Progressive coding of error-diffused bilevel images

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Abstract. We propose a new method to compress error-diffused bilevel images with resolution scalability. This method is a combination of inverse halftoning and rehalftoning. For the inverse halftoning, we combine $2 \times 2$ dots into a single pixel of a resolution-reduced image, where each pixel has a multilevel value of 0, 1, 2, 3, and 4. After the inverse halftoning, the resolution-reduced multilevel image is halftoned by using an error diffusion algorithm. Thus, the resolution of the error-diffused bilevel images can be reduced by repetition of the inverse halftoning and rehalftoning processes. After reducing the image size, we encode an error-diffused bilevel image progressively from the lowest resolution image to the highest resolution image. To encode higher resolution images, we use the information in the previously coded lower resolution images. We encode higher resolution images, we use the information in the previously coded lower resolution image. Though the compression ratios of the proposed algorithm are similar to those of progressive Joint Bilevel Image Processing Group (JBIG), the image quality of the resolution-reduced image from the proposed algorithm is much better than that from the progressive JBIG. © 2003 SPIE and IS&T. [DOI: 10.1117/1.1525794]

1 Introduction

The error diffusion algorithm is a good blue noise generator. Blue noise patterns generated by the error diffusion algorithm enjoy the benefits of aperiodic and uncorrelated structure without low-frequency graininess. Therefore, the neighboring pixels are not correlated to the current pixel in error-diffused bilevel images, and the conventional context-based algorithms do not work well for compression of error-diffused bilevel images.

Ting and Riskin proposed a bilevel image compression method. It took inverse halftoning of a bilevel image, and compressed the inverse-halftoned gray-scale image using vector quantization (VQ). In the decoding process, the gray-scale image was reconstructed by the inverse VQ and halftoning. However, if we compress an error-diffused image using this method, it is impossible to reconstruct the original error-diffused image without information loss, which means it is a lossy compression. Langdon and Riswan used an arithmetic coding for compression of bilevel images. They combined a “Context” algorithm and an arithmetic-coding algorithm. The international standard of bilevel image compression, JBIG (Joint Bilevel Image Processing Group), adopted the context-based arithmetic coding. The JBIG algorithm can code bilevel images using a sequential mode or a progressive mode. In general, there are two kinds of progressive coding modes to compress images: one is the resolution-scalable coding and the other is the quality-scalable coding. The JBIG provides only resolution scalability.

When the JBIG encodes error-diffused bilevel images in a sequential mode, the compression performance is better than that achieved in a progressive mode. Low correlation between a current pixel and its neighboring pixels causes low compression ratios in the error-diffused bilevel images. In addition, for the progressive mode, JBIG reduces the resolutions of bilevel images using deterministic prediction, which is not efficient for compression of highly uncorrelated image such as error-diffused bilevel image.

The authors proposed a two-pass approach to support the quality scalability in compressing bilevel images. This approach is used for our proposed resolution-scalable compression method in this paper. In the proposed method, the lowest resolution image is coded by the two-pass approach and higher resolution images are progressively coded with the information of the lower resolution images.

In this paper, Sec. 2 briefly reviews the two-pass approach proposed by the authors. The proposed progressive coding algorithm is given in Sec. 3 and the experimental results are described in Sec. 4. Section 5 concludes this paper.

2 Brief Review of the Two-Pass Approach

In the two-pass approach, $2 \times 2$ dots (four dots) are grouped into a cell, which is described by two values, a $C$ value and an $S$ value. The $C$ value is a binary representation of the dots in the cell, and the $S$ value is the number of black dots in the cell (Fig. 1), which can be considered as a simple lowpass-filtered value of the cell. If a bilevel image originates from a natural image, the $S$ value of the image has high spatial correlation with the neighbors’ $S$ values.

We express the estimated code length $\hat{L}(x)$ of a sequence of cells $x$ whose cell length is $T$ as follows:
\[ L(x) = - \sum_{i=1}^{T} \log \left( \tilde{p}(x_i) \right), \]  

where \( x_i \) is the \( C \) value of the \( t \)th cell, \( \tilde{p}(x_i) \) is the estimated probability of \( x_i \), and the base of \( \log \) is two.

If we use the Bayes theorem, \( \tilde{p}(x_i) \) can be replaced with 
\[ \prod_{k=0}^{4} \tilde{p}(x_i | C_k) \tilde{p}(C_k), \]  

where \( C_k \) is the partition that includes \( C \) values whose corresponding \( S \) values are \( k \). For example, \( C_2 = \{0011,0101,0110,1001,1010,1100\} \). Since the conditional probability is zero, i.e., \( \tilde{p}(x_i | C_j) = 0 \) when the \( S \) value of \( x_i \) is not \( k \), Eq. (1) can be redefined as follows:

\[ \tilde{L}(x) = - \sum_{i=1}^{T} \log \left( \sum_{k=0}^{4} \tilde{p}(x_i | C_k) \tilde{p}(C_k) \right) \]

\[ = - \sum_{i=1}^{T} \log \left[ \tilde{p}(x_i | C_j) \tilde{p}(C_j) \right] \]

where the \( S \) value of \( x_i \) is \( j \).

Equation (4) shows that the total code length of a sequence of cells is equal to the sum of two terms: the first term represents the code length of the \( S \) values, and the second term represents the code length of the \( C \) values with the given \( S \) values. Therefore, the encoding process can be divided into two passes.

The first pass encodes the \( S \) value of the bilevel image in consideration of four neighboring \( S \) values as an \( S \) value context. After the first pass, the \( C \) value is encoded by using two neighboring \( C \) values and the \( S \) value of the current cell as a \( C \) value context. When the \( S \) value of a cell is 0 or 4, the coder need not handle the \( C \) value of the cell because the \( C \) value is already known as 0000 or 1111, according to the \( S \) value of 0 or 4, respectively. To compress the \( S \) value and the \( C \) value, the two-pass approach uses a context-based arithmetic coding.

Because the \( S \) value is the number of black dots in a cell, we can reconstruct an image having the same size as the original one after decoding the bitstream of the \( S \) value in the first pass. The reconstructed image from the \( S \) value may not be same as the original image. Thus, we call the first pass near-lossless compression. In the second pass, we decode the bit stream of the \( C \) value and reconstruct the original image losslessly.

**Fig. 1** Definition of a \( 2 \times 2 \) cell: \( C \) and \( S \) values.

**Fig. 2** Block diagram of the proposed progressive coding method.
3 Progressive Coding of Error-Diffused Bilevel Images

Figure 2 shows the block diagram of the proposed progressive coding. We reduce the resolution of an image using inverse halftoning and rehalftoning and encode the resolution-reduced image from the lowest resolution image to the highest resolution image. Except for the lowest resolution image, we use the information of the lower resolution image to encode a one-layer-higher resolution image.

3.1 Resolution Reduction by Inverse Halftoning and Rehalftoning

To combine a progressive coding function with the two-pass approach, we must reduce the resolution of error-diffused bilevel images. While the JBIG algorithm uses a template-based method for resolution reduction, we adopt an inverse halftoning and rehalftoning method.

In general, gray-scale images are low-pass filtered and downsampled for resolution reduction. Several resolution reduction methods were also developed for bilevel images. These algorithms work well for document images or ordered dithered images. However, the image quality is degraded when we reduce the resolutions of error-diffused bilevel images using such algorithms.

In the proposed algorithm, we compute the $S$ value of every $2 \times 2$ dots for an inverse halftoning process. The $S$ value is a simple low-pass-filtered value of a cell. In this study, the $S$ value is treated as a gray value of the resolution-reduced image. As we group four dots into a cell, the range of the $S$ value is from 0 to 4. Figure 3(a) shows such an image with five gray levels, which is a resolution-reduced image created from a bilevel image.

There are several methods of mapping each $S$ value onto a binary value of the resolution-reduced bilevel image; simple thresholding [Fig. 3(b)], ordered dithering [Fig. 3(c)], and error diffusion. Among these three binarization methods, we select the error diffusion algorithm for rehalftoning in this paper to inherit advantages of error diffusion.

We call the highest resolution image a layer-0 image for convenience. When we reduce the resolution of an image
3.2 Proposed Progressive Coding Method

After reducing the resolution of an image, we encode the bilevel images progressively from the lowest resolution image to the highest resolution image. We adopt the two-pass approach⁵ to compress each layer image. To encode the lowest resolution image, we use two contexts, namely, the $S$ value context. After coding the lowest resolution image is coded using the $S$ value context and the $C$ approach⁶ to compress each layer image. To encode the age to the highest resolution image. We adopt the two-pass bilevel images progressively from the lowest resolution image. After reducing the resolution of an image, we encode the bilevel values from 0 to 4. We represent this process with the resolution reduction process. After combining four adjacent dots of the bilevel image, we generate an intermediate image (shaded by lines) that has multilevel values from 0 to 4. We represent this process with white arrows in Fig. 4. Using the error diffusion algorithm, we rehalftone the intermediate images.

After coding the lowest resolution image, we must code the next higher resolution image on the previous low-resolution image until we have completed the coding of the original highest resolution image. While we encode the $S$ value of the lowest resolution image only with the causal $S$ value context, we can also use the information of the lower resolution image (that is, encoded previously) to encode the $S$ value of one-layer-higher resolution image. Because the bit value of the lower resolution image is determined by the $S$ value of its one-layer-higher resolution image, we select the $3 \times 3$ dots of the lower resolution bilevel image as the $S$ value context for the $S$ value prediction of the higher resolution image. The center of the $3 \times 3$ dots (point A in Fig. 5) corresponds to the current pixel location (X in Fig. 5) of the intermediate image.

Figure 6 shows a block diagram of the error diffusion process. An error diffusion filter ($H$) is applied to gray-scale pixels $[x(i,j)]$ ranging in $0 \leq x \leq 4$ and the filtered pixels are quantized into bilevel values $[y(i,j)]$. The filtered pixel $[x'(i,j)]$ is described as follows:

$$x'(i,j) = \frac{x(i,j)}{4} - \sum_{(m,n) \in W} h(m,n)e(i-m, j-n),$$

(5)

where $e(i,j)$ is the quantization error, i.e., difference between $x'(i,j)$ and the quantized value of $x'(i,j)$. Thus, magnitude of the quantization error $[e(i,j)]$ is always less than or equal to 0.5 when the threshold value of the quantization process is 0.5. In Eq. (5), the window $W$ should be causal for the error diffusion. In addition, the sum of filter coefficients is usually equal to one. Therefore, if the gray value of a pixel $[0 \leq x(i,j) \leq 4]$ is 0, the quantized value $[y(i,j)\in [0,1]]$ is also 0. On the other hand, if the quantized value is 0 (or 1), then the original gray value cannot be the maximum value (or the minimum value). We use this property to encode $S$ values of one-layer-higher resolution image with information of the lower resolution image. When we encode the $S$ values of higher resolution images, if we know that the binary value of the current pixel is 0 or 1, possible $S$ values can be (0,1,2,3) or (1,2,3,4), respectively. Thus, there are four possible candidates for $S$ value under the known binary value of the current pixel. Two bits for representing four possible candidates are coded by a binary arithmetic coder. The method of encoding the $C$ value of higher resolution images is the same as the second pass of the two-pass approach.

In our work,⁶ we used a multisymbol arithmetic coding⁹ to compress the $S$ and the $C$ values. However, the process used multiplications and divisions to calculate intervals and probabilities of symbols. Such operations are expensive to implement in hardware. In the proposed algorithm, we use binary arithmetic coding for multisymbol data.¹⁰ After describing each value with binary representation, the binary arithmetic coder encodes each binary value with the corresponding contexts.

4 Experimental Results

For the experimental study, we used six bilevel images, each of which has $512 \times 512$ pixels and which are halftoned by error diffusion with a Floyd and Steinberg filter.¹ In the experiment, the compression performance of the proposed algorithm is compared with those of the sequential JBIG and the progressive JBIG with one layer of the resolution reduction. Because the proposed algorithm is designed for progressive coding, we first compare the image quality of the resolution-reduced images. JBIG reduces the image resolution using a template-based method, whereas the proposed algorithm halftones the intermediate images based on $S$ values. We reduced the resolution of the images once.
Then, the resolution-reduced images had 256×256 dots. Figures 7(a) and 7(b) show the resolution-reduced “Barbara” images from the progressive JBIG and the proposed method, respectively. Because the proposed method is composed of inverse halftoning and rehalftoning, Fig. 7(b) is more similar to gray-scale images than is Fig. 7(a). The resolution-reduced image from the deterministic prediction of JBIG [Fig. 7(a)] loses the characteristics of aperiodic and uncorrelated dot patterns in error-diffused images. The images whose resolutions are reduced once again are shown in Fig. 7(c) (the deterministic prediction) and Fig. 7(d) (the proposed method) to show image quality of the resolution-reduced bilevel image by a factor of four both in horizontal and vertical directions.

To measure the quality of the resolution-reduced images, we used a perception-weighted signal-to-noise ratio (PWSNR). We weighted the SNR according to the contrast sensitivity function (CSF) of the human visual system as follows,

\[
PWSNR \text{(dB)} = 10 \log_{10} \left( \frac{\sum_{u,v} |X(u,v)C(u,v)|^2}{\sum_{u,v} [(X(u,v)-Y(u,v)]C(u,v)|^2} \right),
\]

where \(X(u,v)\), \(Y(u,v)\), and \(C(u,v)\) represent the discrete Fourier transforms (DFTs) of the input gray-scale image, the output bilevel image, and the CSF, respectively. We use\(^{11}\) a low-pass CSF as the weight function. We compare PWSNR of resolution-reduced images from the progressive JBIG and the proposed method. We use downsampled gray-scale images as reference images, whose size is 256×256.

Table 1 shows the PWSNR of the test images at different viewing distances. The image quality of the proposed method is better than that of the progressive JBIG as expected from the subjective quality in Fig. 7.

Table 2 shows the compression ratios of the JBIG and the proposed method. As shown in Table 2, the compression ratio of the sequential JBIG is higher than that of our proposed method and the progressive JBIG. This is caused by the fact that if we reduce the resolution of error-diffused images using the deterministic prediction, the correlation between neighboring layers becomes low as shown in Fig. 7(a). However, the sequential coding cannot support the progressive functionality even though it has a high compression ratio. To support the progressive functionality, we must encode images using the progressive JBIG or the proposed method. For fair comparison, we encoded images...
with just one-layer resolution reduction. The compression ratio from the proposed algorithm is slightly higher than that from the progressive JBIG (except with the “Boat” image). Even though the compression ratio of the proposed method is not much better than that of the progressive JBIG, the proposed method offers better quality of the resolution-reduced images than the progressive JBIG.

We used the QM-coder\textsuperscript{12} for arithmetic coding, as used in JBIG. While the JBIG algorithm handles images pixel by pixel, the proposed algorithm handles images using grouped pixels. Because the proposed algorithm performs rehalftoning of $S$ value to reduce the resolution, it requires more calculations for encoding. However, decoders do not require such calculation.

We measured computation time for compression and de-compression. We used the public domain software\textsuperscript{13} “jbig-kit” for the JBIG compression and decompression under Windows 98 with a Pentium-III (450 MHz). To implement the proposed algorithm, we programmed the algorithm using the C language in the same environment. The computation times for encoding and decoding are shown in Table 3. Even though the rehalftoning process takes longer than the deterministic prediction in the progressive JBIG, the computation time of the proposed algorithm is slightly faster than that of the progressive JBIG. This shows that the proposed resolution reduction method is comparable to the deterministic prediction of the JBIG in computation time.

### 5 Conclusions

We proposed a new method to compress error-diffused images with resolution scalability. This algorithm is proposed on the basis of the two-pass approach. In the proposed progressive coding, it performs rehalftoning of the $S$ value with an error diffusion algorithm to reduce the bilevel image resolution. The lowest resolution image is encoded by using the two-pass approach, and the $S$ value of the higher resolution image is encoded with information of the one-layer-lower resolution image. The compression ratios of the proposed algorithm are similar to those of the progressive JBIG. However, the resolution-reduced images generated by the proposed algorithm are better than those generated by the progressive JBIG.

### References


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**Table 2** Compression ratio: (a) JBIG sequential mode, (b) JBIG progressive mode with one layer, and (c) the proposed progressive coding with one layer.

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**Table 3** Computation time (unit: millisecond).

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