Fuzzy Sliding Mode Control for a Robot Manipulator with Passive Joints

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Abstract - A robot manipulator with passive joints is a kind of underactuated system. The control of underactuated manipulator is much more difficult than that of fully-actuated robot manipulator. Though sliding mode control scheme has a robust characteristic and gives systematic approach to design controller, the chattering phenomena is one of major problems. To alleviate chattering phenomena we add fuzzy control scheme which can be designed rather easily and systematically based on phase plane to sliding mode control.

Keywords - underatuated manipulator, fuzzy sliding mode control, chattering phenomena

1. Introduction

Underactuated System is the system in which the dimension of the configuration space exceeds that of the control input space. For example, When one or more of robot manipulators' actuators fail to work properly, it is said to be underactuated which refers to the fact that not all joints or degrees of freedom (DOFs) of the system are equipped with actuators, or are directly controllable.

Control of this type of the underactuated manipulator is important from a fault-tolerance point of view, for sometimes it is necessary that the robot complete its task before repairs can be performed and Research on manipulators with underactuated joints equipped with brakes has been receiving a great deal of attention recently.

Moreover, underactuated systems have some advantages compared to fully-actuated one. First, they weigh less and consume less energy. Because they have smaller number of components than fully-actuated ones. This characteristic is suitable for special machine such as a manipulator attached to space shuttle. Second, Reliable or fault-tolerant design of manipulators is possible for hazardous areas such as space, nuclear power plants, etc. In case one of manipulator's joints has failed in working time, the failed joint -passive joint- still can be controlled via the dynamic coupling with active joints

The dynamics and control schemes have been studied from the 1990's. Because the control of underactuated manipulator is much more difficult than the control of fully-actuated robot manipulator, not much active research about this area has done. H. Arai & S. Tachi, G.Oriolo & Y. Nakamura, E. Papadopoulos & S. Dubowsky [1-3] gave much effort to design controller based on accurate dynamic modeling. But gathering accurate parameter from large scale robot manipulator was not easy, and also load parameter varies according to the what kind of things the robot manipulator is picking. Bergeman applied VSS control scheme to underactuated manipulator for overcoming modeling errors and disturbances[4]. J. Shin & J. Lee has been doing research about robust adaptive control scheme[5].

Sliding mode control gives us systematic approach to get the control law and strong algorithm of control to overcome uncertainty of system and external disturbances. But it is not easy to derive the equation of controller because it needs rigorous mathematical approach. Moreover SMC suffers from chattering phenomena. While Fuzzy control scheme is good for unmodelled plant because it can use I/O behavior of system or the knowledge of expert. But it is less systematic and is not easy to prove stability.

Motivation of our research is as following : If the phase plane of error and the derivative of error can be used to design fuzzy rule base, the more systematic design of fuzzy controller is possible. By using the distance of the representative point from the sliding line as the deciding factor, the magnitude of control is set to be proportional to the distance. Also two regions divided by sliding line are used to determine the sign of control. While upper region can be assigned (-) sign, lower region can be assigned (+) sign to attract the representative point to the sliding line.

Because fuzzy controller can be derived by easy approach, Control law by FSMC(Fuzzy Sliding Mode Controller) are also easy to setup without rigorous mathematical approach. But FSMC is designed based on phase plane, it has the characteristics of SMC like overcoming uncertainties and disturbance. The chattering phenomena can be alleviated because FSMC does not generate drastic change of control.

2. System Dynamics

Using the Lagrangian formulation, the dynamics equation of n-link rigid serial open-chain underactuated robot manipulator with r-actuated joints and p-unactuated joints can be written in joint space as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q) = u+d(t) = \begin{pmatrix} \tau_{a+d_a} \\ 0_{p+d_p} \end{pmatrix} \qquad (1)$$

where joint variables are $q = (q_a^T\ q_p^T)^T \in R^{n=r+p}$, position vector of the active joints are $q_a \in R^r$, position vector of the passive joints are $q_p \in R^p$, inertial matrix is $M(q) \in R^{n\times n}$, centrifugal and Coriolis torques are $C(q,\dot{q})\dot{q} \in R^n$, vector gravitational torques are $G(q) \in R^n$, control torque input vector is $u = (\tau_a^T\ 0_p^T)^T \in R^n$, actual control input is $\tau_a \in R^r$, zero input vector (to passive joints) is $O_p \in R^p$, number of the total joints is $n(=r+p)$, number of active joints is r, number of passive joints is p, a norm-bounded external disturbance vector is $d(t)=(d_a^T\ d_p^T)^T \in R^n$ for which $d_a \in R^r, d_p \in R^p$,

$\| d_a \| \leq d_{am}, \quad \| d_p \| \leq d_{pm},$

Equation (1) can be partitioned as

$$\begin{pmatrix} M_{aa} & M_{ap} \\ M_{pa} & M_{pp} \end{pmatrix} \begin{pmatrix} \ddot{q}_a \\ \ddot{q}_p \end{pmatrix} + \begin{pmatrix} F_a \\ F_p \end{pmatrix} = \begin{pmatrix} \tau_a - d_a \\ 0_p - d_p \end{pmatrix} \quad (2)$$

where $M_{aa} \in R^{r \times r}$, $M_{pp} \in R^{p \times p}$,
and $F(q, \dot{q}) = (F_a^T \; F_p^T)^T = C(q, \dot{q})\,\dot{q} + G(q)$

Some useful properties can be given below.

Property 1 *$M(q)$ is a symmetric, bounded, invertible and positive definite matrix.*

Property 2 *$\dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix.*

Property 3 *There exist positive constants $m_{\min}, m_{\max}, c_{\max}, g_{\max}, f_g$ and f_c such that*

$$m_{\min} \leq \| M(q) \| \leq m_{\max}, \; \| C(q, \dot{q}) \| \leq c_{\max} \| \dot{q} \| \quad \text{and} \quad \| G(q) \| \leq g_{\max}, \; \| F(q, \dot{q}) \| \leq f_g + f_c \| \dot{q}^2 \| \quad (3)$$

where $\| M(q) \|$ and $\| C(q, \dot{q}) \|$ are induced matrix norm, $\| G(q) \|$ and $\| F(q, \dot{q}) \|$ are vector norm.

Property 4 *By Property 1, both $M_{aa} \in R^{r \times r}$ and $M_{pp} \in R^{p \times p}$ are also symmetric, bounded, invertible and positive definite matrices.*

Property 5 *Effective inertial matrices are symmetric invertible positive definite matrices.*

$$\widetilde{M}_{aa} = M_{aa} - M_{ap} M_{pp}^{-1} M_{pa} \quad (4)$$

$$\widetilde{M}_{pp} = M_{pp} - M_{pa} M_{aa}^{-1} M_{ap} \quad (5)$$

Property 6 *Pseudo inverse matrix (for p × r matrix A, where r > p),*

$$AA^{\#} = I_p, \quad A^{\#}A \neq I_r, \quad A^{\#} = A^T (A A^T)^{-1} \quad (6)$$

3. Design of Fuzzy Sliding Mode Controller

Control Objective

Regulation of all the joints to the desired set-points.

Control Procedure

1. Control all passive joints using the dynamic coupling between the active joints and the passive ones.
2. Brake each passive joint as soon as they reach their set-points with zero velocity. Wait until all passive joints are locked.
3. Control all active joints by a new control law.

3.1 Control of Passive Joints

The dynamic equation of the underactuated robot manipulator given in partitioned equation can be rewritten for each joint as

$$\ddot{q}_a = -M_{aa}^{-1}(M_{ap}\,\ddot{q}_p + F_a - \tau_a - d_a) \quad (7)$$

$$\ddot{q}_p = -M_{pp}^{-1}(M_{pa}\,\ddot{q}_a + F_p - d_p) \quad (8)$$

Substituting (7) to (8),

$$M_p\,\ddot{q}_p = M_{\tau_a}\tau_a + M_{F_a}F_a + M_{F_p}F_p + M_{d_a}d_a + M_{d_p}d_p \quad (9)$$

where

$M_p = M_{pp}^{-1}\widetilde{M}_{pp} = M_{pp}^{-1}(M_{pp} - M_{pa}M_{aa}^{-1}M_{ap}) \in R^{p \times p}$

$M_{\tau_a} = -M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}$

$M_{F_a} = M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}, \quad M_{F_p} = -M_{pp}^{-1} \in R^{p \times p}$

$M_{d_a} = -M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}, \quad M_{d_p} = M_{pp}^{-1} \in R^{p \times p}.$

Therefore, the dynamic equation for the passive joints can be rewritten as follows.

$$\ddot{q}_p = M_{p\tau_a}\tau_a + H_p \quad (10)$$

Where

$M_{p\tau_a} = M_p^{-1}M_{\tau_a} = -\widetilde{M}_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r},$

$H_p = M_p^{-1}(M_{F_a}F_a + M_{F_p}F_p + M_{d_a}d_a + M_{d_p}d_p) \in R^p$

Let's define sliding mode controller as

$$\tau_a = \widehat{M}^{\#}_{p\tau_a}(V_{p_r} - \widehat{H}_p) \quad (11)$$

where $\widehat{M}_{p\tau_a} (= -\widehat{\widetilde{M}}_{pp}^{-1}\,\widehat{M}_{pa}\,\widehat{M}_{aa}^{-1})$ with guessed nominal values, $\widehat{H}_p = \widehat{M}_p^{-1}(\widehat{M}_{F_a}\widehat{F}_a + \widehat{M}_{F_p}\widehat{F}_p)$

Applying (11) to (10),

$$\ddot{q}_p = V_{p_r} + \eta_p \quad (12)$$

where the lumped uncertainty term is as follows.

$\eta_p = (M_{p\tau_a}\widehat{M}^{\#}_{p\tau_a} - I_p)V_{p_r} + (H_p - M_{p\tau_a}\widehat{M}^{\#}_{p\tau}\widehat{H}_p)$

The control input V_{pr} is denote as $V_{pr} = V_p + \Delta V_p$ (ΔV_p is a robust control input term)

Tracking error can be given by $e_p = q_p - q_{pd} \in R^p$ (q_{pd} : desired set points vector)

Sliding surface can be given by

$s_p = \dot{e}_p + \Lambda_p e_p \in R^p$ (Λ_p : positive definite diagonal constant gain matrix)

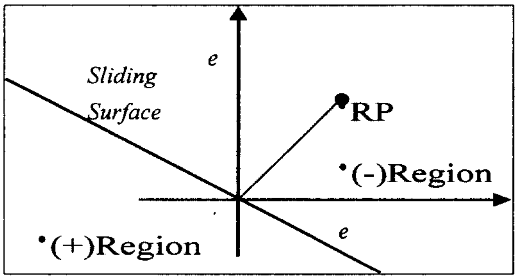
Then outer loop input can be given by

$V_p = \ddot{q}_{p_d} - (K_p + \Lambda_p)\,\dot{e}_p - K_p\Lambda_p e_p$

(K_p : pos. def. diagonal constant matrix)

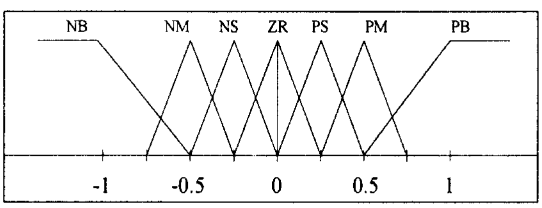
FSMC Strategy for passive control

We use the distance of the representative point from sliding line as the factor for deciding the magnitude of controller. The magnitude of controller can be proportional to the distance. Also We can use two region divided by sliding line to determine the sign of controller. While upper region can be assigned (-) sign, lower region can be assigned (+) sign to attract the representative point to the sliding line.



Fuzzy membership function design

- Same MF for all the fuzzy variable (e_p, $\dot{e}_p$, ∇V_p)
- Normalization Factors : Different
- Inference method : Mamdani min-max
- Defuzzification : COA



Design of Rule table based on phase plane

PB	ZR	NS	NS	NM	NB	NB	NB
PM	PS	ZR	NS	NS	NM	NB	NB
PS	PS	PS	ZR	NS	NS	NM	NB
ZR	PM	PS	PS	ZR	NS	NS	NM
NM	PB	PM	PS	PS	ZR	NS	NS
NS	PB	PB	PM	PS	PS	ZR	NS
NB	PB	PB	PB	PM	PS	PS	ZR
$\dot{e}$ / e	NB	NM	NS	ZR	PS	PM	PB

3.2 Control of active Joints

Equation (7) can be written as

$$\ddot{q}_a = M_{aa}^{-1}\tau_a - M_{aa}^{-1}F_a + M_{aa}^{-1}d_a$$
$$= M_{aa}^{-1}\tau_a + H_a \quad (21)$$
$$where \ \ H_a = -M_{aa}^{-1}(F_a - d_a)$$
$$\ddot{q}_a = M_{aa}^{-1}(\tau_a + d_a - M_{ap}\ddot{q}_p - F_a) \quad (20)$$

Since $\dot{q}_p = \ddot{q}_p = 0$ (by the operation of brakes) tracking error is denoted as follows :

$e_a = q_a - q_{ad} \in R^a$ (q_{ad} : desired set points vector)

Sliding surface design is as follows :

$s_a = \dot{e}_a + \Lambda_a e_a \in R^a$

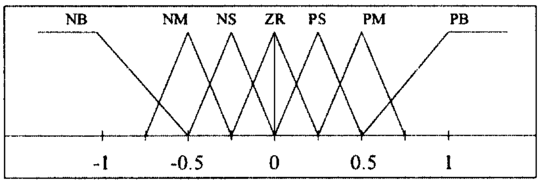
(Λ_a : positive definite diagonal constant gain matrix)

A sliding mode controller is defined :

$\tau_a = -K_a s_a + \Delta V_a \in R^r$ (K_a : pos. def. diagonal const. matrix)

Fuzzy membership function design

- Same MF for all the fuzzy variable (e_a, $\dot{e}_a$, ∇V_a)
- Normalization Factors : Different
- Inference method : Mamdani min-max
- Defuzzification : COA



Design of Rule table based on phase plane

PB	ZR	NS	NS	NM	NB	NB	NB
PM	PS	ZR	NS	NS	NM	NB	NB
PS	PS	PS	ZR	NS	NS	NM	NB
ZR	PM	PS	PS	ZR	NS	NS	NM
NM	PB	PM	PS	PS	ZR	NS	NS
NS	PB	PB	PM	PS	PS	ZR	NS
NB	PB	PB	PB	PM	PS	PS	ZR
$\dot{e}$ / e	NB	NM	NS	ZR	PS	PM	PB

4. Simulation

Simulation is conducted about three-link planar robot arm (n =3). The number of actuated (active) joints is two (r=2) and the number of underactuated (passive) joints (p=1). The passive joint is designated to the third link(q_3). Because r >p condition is satisfied, property 6 is effective. The feature of underactuated robot manipulator is given in fig 1.

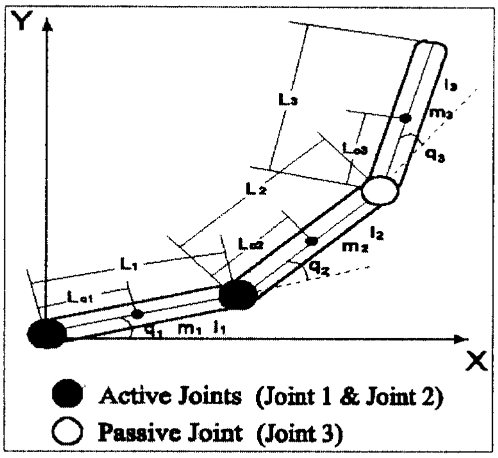


Figure 1. The configuration of underactuated robot manipulator

To consider the uncertainty of robot parameter, nominal values of parameters are selected 70 % of real values. So the ratio of real value vs. nominal value is 70 %. Initial conditions are $q_1(0) = q_2(0) = 0\,\mathrm{deg}$, $q_3(0) = -90\,\mathrm{deg}$. . And desired set points are $q_{1d} = 90\,\mathrm{deg}$, $q_{2d}(0) = q_{3d}(0) = 0\,\mathrm{deg}$. . Constants of gain (or matrix) are given as follows.

$$K_p = 100, \Lambda_p = 15, R_p = 1, K_a = 3, \Lambda_a = 3.$$

So the slope of the sliding line of passive joint in phase portrait is -15, and the slope of the sliding lines of active joints in phase portrait is -3.

Simulation conditions are summarized as follows.

Simulation condition

- **Link Length :** 0.5 m for all joints
- **Link Mass :** 1 Kg for all joints
- **Three-link planar robot arm :** n =3
- **Number of actuated (active) joints :** r = 2
- **Number of underactuated (passive) joints :** p = 1 (third link, q_3)
- **the Ratio of real value vs. nominal value :** 70 %
- **Initial Condition.** $q_1(0) = q_2(0) = 0$, $q_3(0) = -90\,\mathrm{deg}$
- **Desired set points** $q_{1d} = 90\,\mathrm{deg}$, $q_{2d}(0) = q_{3d}(0) = 0$
- $K_p = 100, \Lambda_p = 15, R_p = 1, K_a = 3, \Lambda_a = 3$

For the comparison, the results obtained using sliding mode control are given. The result of SMC ig given first and the result of FSMC follows.

4.1 Joint Response

The passive joint is controlled to desired point in 0.554 sec by SMC, and in 0.518 by FSMC.

4.2 Error of Each Joints

The error converges to zero and the dynamics are similar in two cases.

4.3 Phase Portrait of joint3(passive joint)

The result from two control methods seem to not very much different and both exhibit rather good responses.

4.4 Phase Portrait of joint1 and joint2

For SMC case, we can see the chattering phenomena are severe and it is not desirable in practical situation, With FSMC, the chattering is effectively alleviated because there is no drastic change of control resulted by the term related sgn(s).

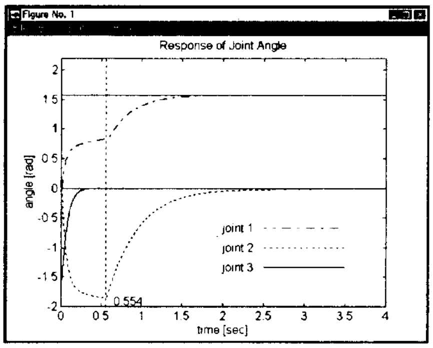


Figure 2. Joint Response by SMC

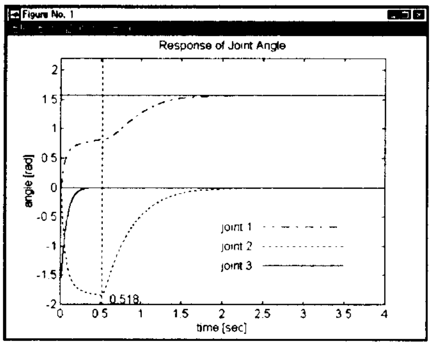


Figure 3. Joint Response by FSMC

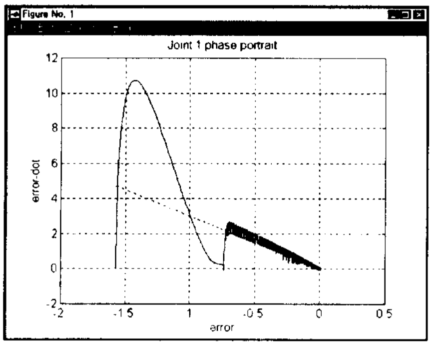


Figure 4. Phase portrait(Joint1) by SMC

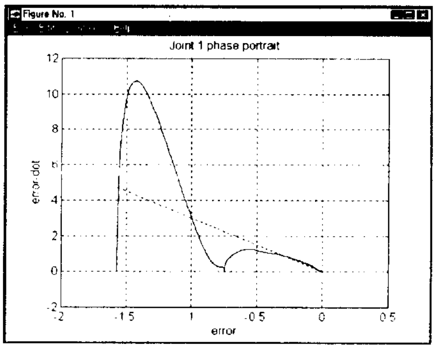


Figure 5. Phase portrait(Joint2) by FSMC

4.5 Generated Torque at joint1 and joint2

For SMC, the control shows severe chattering but this is not with FSMC. Because no excessive control is introduced, the magnitude of torque by FSMC is relatively small than that by SMC.

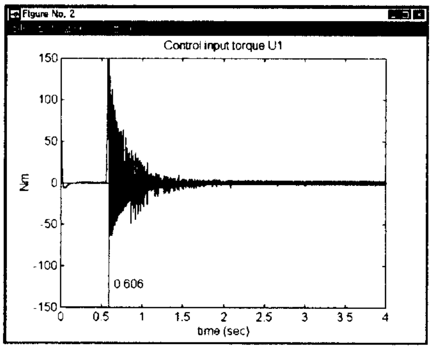


Figure 6. Torque at Joint 1 by SMC

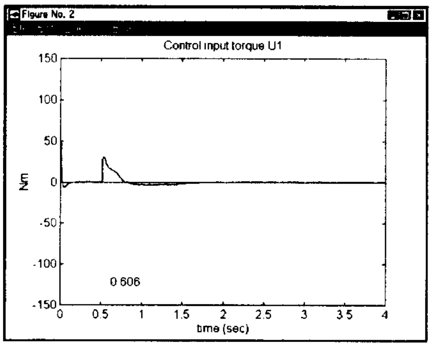


Figure 7. Torque at Joint 1 by FSMC

5. Conclusion

In viewing of simulation results robust control ability overcoming parameter uncertainties (70% real value) in disturbance environment was confirmed. This can be done by very simple controller design algorithm. Because the design is based on the phase plane of e and $\dot{e}$, according to the distance from the sliding surface the tendency (i.e magnitude and sign) of the control input is determined and so we can obtain fuzzy rule relation rather easy and systematically, moreover by introducing fuzzy sliding surface, the chattering phenomenon is effectively resolved which is a major problem of sliding mode control.

6. References

[1] H. Arai, S. Tachi, "Position Control of a Manipulator with Passive Joints Using Dynamic Coupling", IEEE Trans. on Robotics and Automation, Vol.7, No.4, August 1991.

[2] G. Oriolo and Y. Nakamura, "Free-joint manipulators : motion control under second order nonholomonic constraints," IEEE international Workshop on Intelligent robots and systems. 1991, pp 1248-1253.

[3] E. Papadopoulos and S. Dubowsky, "Failure recovery control for space robotic systems." American Control Conference, Vol. 2, 1991, pp 1485-1490

[4] M. Bergerman, "Dynamics and Control of Underactuated Manipulators", Ph.d Thesis, CMU, 1996.

[5] J. Shin, J. Lee, "Robust Adaptive Control for Underactuated Robot Manipulators : Part I. Joints Space Control Using the Brakes at Passive Joints" submitted to IEEE Trans. on Robotics and Automation, 1997

[6] D. Driankov et al, "An Introduction to Fuzzy Control", Reading, Springer-Verlag.

[7] C. Ting et al, "Fuzzy Sliding Mode Control of Nonlinear System", IEEE Int'l Conference on Fuzzy Systems. 1994