{(j)}$ and $U^{(j)}$ are fuzzy sets, e_i^* is a crisp value, and $w^{(j)}$ is a weight.

The weight $w^{(i)}$ is a real number in $[0, 1]$. The weight can be considered as a certainty factor[12], a weighting factor for emphatic and restrained effects on the fuzzy inference result[13], or a possibility of the rule's correctness[14].

The firing of these rules with crisp input values e_j^*, $j = 1, \cdots, m$ gives the truth values $f^{(i)}$, $i = 1, \cdots, n$ of the premises which are obtained from the Cartesian product $E_1^{(i)} \times E_2^{(i)} \times \cdots \times E_m^{(i)}$:

$$f^{(i)} = \min(\mu_{E_1^{(i)}}(e_1^*), \cdots, \mu_{E_m^{(i)}}(e_m^*)) \ .$$

The defuzzification process maps the result of fuzzy logic rule stage to a real number output. In this paper, we use the *height defuzzification method*[15], which is simple to implement and gives relatively good results. The inference result is then given by

$$u^* = \frac{\sum_{i=1}^n c^{(i)} w^{(i)} f^{(i)}}{\sum_{i=1}^n w^{(i)} f^{(i)}}$$

where $f^{(i)}$ is called a height, and $c^{(i)}$ is the peak value of $U^{(i)}$, which is the domain element on u that has degree of membership 1.

In this fuzzy reasoning, the problem may be how to assign weight to each rule. It is hard to figure out the control results by changing the weights of fuzzy rules. Therefore, determination of the postulated weight values is also, in general, based on heuristics. In the next section, we will describe a consistent method for assigning the weight to each fuzzy rule by employing the fuzzy measure and integral.

B. Design of MFC

Now let us consider a problem where a person subjectively evaluates an object according to his preference. Assume that the object in the problem can be divided into l elements or it has l attributes. Let $X = \{x_1, \cdots, x_l\}$ be a set of such elements or attributes.

Let $h : X \to [0, 1]$ be partial evaluation of the object. That is, $h(x_i)$ implies a partial evaluation of the object from the viewpoint of an element x_i. In the applications, a partial evaluation $h(x_i)$ can be determined either objectively or subjectively.

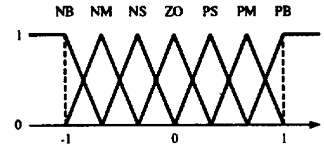

Fig. 1. Membership Functions

Taking a fuzzy integral of h with respect to g, we have

$$w = \int_X h \circ g \ .$$

Above equation represents the aggregation of l partial evaluations, where w can be considered as the overall evaluation of the object.

In the previous fuzzy rules, we choose l attributes and assign partial evaluations $h^{(i)}(x_j)$ to rule R_i from the viewpoint of an attributes x_j. The average value in each rule can be obtained from the fuzzy integral of $h^{(i)}(x_j)$ with respect to preassigned fuzzy measure g. The average value is then used as a weight to each rule. This process is summarized as follows:

$$\begin{array}{lll} R_1 & \Rightarrow h^{(1)}(x_1), \cdots, h^{(1)}(x_l) & \to w^{(1)} = \int_X h^{(1)} \circ g \\ R_2 & \Rightarrow h^{(2)}(x_1), \cdots, h^{(2)}(x_l) & \to w^{(2)} = \int_X h^{(2)} \circ g \\ & \vdots & \\ R_n & \Rightarrow h^{(n)}(x_1), \cdots, h^{(n)}(x_l) & \to w^{(n)} = \int_X h^{(n)} \circ g \end{array}$$

Thus, final consequence u^* is inferred as the weighted average of $c^{(i)}$ by the degree $w^{(i)} f^{(i)}$:

$$u^* = \frac{\sum_{i=1}^n c^{(i)} w^{(i)} f^{(i)}}{\sum_{i=1}^n w^{(i)} f^{(i)}}$$

where $w^{(i)} = \int_X h^{(i)} \circ g$.

In this paper, we choose time-domain specifications including x_1 = *rise time*, x_2 = *overshoot*, and x_3 = *settling time* as the attributes of the system output responses. These three attributes are the main terms of time-domain specification when we design a control system. The collection of *linguistic values* is

$$L = \{NB, NM, NS, ZO, PS, PM, PB\} \ .$$

Figure 1 shows a plot of the membership functions of linguistic values. The "meaning" of each linguistic value should be clear from its mnemonic.

Fuzzy control rules are given in Table 1. $e(k) = y_r(k) - y(k)$ where $y_r(k)$ is the reference input, $y(k)$ is the system output, and $\Delta e(k) = e(k) - e(k-1)$.

As shown in Table 2, each rule in Table 1 contains the three partial evaluations $h^{(i)}(\cdot)$ to each attribute such as *rise time*, *overshoot*, and *settling time*. These three evaluations are based on human reasoning, and the basic idea of assigning these evaluations is very clear and consistent from Figure 2 ~ 4.

Figure 2 shows how the partial evaluations $h^{(i)}(x_1)$ are obtained. Since the fuzzy rules with positive $e(k)$ and

$\Delta u(k)$		$\Delta e(k)$ NB	NM	NS	ZO	PS	PM	PB
$e(k)$	NB				NB (e, a, a)	NM (e, b, b)		
	NM				NM (e, b, b)			
	NS				NS (e, c, c)	ZO (d, d, d)		PM (e, d, b)
	ZO	NB (e, a, a)	NM (e, b, b)	NS (e, c, c)	ZO (d, d, d)	PS (e, d, c)	PM (e, d, b)	PB (e, d, a)
	PS	NM (e, b, b)		ZO (d, d, d)	PS (c, d, c)			
	PM				PM (b, d, b)			
	PB			PM (b, d, b)	PB (a, d, a)			

where a = 1, b = a/2, c = b/2, d = c/2, e = d/2

Table 1. Fuzzy control rules and partial evaluations

ith	$h^{(i)}(x_1)$
Rule	$h^{(i)}(x_2)$
(R_i)	$h^{(i)}(x_3)$

Table 2. Rule table notation

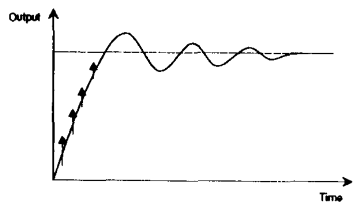

Fig. 2. Partial evaluation about x_1

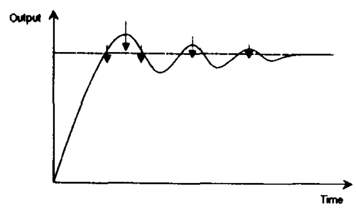

Fig. 3. Partial Evaluation about x_2

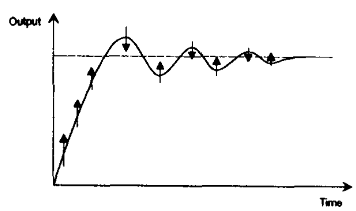

Fig. 4. Partial evaluation about x_3

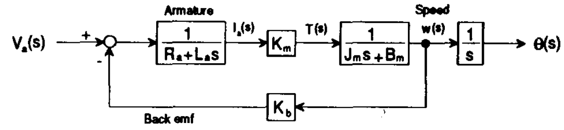

Fig. 5. Armature controlled DC motor

$\Delta u(k)$ effect on *rise time*, these are given higher evaluations. The rules with larger $\Delta u(k)$ effect more on the *rise time*. Thus, higher evaluation are assigned to the rules with larger $\Delta u(k)$ to get a fast *rise time*. We partition the grade of evaluation by 5 steps. The grade $a = 1$ is the highest score and $e = (1/2)^4$ is the lowest score.

The evaluation $h^{(i)}(x_2)$ are chosen to give the effect as in Figure 3. The rules with negative $\Delta u(k)$ have more effect on the *overshoot*. And higher evaluation is assigned to the rule with more negative $\Delta u(k)$ to decrease the *overshoot*.

Figure 4 shows how the evaluations $h^{(i)}(x_3)$ are chosen. As the arrows show, the higher evaluation are given to the rules with larger $|\Delta u(k)|$.

IV. Computer Simulation

Let us consider the armature controlled DC motor position control. The relations for the armature controlled DC motor are shown schematically in Figure 5. We can obtain the transfer function $G(S)$ from Figure 5.

$$
\begin{aligned}
G(s) &= \frac{\Theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a + L_a s)(J_m s + B_m) + K_b K_m]} \\
&= \frac{2.2}{s(8.959 \times 10^{-6} s^2 + 7.268 \times 10^{-3} s + 0.9449)}
\end{aligned}
$$

where the specifications of motor are

- $L_a = 5.27\ mH$
- $R_a = 3.9\ \Omega$
- $K_b = 22.5\ V/Krpm = 0.215\ Vsec$
- $t_m = 14\ msec$
- $K_m = 2.2\ Kgf\text{-}cm/A$
- $J_m = 0.0017\ Kgf\text{-}cm\text{-}sec^2$
- $B_m = J_m/t_m = 0.121\ Kgf\text{-}cm\text{-}sec$.

The actual angle of 360° is normalized to 1 in this simulation. The conditions are as follows: $y_r(k)$ is set to 1.0 which corresponds to 360°, scale factor of $e(k)$ is 1.0, scale factor of $\Delta e(k)$ is 15.0, scale factor of $\Delta u(k)$ is 1.0, and sampling time is 20 *msec*. For the scale factors, we refer the readers to [16]. Figure 6 shows control results using the proposed scheme for four cases. The fuzzy measure means the degree of consideration or importance of the set of the attributes. Here, probability measure is used as a fuzzy measure, which corresponds to the case with $\lambda = 0$ in λ-fuzzy measure. And we use Choquet integral as a fuzzy integral as defined in section II.

Compared to the well-tuned fuzzy output response as in Figure 6(a), we can get an output response with a fast *rise time* or in Figure 6(b) by assigning $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, and $g(\{x_3\}) = 0$. Similarly, by changing

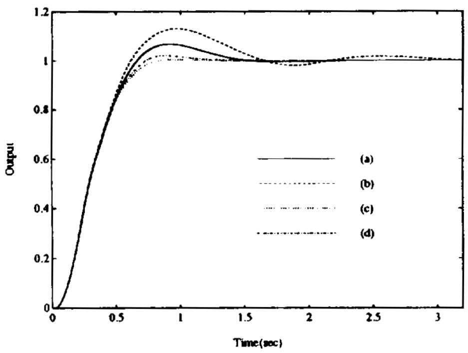

Fig. 6. Simulation results (a) Well-tuned fuzzy output (b) $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 0$ (c) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 1$, $g(\{x_3\}) = 0$ (d) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 1$

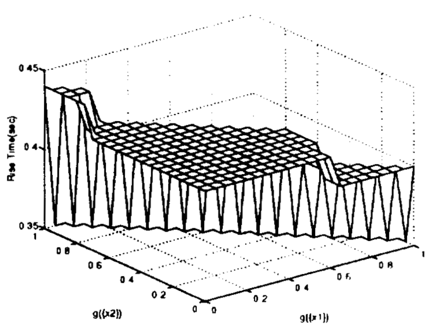

Fig. 7. Rise time

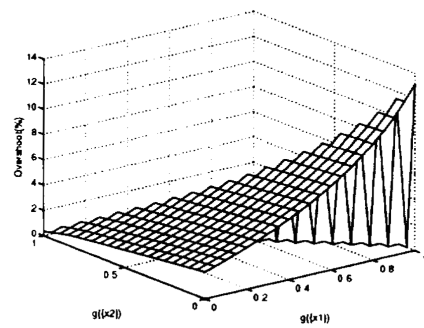

Fig. 8. Overshoot

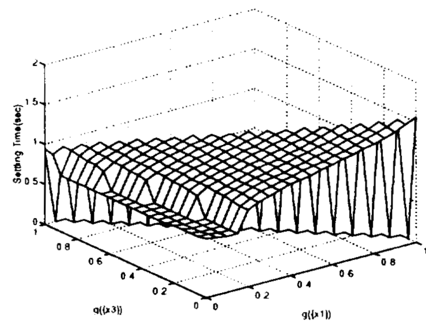

Fig. 9. Settling time

the degree of importance, we can get an output response with a decreased *overshoot* as in Figure 6(c) or that with a fast *settling time* as in Figure 6(d). We can conclude that our scheme can be used in the fine control problem.

To show the effect of the fuzzy measure on the system output, Figure 7 ~ 9 are given. Here, *rise time* is defined as the 10-90% *rise time*. The *overshoot* is the percent overshoot, *P.O.* which is defined as $P.O. = (M_p - fv)/fv \times 100\%$ where M_p is the peak value of the time response and fv is the final value of the response. The *settling time* is defined as the time required for the system to settle within 2% of the input amplitude. Figure 7 shows the *rise time* variation, where $g(\{x_1\})$ and $g(\{x_2\})$ are continuously changed from 0 to 1 with the condition $g(\{x_1\}) + g(\{x_2\}) + g(\{x_3\}) = 1$. Similarly, Figure 8 and Figure 9 show the *overshoot* variation and the *settling time* variation according to the change of the fuzzy measure. From Figure 7 ~ 9, we can see that as $g(\{x_i\})$ becomes larger, the value of the attribute x_i decreases. The effectiveness of the MFC scheme is clear from these plots.

V. Experiment

Figure 10 shows a diagram of the control system. The specifications of the motor are the same as those for computer simulation in Section IV. The position of the DC servomotor is controlled via an IBM PC/AT with a 12MHz Intel 80286 microprocessor and a 80287 coprocessor. The PC/AT is interfaced to the chopper circuit and the motor shaft encoder through a custom card containing a 82C54 programmable interval and time(PIT) and a 82C55 programmable peripheral interface(PPI). The computed control input is applied to the motor by adjusting the duty ratio of PWM signal, which is fed to the PWM chopper circuit.

We use Choquet integral as a fuzzy integral and probability measure as a fuzzy measure. The actual angle of 360° is normalized to 1 in this experiment. The conditions are as follows: $y_r(k)$ is set to 1.0 which corresponds to 360°, scale factor of $e(k)$ is 1.0, scale factor of $\Delta e(k)$ is 6.7, scale factor of $\Delta u(k)$ is 0.1, and sampling time is 20 *msec*.

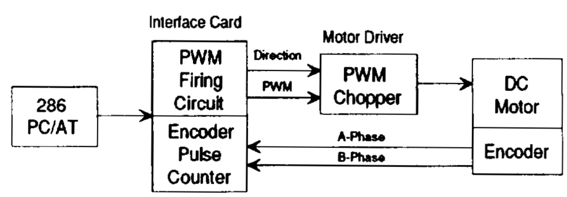

Fig. 10. DC servomotor position control system

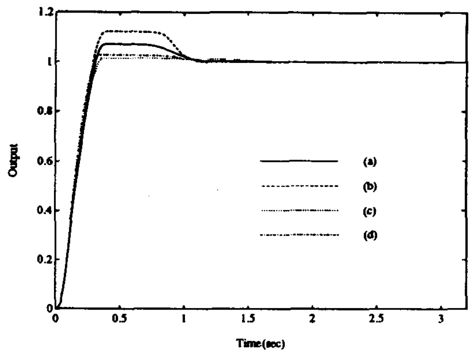

Fig. 11. Output responses (a) Well-tuned fuzzy output (b) $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 0$ (c) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 1$, $g(\{x_3\}) = 0$ (d) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 1$

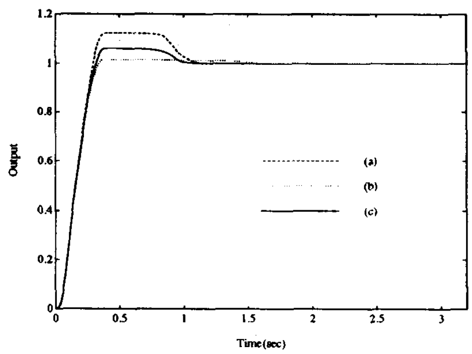

Fig. 12. Output responses (a) $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 0$ (b) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 1$, $g(\{x_3\}) = 0$ (c) $g(\{x_1\}) = 0.5$, $g(\{x_2\}) = 0.5$, $g(\{x_3\}) = 0$

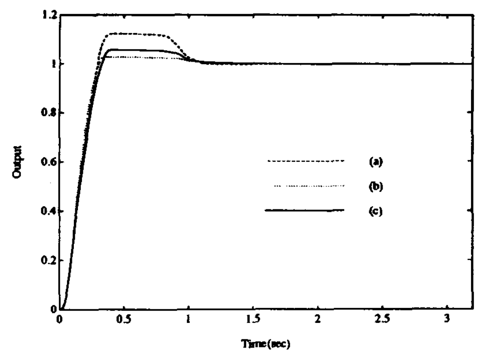

Fig. 13. Output responses (a) $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 0$ (b) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 1$ (c) $g(\{x_1\}) = 0.5$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 0.5$

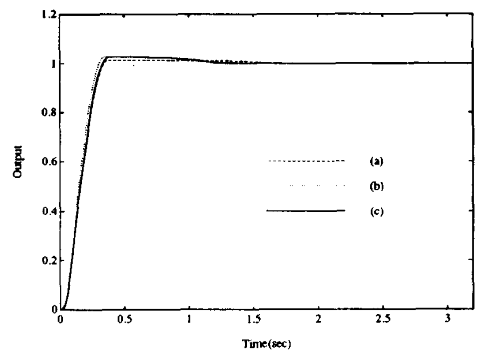

Fig. 14. Output responses (a) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 1$, $g(\{x_3\}) = 0$ (b) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 0$, $g(\{x_3\}) = 1$ (c) $g(\{x_1\}) = 0$, $g(\{x_2\}) = 0.5$, $g(\{x_3\}) = 0.5$

Compared to the well-tuned fuzzy output response as in Figure 11(a), we can get an output response with a fast *rise time* as in Figure 11(b) by assigning $g(\{x_1\}) = 1$, $g(\{x_2\}) = 0$, and $g(\{x_3\}) = 0$. Similarly, by changing the degree of importance, we can get an output response with a decreased *overshoot* as in Figure 11(c) or that with a fast *settling time* as in Figure 11(d).

Figure 12 $\sim$ 14 show the effect of the fuzzy measure on the system output. By assigning $g(\{x_1\}) = 0.5$, $g(\{x_2\}) = 0.5$, and $g(\{x_3\}) = 0$, we can get a output response Figure 12(b) which lies between Figure 12(a) and (c). Similarly, Figure 13 and Figure 14 show the output response variations according to the fuzzy measure values. From Figure 12 $\sim$ 14, we can see that the MFC can adjust subtly control results by changing the fuzzy measure values.

VI. Conclusions

In this paper, we have proposed a multicriteria fuzzy controller(MFC) using fuzzy measure and integral theory. In this controller, three items of time-domain specifications including *rise time*, *overshoot*, and *settling time* were considered as the attributes of the fuzzy measures and integrals. Good control performance has been obtained by the MFC since this controller can adjust subtly control results by changing the weights of fuzzy control rules with the fuzzy integral according to the preassigned fuzzy measure to the set of attributes. We have demonstrated the performance of MFC via computer simulations and experiments.

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