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Performance Analysis of M-QAM Scheme Combined With Multiuser Diversity Over Nakagami-m Fading Channels

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***Abstract*—In this paper, we consider an M-ary quadrature amplitude modulation (M-QAM) scheme combined with multiuser diversity over Nakagami-m fading channels. Assuming that delayed but error-free signal-to-noise ratio (SNR) feedback is available, we derive closed-form formulas for the average transmission rate and the average bit error rate (BER), which are also shown to be generalizations of many previous results. Through numerical studies and simulations, we check the validity of our analysis. In addition, we investigate the impact of the Nakagami fading parameter m and feedback delay on system performance.**

***Index Terms*—Average bit error rate (BER), average transmission rate, feedback delay, M-ary quadrature amplitude modulation (M-QAM), multiuser diversity, Nakagami-m fading.**

I. Introduction

The demand for wireless communication services has tremendously been increasing, but the available radio spectrum is scarce. Accordingly, the problem of enhancing spectral efficiency is a key issue in the design of future wireless networks. To solve this problem, multiuser diversity has been studied for a wireless network with multiple users [1], [2]. Multiuser diversity comes from independent channel variations between different users and can be exploited by allocating the channel to the user with the best channel condition. By utilizing multiuser diversity, we can maximize the total information-theoretic capacity of a wireless network [1], [2]. Another popular approach to enhance spectral efficiency is to apply M-ary quadrature amplitude modulation (M-QAM) schemes that adapt the transmission rate to a time-varying fading channel [3]–[5].

In this paper, we consider an M-QAM scheme combined with multiuser diversity over Nakagami-m fading channels. We use the Nakagami-m model because it represents a broad class of fading channels [3], [6]. Assuming that delayed but error-free signal-to-noise ratio (SNR) feedback is available, we derive closed-form formulas for the average transmission rate and the average bit error rate (BER). Note that the average transmission rate and the average BER are frequently used for performance analysis in the literature (e.g., [1]–[4] and [7]).

Our results are generalizations of previous works in [2] and [3]. In [3], Alouini and Goldsmith analyze the performance of an M-QAM scheme over Nakagami-m fading channels. They derive closed-form formulas for the spectral efficiency and the average BER under perfect channel estimation and without feedback delay. They also analyze the impact of feedback delay on the average BER, but multiuser diversity is not considered. In [2], Ma *et al.* consider a wireless network with multiple users over Rayleigh fading channels and derive closed-form formulas for the average transmission rate and the average BER of an M-QAM scheme combined with multiuser diversity, assuming that delayed but error-free SNR feedback is available.

The remainder of this paper is organized as follows. We describe the system characteristics in Section II. We derive closed-form formulas for the average transmission rate and the average BER in Sections III and IV, respectively. Numerical studies based on our analysis and simulation results are given in Section V. Finally, we give our conclusions in Section VI.

II. System Description

A. Basic Assumptions

We consider a downlink transmission from a base station (BS) to N mobile stations (MSs) with a constant transmission power. The basic assumptions regarding system modeling are the following.

1) Gray-coded finite M_n-ary QAM modes with $M_n := 2^n (n = 1, \ldots, J-1)$ are used for transmission without forward error correction.
2) The received complex envelopes at MSs are independent and identical wide-sense stationary (WSS) random processes.
3) The signal at each MS is perturbed by additive white Gaussian noise (AWGN) with zero mean and variance N_0, which is independent of the received complex envelope.
4) Perfect channel estimation is possible at each MS, and the estimated SNR is transmitted through a delayed but error-free feedback path from each MS to the BS with a delay time τ.

Under these assumptions, the received signal $r_n(t)$ of the nth MS at time t can be expressed as $r_n(t) = g_n(t)b(t) + w_n(t)$, where $g_n(t)$ is the channel gain, i.e., the received complex envelope, between the BS and the nth MS, $b(t)$ is the signal broadcast from the BS, and $w_n(t)$ represents the AWGN in the channel of the nth MS [2].

We define the instantaneous SNR $\gamma_n(t)$ of the nth MS at time t by $|g_n(t)|^2/N_0$. We denote the average received SNR $\mathrm{E}[\gamma_n(t)]$ by $\overline{\gamma}$.

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Note that all MSs in the system have the same average received SNR $\overline{\gamma}$ since we assume that the received complex envelopes $g_n(t)$ are all identical and that $\overline{\gamma}$ is independent of time t due to the WSS assumption of $g_n(t)$.

Since we assume that $g_n(t)$'s are all identical, we use $\gamma(t)$ to denote the generic random process for $\gamma_n(t)$. In the Nakagami-m model, the probability density function (pdf) of $\gamma := \gamma(t)$ at an arbitrary time t is given by [8]

$$p_\gamma(\gamma) = \left(\frac{m}{\overline{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right)$$

where m is the Nakagami fading parameter $(m \geq 1/2)$, and $\Gamma(m)$ is the Gamma function defined by $\Gamma(m) = \int_0^\infty y^{m-1}\exp(-y)dy$.

B. M-QAM Scheme Combined With Multiuser Diversity

In this strategy, the BS transmits data only to the MS that has the best instantaneous SNR, which is called the best MS, each time. Since we assume an error-free feedback path, the received feedback SNRs at the BS are equal to the estimated SNRs at the MSs. Let $\hat{\gamma}(t)$ denote the feedback SNR of the best MS at time t. Then, considering the feedback delay time τ, $\hat{\gamma}(t)$ is computed as $\hat{\gamma}(t) = \max_n \gamma_n(t-\tau)$.

For the M-QAM scheme considered in this paper, we have a finite set of constellation sizes $M := \{M_0, \ldots, M_{J-1}\}$, where $M_0 := 0$ implies no transmission. Given a set of switching thresholds $\mathbf{t} := \{t_0, \ldots, t_J\}$, where $t_0 := 0$ and $t_J := \infty$, the QAM with the constellation size M_j is used for transmission at time t if the best MS's SNR $\hat{\gamma}(t)$ at time t is in the range $[t_j, t_{j+1})$.

III. Average Transmission Rate

In this section, we derive the average transmission rate, i.e., the average bit per symbol (BPS) rate, of the multiuser system considered in Section II. When we use the switching threshold $\mathbf{t}$, the average BPS rate can be formulated as the weighted sum of the individual M-QAM rates using their probabilities as follows [2]:

$$\overline{R}(\mathbf{t}) = \sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} R_j p_{\hat{\gamma}}(x)dx \tag{1}$$

where $R_0 := 0$, $R_j := \log_2 M_j (j = 1, \ldots, J-1)$, and $p_{\hat{\gamma}}(x)$ represents the pdf of $\hat{\gamma} := \hat{\gamma}(t)$ at an arbitrary time t. The average BPS rate is derived in the following theorem.

Theorem 1:

$$\overline{R}(\mathbf{t}) = \sum_{j=0}^{J-1} \frac{R_j}{[\Gamma(m)]^N} \times \left\{ \left[\tilde{\Gamma}\left(m, \frac{m}{\overline{\gamma}}t_{j+1}\right)\right]^N - \left[\tilde{\Gamma}\left(m, \frac{m}{\overline{\gamma}}t_j\right)\right]^N \right\}$$

where $\tilde{\Gamma}(m,x)$ is the lower incomplete Gamma function defined by $\tilde{\Gamma}(m,x) = \int_0^x y^{m-1}\exp(-y)dy$.

Proof: Based on the independent and identical WSS assumption of $\gamma_n(t)$, we have

$$P(\hat{\gamma} \leq x) = [F_\gamma(x)]^N \tag{2}$$

where $F_\gamma(x) := \int_0^x p_\gamma(\gamma)d\gamma = \tilde{\Gamma}(m, (m/\overline{\gamma})x)/\Gamma(m)$. By combining (1) and (2), our theorem follows immediately. ■

Note that the feedback delay time τ is irrelevant in the above derivation of Theorem 1. The reason for this is as follows. The constellation size of the M-QAM scheme at time t is determined by the value of $\hat{\gamma}(t) = \max_n \gamma_n(t-\tau)$. Therefore, the BPS rate changes over time. However, the average BPS rate at an arbitrary time depends on $F_\gamma(x) = P(\gamma \leq x) = P(\gamma_n(t-\tau) \leq x)$, $1 \leq n \leq N$, which is time invariant due to the WSS assumption of the channel gain process. Accordingly, the average BPS rate is irrelevant to time and feedback delay.

Note that our result in Theorem 1 includes previously known results for special cases, e.g., the multiuser system over Rayleigh fading channels [2] and the single-user system over Nakagami-m fading channels [3].

IV. Average BER

A. BER of M-QAM Over AWGN Channels

The BER of Gray-coded M-QAM over AWGN channels can well be approximated by $\text{BER}(M,\gamma)$, which is a function of constellation size M and the received SNR γ, as follows [4]:

$$\text{BER}(M,\gamma) \approx 0.2\exp\left\{-\frac{3\gamma}{2(M-1)}\right\}. \tag{3}$$

Since using the approximated BER expression (3) is justified for both practical and analytical purposes as in [3], we will use (3) in the derivation of the average BER.

B. Feedback Delay Impact

The BER is a function of constellation size and the received SNR, as given in (3). When the feedback information is delayed, the constellation size is selected based on the outdated SNR rather than the received SNR. Hence, in contrast to the average BPS rate, the BER and the average BER are affected by feedback delay.

Given the estimate $\hat{\gamma} := \hat{\gamma}(t) = \max_n \gamma_n(t-\tau)$ of the best MS selected at time t, we need to know the conditional pdf $p_{\gamma_\tau|\hat{\gamma}}(\gamma_\tau|\hat{\gamma})$ of the received SNR $\gamma_\tau := \gamma(t)$ of the best MS selected at time t. In [3], the conditional probability $p_{\gamma_\tau|\hat{\gamma}}(\gamma_\tau|\hat{\gamma})$ is derived based on the work of Nakagami [9] as follows:

$$p_{\gamma_\tau|\hat{\gamma}}(\gamma_\tau|\hat{\gamma}) = \frac{m}{(1-\rho)\overline{\gamma}}\left(\frac{\gamma_\tau}{\rho\hat{\gamma}}\right)^{(m-1)/2} I_{m-1}\left(\frac{2m\sqrt{\rho\hat{\gamma}\gamma_\tau}}{(1-\rho)\overline{\gamma}}\right) \times \exp\left(-\frac{m(\rho\hat{\gamma}+\gamma_\tau)}{(1-\rho)\overline{\gamma}}\right) \tag{4}$$

where $I_{m-1}(\cdot)$ is the $(m-1)$th-order modified Bessel function of the first kind, $\rho := J_0^2(2\pi f_d\tau)$ is the correlation factor between $\hat{\gamma}$ and γ_τ, f_d is the maximum Doppler frequency, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

C. Average BER

Given a set of switching thresholds $\mathbf{t}$, the average BER is defined by the ratio of the average number of bits in error to the average number of transmitted bits [2], [3], [7], i.e.,

$$\overline{\text{BER}}(\mathbf{t}) = \frac{1}{\overline{R}(\mathbf{t})}\sum_{j=0}^{J-1} R_j \overline{\text{BER}}_j \tag{5}$$

where

$$\overline{\text{BER}}_j := \int_{t_j}^{t_{j+1}}\int_0^\infty \text{BER}(M_j,\gamma_\tau)\, p_{\gamma_\tau|\hat{\gamma}}(\gamma_\tau|\hat{\gamma})d\gamma_\tau p_{\hat{\gamma}}(\hat{\gamma})d\hat{\gamma}. \tag{6}$$

We denote the inner integral in (6) by $\widehat{\mathrm{BER}}(j,\hat{\gamma})$ and simplify (6) as follows:

$$\widehat{\mathrm{BER}}(j,\hat{\gamma}) := \int_0^\infty \mathrm{BER}(M_j,\gamma_\tau) p_{\gamma_\tau|\hat{\gamma}}(\gamma_\tau|\hat{\gamma}) d\gamma_\tau \tag{7}$$

$$\overline{\mathrm{BER}}_j = \int_{t_j}^{t_{j+1}} \widehat{\mathrm{BER}}(j,\hat{\gamma}) p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}. \tag{8}$$

The following proposition gives the closed-form formula for $\widehat{\mathrm{BER}}(j,\hat{\gamma})$.

Proposition 1:

$$\widehat{\mathrm{BER}}(j,\hat{\gamma}) = 0.2 \left\{ \frac{2C(M_j-1)}{3\rho} \right\}^m \exp(-C\hat{\gamma}) \tag{9}$$

where $C := C(m,\rho,\overline{\gamma},M_j) = (3m\rho/3\overline{\gamma}(1-\rho)+2m(M_j-1))$.

Proof: See Appendix A.

Then, combining (8) and (9) yields

$$\overline{\mathrm{BER}}_j = 0.2 \left\{ \frac{2C(M_j-1)}{3\rho} \right\}^m \int_{t_j}^{t_{j+1}} \exp(-C\hat{\gamma}) p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}. \tag{10}$$

To compute the integral on the right-hand side of (10), we need Proposition 2, shown at the bottom of the page. The proof of Proposition 2 is given in Appendix B. Note that if $N=1$, the second term in Proposition 2 is defined to be zero.

By using (10) and Proposition 2 with $a := C$, we can calculate $\overline{\mathrm{BER}}_j$, shown in Theorem 2 at the bottom of the page. Finally, the average BER is computed from Theorem 2 and (5).

For a special case, we consider a multiuser system over Rayleigh fading channels. Then, from Theorem 2 and (5) with $m=1$, we have

$$\overline{\mathrm{BER}}(\mathbf{t}) = \frac{1}{\overline{R}(\mathbf{t})} \sum_{j=0}^{J-1} R_j \sum_{l=0}^{N-1} (-1)^l \binom{N-1}{l} C_j(l) \times \left[\exp\left(-\frac{t_j}{A_j(l)}\right) - \exp\left(-\frac{t_{j+1}}{A_j(l)}\right) \right]$$

where

$$C_j(l) := \frac{0.4N(M_j-1)}{2(1+l)(M_j-1)+3(1+l-l\rho)\overline{\gamma}}$$

$$A_j(l) := \frac{[2(M_j-1)+3\overline{\gamma}(1-\rho)]\,\overline{\gamma}}{2(1+l)(M_j-1)+3(1+l-l\rho)\overline{\gamma}}.$$

It can easily be checked that the above result is identical to the result obtained in [2], which partially verifies the validity of our derivation.

Remark: Our average BER formula can easily be generalized to consider cases where MSs experience different feedback delays as follows. Let τ_n be the feedback delay experienced by the nth MS. From the fact that, for $\hat{\gamma} = \max_n \gamma_n(t-\tau_n)$

$$\begin{aligned} P\left(\hat{\gamma} = \gamma_k(t-\tau_k), \gamma_k(t-\tau_k) \le x\right) &= \int_0^x P\left(\hat{\gamma} \le u | \gamma_k(t-\tau_k) = u\right) \cdot p_\gamma(u) du \\ &= \int_0^x F_\gamma(u)^{N-1} \cdot p_\gamma(u) du \end{aligned}$$

Proposition 2: For $a \ge 0$ and a positive integer m,

$$\int_{t_j}^{t_{j+1}} \exp(-ax) p_{\hat{\gamma}}(x) dx = \frac{N}{\Gamma(m)} \left[G(\tilde{a}+1, m-1, \tilde{t}_j) - G(\tilde{a}+1, m-1, \tilde{t}_{j+1}) \right] + \frac{N}{\Gamma(m)} \sum_{l=1}^{N-1} \binom{N-1}{l} (-1)^l \times \sum_{k_1=0}^{m-1}, \ldots, \sum_{k_l=0}^{m-1} \frac{G(\tilde{a}+1+l, \sum_i k_i + m - 1, \tilde{t}_j) - G(\tilde{a}+1+l, \sum_i k_i + m - 1, \tilde{t}_{j+1})}{(k_1)!(k_2)!, \ldots, (k_l)!}$$

where $G(\mu,n,x) := \exp(-\mu x) \sum_{k=0}^n \frac{n!}{k!} \frac{x^k}{\mu^{n-k+1}}$, $\tilde{t}_j := m \cdot t_j/\overline{\gamma}$, and $\tilde{a} := \overline{\gamma} \cdot a/m$.

Theorem 2: $\overline{\mathrm{BER}}_j$ is computed as

$$\overline{\mathrm{BER}}_j = 0.2 \left\{ \frac{2C(M_j-1)}{3\rho} \right\}^m \cdot \left\{ \frac{N}{\Gamma(m)} \left[G(\tilde{C}+1, m-1, \tilde{t}_j) - G(\tilde{C}+1, m-1, \tilde{t}_{j+1}) \right] + \frac{N}{\Gamma(m)} \sum_{l=1}^{N-1} \binom{N-1}{l} (-1)^l \times \sum_{k_1=0}^{m-1}, \ldots, \sum_{k_l=0}^{m-1} \frac{G(\tilde{C}+1+l, \sum_i k_i + m - 1, \tilde{t}_j) - G(\tilde{C}+1+l, \sum_i k_i + m - 1, \tilde{t}_{j+1})}{(k_1)!(k_2)!, \ldots, (k_l)!} \right\}$$

where $G(\mu,n,x) := \exp(-\mu x) \sum_{k=0}^n \frac{n!}{k!} \frac{x^k}{\mu^{n-k+1}}$, $\tilde{t}_j := m \cdot t_j/\overline{\gamma}$, and $\tilde{C} := \overline{\gamma} \cdot C/m$.

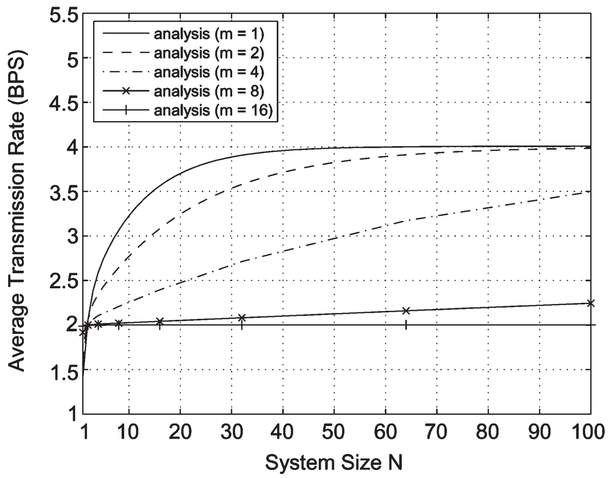

Fig. 1. Average transmission rate ($\overline{\gamma} = 15$ dB).

the joint pdf $p_{\hat{\gamma},k}(x)$ of $\hat{\gamma}$ and the event that the kth MS is the best MS is given by

$$p_{\hat{\gamma},k}(x) = F_{\gamma}(x)^{N-1} \cdot p_{\gamma}(x).$$

Then, the expression of $\overline{\mathrm{BER}}_j$ in (6) changes to

$$\overline{\mathrm{BER}}_j = \sum_{k=1}^{N} \int_{t_j}^{t_{j+1}} \int_{0}^{\infty} \mathrm{BER}(M_j, \gamma_{\tau_k}) \times p_{\gamma_{\tau_k}|\hat{\gamma}}(\gamma_{\tau_k}|\hat{\gamma}) d\gamma_{\tau_k} \cdot p_{\hat{\gamma},k}(\hat{\gamma}) d\hat{\gamma}.$$

Hence, by using the same arguments as in the derivations of Propositions 1 and 2 and Theorem 2, we can obtain the average BER for this case.

V. Numerical Studies

In this section, we consider three scenarios to investigate the behaviors of the average transmission rate and the average BER. In all scenarios, the target BER, which is denoted by BER_0, is set to 10^{-4}, and a set of constellation sizes $M = \{0, 4, 16, 64, 256\}$ is considered for adaptive modulation. Unless otherwise mentioned, the average received SNR $\overline{\gamma}$ is set to 15 dB. We use the *generic* set of switching thresholds $\mathbf{t}^g = \{t_0^g, t_1^g, \ldots, t_J^g\}$, as given in [2]. That is, $t_0^g := 0$, $t_J^g := \infty$, and $t_j^g (j = 1, \ldots, J-1)$ is set to the SNR so that $\mathrm{BER}(M_j, t_j^g) = \mathrm{BER}_0$, i.e., $t_j^g := -(2(M_j - 1)/3) \ln(5 \cdot \mathrm{BER}_0)$. Note that for the QAM with a constellation size $M \geq 4$ and for $\mathrm{BER}_0 \leq 10^{-2}$, the generic thresholds guarantee that the instantaneous BER remains below the target BER when there is no feedback delay [2]. To verify our analysis, we provide simulation results by using the Nakagami-m fading simulation model in [10]. All simulation results are obtained by averaging values from five simulation runs with $f_d = 100$ Hz.

A. Scenario 1

In Scenario 1, we examine the impact of system size (or the number of MSs) N and the Nakagami fading parameter m on the average BPS rate. To do this, we consider a system with $m = 1, 2, 4, 8, 16$, and change the value of the system size from $N = 1$ to $N = 100$. The results are plotted in Fig. 1. We see that the effect of multiuser diversity becomes less significant as m increases. In addition, the average BPS rates converge to different values for each value of

TABLE I
System Size to Optimize the Benefit From Multiuser Diversity ($\overline{\gamma} = 15$ dB)

m	1	2	3	4	5
N	45	80	155	285	520

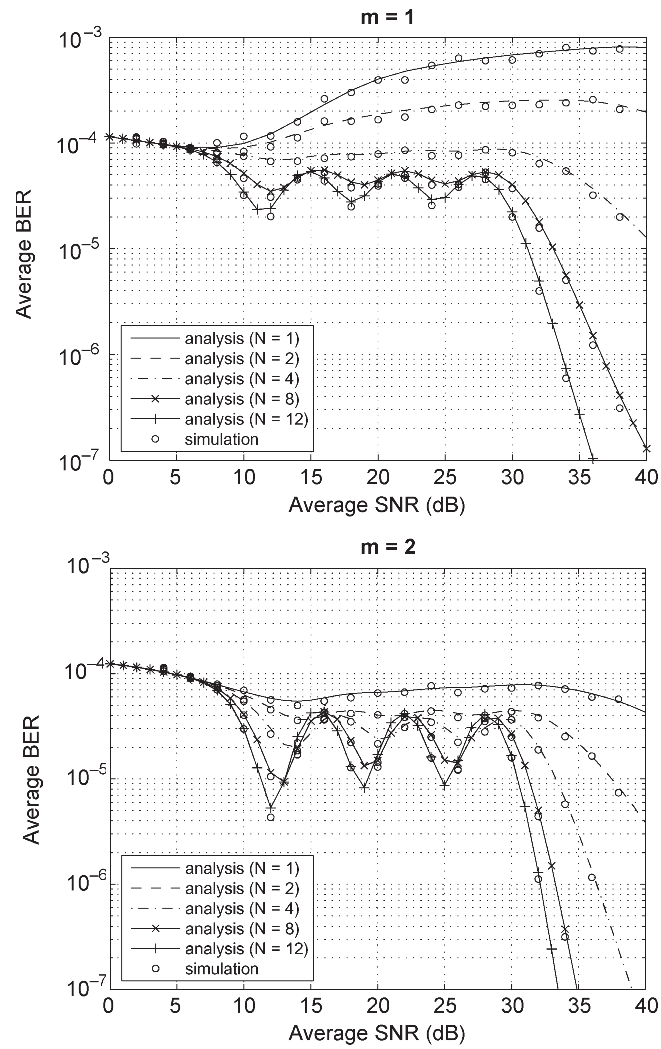

Fig. 2. Average BER ($f_d\tau = 0.05$, $\mathrm{BER}_0 = 10^{-4}$).

m, and the convergence rate decreases with an increase in m. This implies that the system size, which can optimize the benefit from multiuser diversity, is different for each fading environment, i.e., the Nakagami fading parameter m. We define the optimal system size as the *minimum* number of MSs with which we can achieve 99% of the average BPS rate of the system with a sufficiently large number of MSs, e.g., $N = 10^3$. We summarize the resulting optimal system sizes for different values of m in Table I.

B. Scenario 2

As shown in [2], the use of the generic set of switching thresholds guarantees the target BER, assuming no feedback delay. However, we do not know if the use of generic thresholds can also guarantee the target BER when a feedback delay exists over Nakagami-m fading channels. In this scenario, we examine this question. For this, the normalized feedback delay $f_d\tau$ is set to 0.05, and we calculate the average BER as we change the average SNR. Fig. 2 shows the results

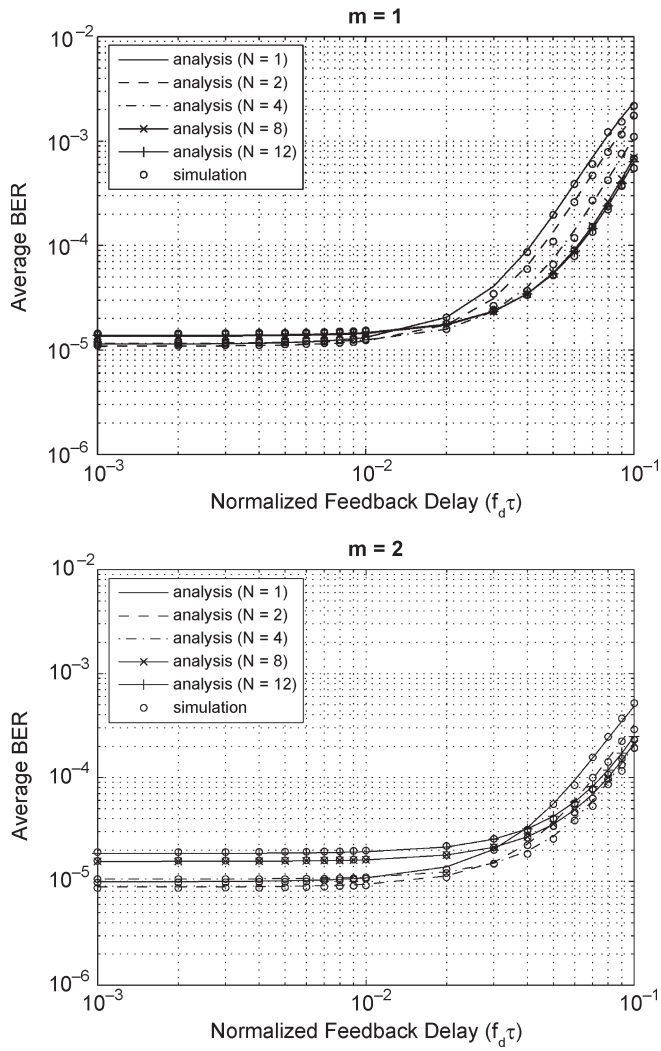

Fig. 3. Impact of feedback delay on the average BER ($\overline{\gamma} = 15$ dB, $\mathrm{BER}_0 = 10^{-4}$).

when $m = 1, 2$. The fluctuation of the average BER in Fig. 2 results from the use of a discrete-rate M-QAM. In a neighborhood of a threshold, the received SNR value higher than the threshold has a higher BER than the received SNR value lower than the threshold. So, for *some* average SNR values $\overline{\gamma}$, e.g., $13 < \overline{\gamma} < 15$ in Fig. 2, the increase in $\overline{\gamma}$ increases the probability of having SNR values higher than the threshold and decreases the probability of having SNR values lower than the threshold. This results in the increase in the average BER. Note that this effect is alleviated if we use a set of QAMs where the differences between adjacent QAM sizes are small.

As shown in the figures, when $m = 1$, the use of generic thresholds does not always satisfy the target BER constraint. However, when $m = 2$, the BER constraint is satisfied for a wide range of SNRs (> 5 dB). For other values of $m(\geq 3)$, we observed the same behavior, and the detailed results are omitted here. Hence, we conclude that we should be careful in selecting a set of switching thresholds when a feedback delay exists.

C. Scenario 3

In this scenario, we investigate the impact of feedback delay on the average BER in detail. Fig. 3 shows the average BER as we change the normalized feedback delay $f_d\tau$ from 10^{-3} to 10^{-1} for system sizes $N = 1, 2, 4, 8, 12$ with the fading parameter $m = 1, 2$.

TABLE II
NORMALIZED FEEDBACK DELAY THRESHOLD
$f_d\tau(\overline{\gamma} = 15\ \mathrm{dB}, \mathrm{BER}_0 = 10^{-4})$

m	1	2	3
$f_d\tau$	$2 \cdot 10^{-2}$	$3 \cdot 10^{-2}$	$4 \cdot 10^{-2}$

In each case, we see that a delay threshold exists up until the average BER is approximately flat, regardless of the system size, and, after which, there occurs a rapid degradation in the average BER. This delay threshold depends on the value of m and $\overline{\gamma}$ and is summarized in Table II for $\overline{\gamma} = 15$ dB. From the table, we see that the delay threshold increases with an increase in m. This implies that the effect of feedback delay on the average BER becomes less significant as m increases.

The fact that the delay threshold depends on the Nakagami fading parameter m and the average received SNR $\overline{\gamma}$ is quite useful when we design an SNR feedback protocol over the Nakagami-m fading channel. That is, from the estimation of the fading parameter m and the average SNR $\overline{\gamma}$, we can find a tolerable value of feedback delay up until the average BER is not degraded. For the estimation of the Nakagami fading parameter m, see [11] and the references therein.

VI. CONCLUSION

In this paper, we have analyzed the performance of an M-QAM scheme combined with multiuser diversity over Nakagami-m fading channels. Assuming that delayed but error-free SNR feedback is available, we derived closed-form formulas for the average transmission rate and the average BER, which are shown to be well matched with the simulation results. We also provided numerical examples to observe the impact of feedback delay on system performance.

APPENDIX A
PROOF OF PROPOSITION 1

We first substitute (3) and (4) into (7) to obtain

$$\widehat{\mathrm{BER}}(j, \hat{\gamma}) = C_1 \int_0^\infty \gamma_\tau^{\frac{m-1}{2}} \exp(-C_2\gamma_\tau) I_{m-1}(C_3\sqrt{\gamma_\tau}) d\gamma_\tau$$

where

$$C_1 := C_1(m, \rho, \overline{\gamma}, \hat{\gamma}) = \frac{0.2m}{(1-\rho)\overline{\gamma}} \left(\frac{1}{\rho\hat{\gamma}}\right)^{\frac{m-1}{2}} \exp\left\{-\frac{m\rho\hat{\gamma}}{(1-\rho)\overline{\gamma}}\right\}$$

$$C_2 := C_2(m, \rho, \overline{\gamma}, M_j) = \frac{3}{2(M_j - 1)} + \frac{m}{(1-\rho)\overline{\gamma}}$$

$$C_3 := C_3(m, \rho, \overline{\gamma}, \hat{\gamma}) = \frac{2m\sqrt{\rho\hat{\gamma}}}{(1-\rho)\overline{\gamma}}.$$

Let $x := \sqrt{2C_2\gamma_\tau}$. By the change of variables, we obtain

$$\begin{aligned}\widehat{\mathrm{BER}}&(j, \hat{\gamma}) \\ &= \frac{C_1}{C_2}\left(\frac{1}{\sqrt{2C_2}}\right)^{m-1} \int_0^\infty x^m \exp\left(-\frac{x^2}{2}\right) I_{m-1}(C_4 x) dx \\ &= \frac{C_1}{C_2}\left(\frac{1}{\sqrt{2C_2}}\right)^{m-1} C_4^{m-1} \exp\left(\frac{C_4^2}{2}\right) Q_m(C_4, 0) \qquad (11)\end{aligned}$$

where $C_4 := C_3/\sqrt{2C_2}$. Here, $Q_m(\alpha,\beta)$ is the generalized Marcum Q-function of order m defined by $Q_m(\alpha,\beta) = (1/\alpha^{m-1})\int_\beta^\infty x^m \exp(-(x^2+\alpha^2/2))I_{m-1}(\alpha x)dx$ [8]. Note that $Q_m(\alpha,\beta) = 1-\exp(-(\alpha^2+\beta^2/2))\sum_{r=m}^\infty((\beta/\alpha))^r I_r(\alpha\beta)$ [8]. Hence, (11) is reduced to

$$\begin{aligned}\widehat{\mathrm{BER}}(j,\hat{\gamma}) &= \frac{C_1}{C_2}\left(\frac{1}{\sqrt{2C_2}}\right)^{m-1} C_4^{m-1}\exp\left(\frac{C_4^2}{2}\right)\\ &= 0.2\left\{\frac{2C(M_j-1)}{3\rho}\right\}^m \exp(-C\hat{\gamma})\end{aligned}$$

where $C := C(m,\rho,\overline{\gamma},M_j) = (3m\rho/3\overline{\gamma}(1-\rho)+2m(M_j-1))$. ■

Appendix B
Proof of Proposition 2

We start with (2) to obtain $p_{\hat{\gamma}}(x)$ as $p_{\hat{\gamma}}(x) = N[F_\gamma(x)]^{N-1}p_\gamma(x)$. It then follows that

$$\begin{aligned}&\int_{t_j}^{t_{j+1}} \exp(-ax)p_{\hat{\gamma}}(x)dx\\ &= \int_{t_j}^{t_{j+1}} \exp(-ax)N\left[F_\gamma(x)\right]^{N-1}p_\gamma(x)dx\\ &= \frac{N}{[\Gamma(m)]^N}\left(\frac{m}{\overline{\gamma}}\right)^m \int_{t_j}^{t_{j+1}} \exp\left(-\left(a+\frac{m}{\overline{\gamma}}\right)x\right)x^{m-1}\\ &\quad\times\left[\tilde{\Gamma}\left(m,\frac{m}{\overline{\gamma}}x\right)\right]^{N-1}dx.\end{aligned}$$

Let $t := m\cdot x/\overline{\gamma}$. By the change of variables, we obtain

$$\begin{aligned}&\int_{t_j}^{t_{j+1}} \exp(-ax)p_{\hat{\gamma}}(x)dx\\ &= \frac{N}{[\Gamma(m)]^N}\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1)t\right)t^{m-1}\left[\tilde{\Gamma}(m,t)\right]^{N-1}dt \qquad (12)\end{aligned}$$

where $\tilde{t}_j := m\cdot t_j/\overline{\gamma}$, and $\tilde{a} := \overline{\gamma}\cdot a/m$.

For a positive integer m, the lower incomplete gamma function can be expressed as [12]

$$\tilde{\Gamma}(m,x) = \Gamma(m)\left[1-\exp(-x)\sum_{k=0}^{m-1}\frac{x^k}{k!}\right]. \qquad (13)$$

Substituting (13) into (12) and then using Binomial expansion, we obtain

$$\begin{aligned}&\int_{t_j}^{t_{j+1}} \exp(-ax)p_{\hat{\gamma}}(x)dx\\ &= \frac{N}{\Gamma(m)}\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1)t\right)t^{m-1}\\ &\quad\times\left[1-\exp(-t)\sum_{k=0}^{m-1}\frac{t^k}{k!}\right]^{N-1}dt\\ &= \frac{N}{\Gamma(m)}\sum_{l=0}^{N-1}\binom{N-1}{l}(-1)^l\\ &\quad\times\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1+l)t\right)t^{m-1}\left(\sum_{k=0}^{m-1}\frac{t^k}{k!}\right)^l dt.\end{aligned}$$

Note that for $u_1, u_2 > 0, \operatorname{Re}\mu > 0$ [13]

$$\begin{aligned}\int_{u_1}^{u_2} x^n\exp(-\mu x)dx &= \exp(-u_1\mu)\sum_{k=0}^n\frac{n!}{k!}\frac{u_1^k}{\mu^{n-k+1}}\\ &\quad-\exp(-u_2\mu)\sum_{k=0}^n\frac{n!}{k!}\frac{u_2^k}{\mu^{n-k+1}}\\ &= G(\mu,n,u_1)-G(\mu,n,u_2)\end{aligned}$$

where $G(\mu,n,x) := \exp(-x\mu)\sum_{k=0}^n(n!/k!)(x^k/\mu^{n-k+1})$.

Suppose $l=0$. Then

$$\begin{aligned}&\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1+l)t\right)t^{m-1}\left(\sum_{k=0}^{m-1}\frac{t^k}{k!}\right)^l dt\\ &= G\left(\tilde{a}+1,m-1,\tilde{t}_j\right)-G\left(\tilde{a}+1,m-1,\tilde{t}_{j+1}\right). \qquad (14)\end{aligned}$$

Now, for $l = 1,\ldots,N-1$

$$\begin{aligned}&\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1+l)t\right)t^{m-1}\left(\sum_{k=0}^{m-1}\frac{t^k}{k!}\right)^l dt\\ &= \sum_{k_1=0}^{m-1},\ldots,\sum_{k_l=0}^{m-1}\int_{\tilde{t}_j}^{\tilde{t}_{j+1}} \exp\left(-(\tilde{a}+1+l)t\right)t^{m-1}\\ &\quad\times\frac{t^{k_1}t^{k_2},\ldots,t^{k_l}}{(k_1)!(k_2)!,\ldots,(k_l)!}dt\\ &= \sum_{k_1=0}^{m-1},\ldots,\sum_{k_l=0}^{m-1}\left\{\frac{G(\tilde{a}+1+l,\sum_i k_i+m-1,\tilde{t}_j)}{(k_1)!(k_2)!,\ldots,(k_l)!}\right.\\ &\quad\left.-\frac{G(\tilde{a}+1+l,\sum_i k_i+m-1,\tilde{t}_{j+1})}{(k_1)!(k_2)!,\ldots,(k_l)!}\right\}. \qquad (15)\end{aligned}$$

By combining (14) and (15), we finally obtain

$$\int_{t_j}^{t_{j+1}} e^{-ax} p_{\hat{\gamma}}(x) dx$$
$$= \frac{N}{\Gamma(m)} \left[G(\tilde{a}+1, m-1, \tilde{t}_j) - G(\tilde{a}+1, m-1, \tilde{t}_{j+1}) \right]$$
$$+ \frac{N}{\Gamma(m)} \sum_{l=1}^{N-1} \binom{N-1}{l} (-1)^l$$
$$\times \sum_{k_1=0}^{m-1}, \ldots, \sum_{k_l=0}^{m-1} \left\{ \frac{G(\tilde{a}+1+l, \sum_i k_i + m - 1, \tilde{t}_j)}{(k_1)!(k_2)!, \ldots, (k_l)!} - \frac{G(\tilde{a}+1+l, \sum_i k_i + m-1, \tilde{t}_{j+1})}{(k_1)!(k_2)!, \ldots, (k_l)!} \right\}.$$

■

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A Cross-Layer Approach for Cooperative Networks

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***Abstract*—This paper deals with a cross-layer approach for cooperative diversity networks, which use a combination of amplify-and-forward (AF) and decode-and-forward (DF) as a relaying strategy. Based on a well-selected ad hoc configuration, the proposed approach combines the cooperative diversity concept with a simultaneous optimization of physical, network, and multiple access control layers. The considered optimization problem requires an appropriate distribution of three roles among the network nodes, which are the diversity relays (AF concept), the intermediate router (DF and routing), and the destination (scheduling). The proposed role assignment is based on the instantaneous channel conditions between the links and jointly supports performance optimization and a long-term fairness concept. To minimize the required complexity, a partial and quantized channel feedback is also proposed. The proposed cross-layer solution is compared with conventional approaches by computer simulations and theoretical studies, and we show that it achieves an efficient performance–complexity tradeoff.**

***Index Terms*—Ad hoc networks, amplify-and-forward (AF), cooperative systems, cross-layer design, relay channels.**

I. Introduction

Cooperative diversity is a new diversity technique that was proposed in the literature as a form of a spatial diversity system [1]–[6]. It uses the broadcast nature of the wireless medium, and its basic idea is that the terminals, which are in the coverage area of a transmitter, can forward an "overheard" version of the transmitted signal. This virtual antenna array produces the desired diversity gain at the destination. Among the proposed reforward strategies, the amplify-and-forward (AF), which involves a simple scale and retransmission of the received signal by the relay nodes, seems to give a good performance and complexity tradeoff [7].

Currently, there has been a lot of interest in AF cooperative schemes, where the relay nodes are selected according to some well-defined system parameters [8]–[11]. In [12], the authors proposed a distributed relay selection for a two-hop AF system, where the selected criterion is the best instantaneous signal-to-noise ratio (SNR) composed of the SNR across the two hops. This solution extracts a diversity gain on the order of the number of relays [13] with a lower complexity than the complicated distributed space–time codes. However, a distributed relay selection is time sensitive and requires perfect time synchronization among the nodes, which is a crucial issue for practical systems. On the other hand, centralized approaches require continuous channel feedback from all the links of the network and, therefore, result in a high power consumption [14]. The related complexity is increased as the number of nodes and hops is increased. In resource-constrained wireless systems such as sensor networks [15], by monitoring the

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