On Spherical Linear Interpolation for MIMO-OFDM Beamforming Systems

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SUMMARY We analyze linear channel estimation for MIMO-OFDM systems and propose a spherical linear interpolator in closed-form for the beamforming codewords. We also suggest a hybrid interpolator using a simplified version of the derived interpolator. Simulation results show that the proposed schemes are efficient and competitive with respect to the feedback overhead and have low complexity.

key words: spherical linear interpolation, beamforming, MIMO-OFDM

1. Introduction

The multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system is the best-known candidate for high-speed wireless communication. In particular, transmit beamforming, which is used in MIMO systems, is a simple and powerful technique for improving performance. However, transmit beamforming usually requires that channel state information (CSI) be known at the transmitter. In order to reduce the feedback overhead caused by ensuring that CSI is sent to the transmitter, it was proposed in [1] and [2] that methods based on limited feedback be used. In [3] and [4], it was shown that Grassmannian line packing provides a good codebook criterion for beamforming.

However, if limited feedback is used directly in MIMO-OFDM systems, the feedback overhead increases enormously, thereby rendering it difficult to implement limited feedback in practical applications. To mitigate the feedback overhead, it is possible to use the clustering method (pure clustering), in which the beamforming vector for the center subcarrier is reused for the other subcarriers in a cluster. However, the performance of clustering is unsatisfactory, due to the severe mismatch between the channel direction and beamforming vector.

Given the failure of the aforementioned attempts to reduce the feedback overhead, a spherical linear interpolator (SLI) was proposed in [5]. Due to the fact that the SLI requires rotating the phase of codewords to maximize the minimum effective channel gain, the authors suggested that feedback about the phase rotations be gathered and used. In order to avoid the need to use feedback about phase rotations, a geodesic interpolation and Karcher mean clustering methods were introduced in [6].

In this letter, we investigate the linear channel estimator and derive an SLI in closed form. From the obtained closed form solution, we suggest an approximated solution for a proposed SLI. Furthermore, we propose a hybrid interpolation method that performs better than the conventional SLI.

2. Investigation of SLI

2.1 System Model

The OFDM system may be considered as having parallel flat-fading channels. If perfect CSI at the transmitter is available, optimal beamforming vectors can be applied to each subcarrier, thereby yielding the best performance. However, due to the fact that perfect CSI at the transmitter is usually not possible, the receiver sends the indices of the best beamforming codewords to the transmitter via a feedback channel.

For the $k$-th subcarrier, the input-output relationship for a multiple-input single-output (MISO) system can be written as $y(k) = \mathbf{h}^T(k)w(k)s + n(k)$, and for a MIMO system as $y(k) = \mathbf{H}(k)w(k)s + n(k)$, where $y (y)$ denotes the received signal(s), $\mathbf{h}^T (\mathbf{H})$ denotes the channel, $\mathbf{w}$ denotes the beamforming vector, $s$ denotes the transmitted signal, and $n (n)$ denotes additive white Gaussian noise.

2.2 MISO Case

For a practical OFDM system, a linear interpolation is often used for channel estimation as follows:

$$\tilde{\mathbf{h}}(p_t + k) = (1 - c(k))\mathbf{h}(p_t) + c(k)\mathbf{h}(p_{t+1}),$$

(1)

where $p_t$ denotes the $l$-th pilot position, $p_{t+1} = p_t + K$, and $c(k) = k/K, k = 0, 1, \cdots, K - 1$. For the sake of simplicity, we define $\mathbf{h}_k \equiv \mathbf{h}(p_t + k), \mathbf{h}_0 \equiv \mathbf{h}(p_t)$, and $\mathbf{h}_K \equiv \mathbf{h}(p_{t+1})$.

Without loss of generality, we can assume that the cluster size is $K$. Let $\mathbf{w}_0$ and $\mathbf{w}_K$ be the chosen codewords for the channel $\mathbf{h}_0$ and $\mathbf{h}_K$, respectively. Since the optimal beamforming vector is written as $e^{j\tilde{\theta}_0}h_{0,K}^* / ||h_{0,K}||$ with an arbitrary phase $\tilde{\theta}_0$, we have the error vector

$$\mathbf{e}_{0,K} = \mathbf{w}_{0,K} - e^{j\tilde{\theta}_0}h_{0,K}^* / ||h_{0,K}||.$$

(2)

From (2), the channel vector $h_{0,K}^*$ can be expressed as $h_{0,K}^* = \ldots$
\[ e^{-j\theta_{0,k}}\|h_{0,k}\| (w_{0,k} - e_{0,k}) \] by letting \( \theta_{0,k} = \arg \min_{\theta_{0,k}} \|e_{0,k}\| \).

The beamforming vector under a linear channel estimator before normalization is thus obtained as follows:

\[
\begin{align*}
\tilde{w}_{\text{link}_k} & \approx \tilde{h}_k = (1 - c)h_{0,k}^* + ch_{K}^* \\
& = \left((1 - c)w_0 + ce^{-j\theta_{0,k}}\|h_{0,k}\|\|w_K\|\right)e^{-j\theta_{0,k}}\|h_{0,k}\| + e,
\end{align*}
\]  

(3)

where \( e \in -1(1 - c)e^{-j\theta_{0,k}}\|h_{0,k}\|e_0 - ce^{-j\theta_{0,k}}\|h_{0,k}\|e_K\) and \( c \in c(k)\).

From (3), the beamforming vector can be obtained by normalization as \( w_{\text{link}_k} = \tilde{w}_{\text{link}_k}/\|\tilde{w}_{\text{link}_k}\|\). If we assume that the codebook has large cardinality, the error vector \( e \) can be negligible, because the angle \( \theta_{0,k} \) is chosen to minimize the norm of the error vector \( e_{0,k}\). Equation (3) can then be approximated by the phase invariance of the beamforming vector and the postnormalization process as \( \tilde{w}_{\text{link}_k} \approx (1 - c)w_0 + c e^{-j\theta_{0,k}}w_K\), where \( t \in [|h_K|]/|h_0|\) and \( \theta_t \equiv \theta_{0,k} - \theta_0 \). Therefore, the final beamforming vector \( w_{\text{link}_k} \) can be approximated as follows:

\[
w_{\text{link}_k} \approx \frac{(1 - c)w_0 + c e^{-j\theta_{0,k}}w_K}{\|1 - c)w_0 + c e^{-j\theta_{0,k}}w_K\|}.
\]  

(4)

Note that (4) looks similar to (10) in [5], except for parameter \( t \). The term \( e^{-j\theta_{0,k}} \) is a complex number that represents the relationship between two channels \( h_0 \) and \( h_K \). However, in order to exploit (4), sending the parameter \( te^{-j\theta_{0,k}} \) back to the transmitter is equivalent to full channel feedback. If we assume that the channels of two subcarriers are highly correlated, parameter \( t \) tends to 1. Therefore, [5] can be considered as an effort to search for an optimal \( \theta_t \) with an assumption that \( t = 1 \), even if it is not expressed explicitly. However, given that [5] guarantees the maximization of the minimum channel gain irrespective of the cluster size, it alone is worthy of note, despite the complexity of its search procedure. In this letter, we suggest a scheme to approach the performance of [5] without additional feedback overhead.

Given the assumption that the channels in the frequency domain are highly correlated, the difference norm between two channels \( h_0 \) and \( h_K \) is expected to be small; this leads to the following relations:

\[
\begin{align*}
\|h_0 - h_K\| & \rightarrow 0 \implies \|h_0^* - h_K^*\| \rightarrow 0 \\
& \implies \|w_0 - e^{-j(\theta_{0,k})}h_{0,k}^*/\|w_K\|\|w_K\| \rightarrow 0 \\
& \implies \|w_0 - e^{-j\theta_{0,k}}w_K\| \rightarrow 0.
\end{align*}
\]  

(5)

From (5), we propose a suboptimal phase term \( \theta_{\text{pro}} \) for \( \theta_t \) in (4) as follows:

\[
\theta_{\text{pro}} = \arg \min_{\theta} \|w_0 - e^{-j\theta}w_K\|,
\]  

(6)

and the closed-form solution is obtained by \( \theta_{\text{pro}} = c(w_K^Hw_K)^{-1/2} \). The proposed SLI is thus written as

\[
w_{\text{pro},k} = \frac{(1 - c)w_0 + ce^{-j\theta_{\text{pro}}}w_K}{\|1 - c)w_0 + ce^{-j\theta_{\text{pro}}}w_K\|}.
\]  

(7)

where \( e^{-j\theta_{\text{pro}}} = w_K^Hw_0/\|w_K^Hw_0\|. \) Interestingly, the solution does not depend on the channels, but the codewords. This can lead to a reduction in the feedback by creating a table that contains the solution \( \theta_{\text{pro}} \) for every codeword pair. It is noteworthy to mention that the proposed SLI pursues the same strategy as the geodesic interpolation in [6], in the sense of minimizing the Euclidean distance between two codewords.

2.3 MIMO Case

The interpolation method can be extended easily from the MISO case to the MIMO case. The linearly interpolated channel can be written as \( \tilde{H}_k = (1 - c)H_0 + cH_K \). By singular-value decomposition (SVD), this can be rewritten as follows:

\[
\begin{align*}
\tilde{H}_k &= (1 - c)H_0 + cH_K \\
& = (1 - c)U_0\Sigma_0V_0^H + cU_K\Sigma_KV_K^H \\
& = \sum_{i=1}^{r} (1 - c)\lambda_0, u_{0,i}V_{0,i}^H + c\lambda_K, u_{K,i}V_{K,i}^H.
\end{align*}
\]  

(8)

(9)

(10)

where \( r \) denotes the rank of the channel matrix \( H_{0,K} \). By multiplying the receive filter \( u_{0,1}^H \), we have

\[
\begin{align*}
u_{0,1}^H\tilde{H}_k & \approx (1 - c)\lambda_0, u_{0,1}^H + c\lambda_K, u_{K,1}^H
\end{align*}
\]  

(11)

By letting \( \tilde{h}_k \equiv \tilde{H}_k^Hu_{0,1}^H, h_0 \equiv \lambda_0, v_{0,1}, \) and \( h_K \equiv (\lambda_K, u_{K,1}^H, v_{K,1}^H) \), (11) is analogous to (1), which is a MISO channel model. Therefore, the same interpolation methods using (4) and (7) can be applied to the MIMO case.

3. Hybrid Interpolation

If the cluster size is too large for the high correlation between two subcarriers to be maintained, the proposed SLI and geodesic interpolation can fail. To avoid this eventuality, we now propose a hybrid interpolation method to complement the proposed SLI.

In the proposed interpolation, a cluster is divided into two parts, which are supported by the Karcher mean clustering and the proposed SLI, respectively. The Karcher mean clustering chooses a representative codeword that minimizes the mean distance given the optimal beamforming vectors for every subcarrier in each cluster. Therefore, if a portion of subcarriers in a cluster is supported by the Karcher mean clustering, the number of remaining subcarriers is reduced, and it leads that the channels at both edges for interpolation becomes highly correlated.

We denote the size of a cluster by \( K \), and the number of subcarriers supported by the proposed SLI by \( I(K) \). The value of \( I \) can be determined as a limit of the size of the cluster for which the proposed SLI works satisfactorily. With \( K \) and \( I \), the hybrid interpolation is formulated as follows:

\[
w_{\text{hyb},k} = \begin{cases} w_{\text{Karcher}}, & 0 \leq k < K - I, \\ w_{\text{SLI}(k)}, & K - I \leq k \leq K - 1. \end{cases}
\]  

(12)
where \( \mathbf{w}_{\text{Karcher}} = \arg \max_{\mathbf{w}} \sum_{k=0}^{K-1} \alpha(k) |w^H w_k| \), and \( \mathbf{w}_{\text{SLI}}(k) = \frac{(1-(c(k,I)\mathbf{w}_{\text{pro}})+c(k,I)\mathbf{w}_{\text{pro}})}{\|I\|_{\mathbf{w}_{\text{pro}}}} \). The vector \( \mathbf{w}_{\text{pro}} \) is an optimal beamforming vector for the \( k \)-th subcarrier, \( \alpha(k) \) is the weighting factor, \( c(k,I) = (k - K + I + 1)/(I + 1) \), and \( \mathbf{w}_{\text{high},K} \) denotes the beamforming vector of the next cluster.

The beamforming vectors that are obtained by a simple Karcher mean are not preferable, because the proposed SLI requires that two beamforming vectors should be close in a Euclidean sense at both edges. We thus propose the following weighting factor in order to enhance both edges:

\[
\alpha(k) = (k - (K - I)/2)^p + 1, \quad (13)
\]

where \( p \) is a positive exponent to decide the weighting factor.

4. Simulation Results

To demonstrate the efficacy of the proposed algorithms, the following parameters were used: the number of transmit antennas = 4, the number of subcarriers = 64, cyclic prefix length = 16, sampling frequency = 40 MHz, and rms delay spread = 100 ns. The result of the interpolation method depends strongly on the system environment, with respect to such factors as codebook size, delay spread, and sampling frequency. Therefore, the parameters regarding the interpolation must be reconsidered when the channel environment, especially such as a coherent bandwidth, is changed.

In order to investigate the effect of the cluster size, the performance versus the cluster size was analyzed numerically by computing the average value of the correlation \( |\mathbf{h}(k)^T \mathbf{w}(k)|/|\mathbf{h}(k)||\mathbf{w}(k)|/1000 \) through 1000 channel realizations as shown in Fig. 1. The figure on the left represents the case when one receive antenna is used, and that on the right is when four receive antennas are used. The size of the beamforming codebook was set to 5 bits during this simulation. If the number of transmit antennas is increased, the codebook cardinality should be increased to reduce the error due to the mismatch between the channel direction and the codeword. The conventional SLI denotes [5], which requires the quantized phase value to maximize the minimum gain in a cluster, and an additional 2 bits per cluster were used for the feedback of the phase term. It can be therefore considered that the conventional SLI uses a 7-bit codebook for each cluster.

When \( K = 2, 4 \) (a strongly correlated case), the interpolation methods outperform the full feedback case (which is the case where every subcarrier takes the best-matched codeword), because interpolation can have a continuous beamforming vector, whereas full feedback cannot. Equation (4) performs best at \( K = 2 \), but its performance degrades rapidly because the assumption of high correlation ceases to hold as the size of the cluster increases. As predicted in Sect. 2, the performance of the geodesic interpolation and the proposed interpolation is almost the same. The performance gain of the conventional SLI can be inferred from the additional 2 bits of information at the transmitter. When four receive antennas are used, it can be seen that (4) for the MIMO case performs properly as well, and that the array gain is also obtained.

In the simulation for the hybrid method, the size of the interpolation section \( I \) and the edge enhancement value \( p \) were set to 3 and 4, respectively, by exhaustive search. It is left for future work to find an analytic method for determining \( I \) and \( p \), but the following can be surmised.

1. \( I \) should be determined within the size where the interpolation scheme works better than using Karcher mean clustering.
2. When the size \( K - I \) for the Karcher mean clustering is too large, resulting from the channel at one edge being too different from that at another edge, \( \mathbf{w}_{\text{Karcher}} \) cannot represent both edge channels, which can lead to the failure of the hybrid method.
3. When the delay spread increases, the coherent bandwidth decreases, and so the size \( I \) for the interpolation section should be decreased.

Figure 2 shows the bit error rate (BER) performance of several schemes with a 1/2-rate convolutional code when \( K = 4 \) and 8. For this simulation, four transmit antennas and one receive antenna are used. When \( K = 4 \), we omit...
the simulation of the hybrid interpolation, because it works properly for large $K$, as mentioned before. For the full-feedback case, $320 (= 5 \times 64)$ bits are required, for the conventional SLI, 112 bits are required, and for other schemes, 80 bits are required for the feedback. As was expected in the simulation of the performance versus the cluster size, for $K = 4$ (highly correlated case), the interpolation schemes (the conventional and proposed SLI) perform as well as the full feedback scheme. Furthermore, the proposed SLI outperforms pure clustering or the Karcher mean clustering. To consider the fairness with respect to the feedback overhead, increasing the codebook size to 7 bits for the proposed SLI yields a very strong increase in performance. We can see that the BER curve approaches the optimal beamforming very closely.

When $K = 8$, for the conventional SLI, 56 bits are required for the feedback, whereas for the other schemes, 40 bits are used. When $K = 8$, the Karcher mean clustering performs better than the proposed SLI, but worse than the proposed hybrid interpolation with $I = 3$. A simulation similar to that for previous scenarios was performed for the proposed SLI and the proposed hybrid interpolation, with codebook size increased to 7 bits. From the results, we can see that the proposed hybrid method outperforms the conventional SLI. This increase in performance comes from the fact that the proposed methods can use the remaining feedback resource to increase the size of the codebook while enabling interpolation without any additional feedback, which the conventional SLI cannot do.

5. Conclusion

We formulated a spherical linear interpolator (SLI) with a closed-form solution for the phase, and proposed a new SLI method that uses the approximated phase term to reduce the feedback overhead. In addition, a hybrid interpolation method was proposed. This method exploits the fact that the proposed SLI performs satisfactorily for a highly correlated channel, but the Karcher mean clustering performs better than the interpolation methods in a slightly correlated channel. Numerical simulations showed that the proposed hybrid interpolation outperforms the proposed SLI or Karcher mean clustering.

Due to the fact that the proposed interpolation method does not require additional feedback, the proposed algorithms show more efficient and better performance in comparison with existing schemes. It is to be expected that further gains in performance can be achieved if the saved feedback is used to increase the codebook size or to use other feedback schemes. In order to achieve further reduction in the feedback overhead, it will be fruitful to exploit [7]. In the future, we will perform a quantitative analysis on the performance tradeoff between cluster size and the size of the codebook.

References