A PCI (Process Capability Index)-based desirability function approach for multi-characteristics parameter design problems is presented. Three types of PCI’s, including the one newly developed for the-nominal-the-best characteristic, are employed depending on the type of performance characteristics, and a new desirability function (i.e., a logistic function) is proposed for the PCI’s. The weighted geometric mean of the desirabilities for PCI’s is then used as an aggregated performance measure for compromising the conflicts among the levels of a design parameter. Using PCI’s and a logistic desirability function for solving a multi-characteristics parameter design problem is relatively simple and easy to implement. In addition, the proposed approach yields dispersion-sensitive results regardless of the type of performance characteristics. The developed approach is illustrated with an example from a drawing process of optical fiber, and compared with the existing desirability function approaches.

**Keywords:** Multiple Performance Characteristics; Parameter Design; Process Capability Index; Desirability Function; Compromising Conflicts.

1. **Introduction**

Designing a high-quality product or a process at a low cost is one of the most important objectives of systems designers. Parameter design approaches such as those proposed by Taguchi\(^1,2\) and others have been effectively utilized to achieve this objective. The purpose of parameter design is to experimentally determine appropriate levels of design
parameters such that the performance characteristics become robust against noise such as manufacturing variations and environmental disturbances. However, most of the existing parameter design techniques deal with the case of a single performance characteristic, although the case of multiple characteristics appears more common in product or process design. One difficulty involved in dealing with multi-characteristics parameter design (MCPD) problems is that there may exist a conflict among the levels of a design parameter. For instance, a certain level of design parameter \( A \) is best for performance characteristic \( Y_1 \) while, for performance characteristic \( Y_2 \), other levels of \( A \) might be better. Therefore, an important problem in dealing with an MCPD problem is to determine the optimum levels under such conflicts.

One of the most popular approaches to multi-characteristics decision making problems is to aggregate the multiple performance characteristics into a single performance measure. For instance, ‘desirability’ has been frequently used as such a performance measure due to its simplicity and versatility. Harrington\(^3\) originally proposes the desirability function approach for solving a multi-characteristics decision making problem. The Harrington desirability function is subsequently modified by Derringer and Suich\(^4\) and Derringer,\(^5\) and these modified desirability functions are used in most desirability function approaches.

Several authors develop methods for incorporating the concept of robustness against noise into the desirability function approach for solving the MCPD problem. For instance, Seo and Choi\(^6\) develop a method in which the Taguchi SN ratio is calculated for each performance characteristic at each design parameter combination and converted to the individual desirability. Then, the overall desirability is calculated at each design parameter combination using the method in Derringer,\(^5\) and analyzed to obtain an optimal
setting of design parameters. In the Phillips and Kim\textsuperscript{7} approach, (i) each performance characteristic value is converted to the individual desirability, (ii) the overall desirability \( (D) \) is calculated at each noise condition imposed on each design parameter combination, (iii) the mean \( (\bar{D}) \) and the standard deviation \( (S) \) of \( D \)'s are calculated at each design parameter combination, and the relationship between \( \bar{D} \)'s and \( S \)'s are examined to determine if a transformation of \( D \)'s is necessary, and (iv) the mean square error (MSE) of (transformed) \( D \)'s at each design parameter combination is used as a performance statistic for optimization. Further details of these two contributions are introduced in the Appendix.

In this paper, a new desirability function approach based on process capability indices (PCI's) and a logistic desirability function is developed for solving the MCPD problem arising in process design or improvement. The motivation for the new approach is to alleviate the difficulties frequently encountered by engineers in selecting appropriate shape parameter values in the traditional desirability function approach and to provide a simpler approach based on familiar PCI's. Like ‘desirability’, the PCI is a dimensionless, standardized quantity, and therefore, can be used as a common measure in order to aggregate or compare different characteristics. Besides, the concept of robustness is already reflected in the PCI since it involves the standard deviation of a performance characteristic. In addition, using the PCI for the MCPD problem has the following advantages:

- Since each performance characteristic is converted to a PCI regardless of its type, and the PCI is the-larger-the-better quantity, the overall process of converting PCI’s to individual desirabilities can be simplified. In addition, most process engineers are familiar with PCI’s. Therefore, it is relatively easy to determine the parameter values
necessary for implementing a desirability function approach.

- A Monte Carlo simulation study conducted by the authors shows that, among the three approaches, only the proposed PCI-based approach yields dispersion-sensitive results regardless of the type of performance characteristics. This is an important advantage of the proposed approach over the others since the dispersion of a performance characteristic is in general more difficult to deal with than the mean.

  Detailed simulation results are available from the authors upon request.

Although the present investigation is concerned with the desirability function approach, it is worth noting that other approaches have been proposed in the literature. They include: ‘β-technique’ for transformation and a linear programming approach by Logothetis and Haigh; loss function approaches by Pignatiello, Ribeiro and Albin; Ribeiro and Elsayed; and Tsui; the PerMQ (Performance Measure on Quality) approach by Elsayed and Chen; PCA(Principal Component Analysis)-based approaches by Su and Tong and Antony; and fuzzy approaches by Lai and Chang, Lai, and Tong and Su, among others.

The remainder of the paper is organized as follows. Section 2 presents a motivational example from an optical fiber fabrication process, and Section 3 reviews various existing PCI’s. In Section 4, a desirability function approach based on PCI’s is developed, and the motivational example is solved in Section 5. The proposed and existing desirability function approaches are compared using the motivational example in Section 6, and future research directions are discussed in Section 7.

2. Motivational Example
In the communication industry, optical fibers are often used to provide means for guiding the visible and infrared light over a long distance. Fabrication of optical fibers consists of two main operations: MCVD (modified chemical vapor deposition) and drawing. The MCVD operation produces high-silica glass preforms, each of which is a rod of glass containing a cladding surrounding the core material. A single fiber is then drawn from a preform and rolled into spools. Finally, the fiber in each spool is inspected to measure its performance characteristics. Engineers are mainly interested in three performance characteristics \( Y_i \sim Y_i \), among which \( Y_1 \) and \( Y_2 \) are of the-nominal-the-best (NB) type and \( Y_3 \) is of the-smaller-the-better (SB) type. Table 1 summarizes the specification limits of the three performance characteristics. Due to proprietary requirements, variable names are not explicitly stated and their values are properly adjusted.

<table>
<thead>
<tr>
<th>Performance characteristics</th>
<th>Specification limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1 )</td>
<td>13.4 – 13.6</td>
</tr>
<tr>
<td>( Y_2 )</td>
<td>140 – 142</td>
</tr>
<tr>
<td>( Y_3 )</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

To optimize the drawing operation experimentally with respect to the three performance characteristics, four design parameters \( A \sim D \) are selected. Three levels are considered for each design parameter, and experimental runs (or design parameter combinations) are determined using an orthogonal array \( L_9(3^4) \) as shown in Table 2 (see Taguchi\(^1\)). Only main effects of design parameters are considered since interaction effects are believed to be negligible. The existing process condition corresponds to the 2nd experimental run in which design parameters \( A, B, C \), and \( D \) are set to the 1st, 2nd, 2nd, and 2nd levels, respectively (i.e., \( A_1B_2C_2D_2 \)).
Four preforms are processed at every experimental run and several spools are made per preform. Finally, each of the three characteristics is measured once for every spool.

The means and standard deviations of the three characteristics at each experimental run are shown in Tables 3 and 4, respectively.

It is of interest to determine levels of design parameters such that the three performance characteristics may be simultaneously optimized. Since multiple
performance characteristics are involved in this experiment, it is expected that there might exist conflicts among the levels of the design parameters. Considering the advantages mentioned in Section 1, we develop a PCI-based desirability function approach and apply it to the above MCPD problem as described in the following sections.

3. Review of Process Capability Indices

Sullivan\textsuperscript{19,20} observes PCI’s in use at Japanese manufacturing facilities and describes five such indices as $C_p$, $C_{pk}$, $k$, $C_{ps}$, and $C_{pl}$. For an NB characteristic $Y$, the PCI is given by

$$C_p = \frac{USL - LSL}{6\sigma}$$

where $USL$ and $LSL$ are respectively the upper and lower specification limits, and $\sigma$ is the standard deviation of $Y$. Note that $C_p$ considers only the variability of $Y$. The potential performance of the process becomes worse as the mean of $Y$ deviates from the midpoint of the specifications. To deal with this drawback, $C_{pk}$ and $k$ are developed as follows:

$$C_{pk} = \frac{\text{Min} \{USL - \mu, \mu - LSL\}}{3\sigma} = (1-k)C_p$$

$$k = \frac{|\mu - M|}{(USL - LSL)/2}$$

where $\mu$ is the mean of $Y$ and $M$ is the specification midpoint, i.e. $M = (USL + LSL)/2$. The index $k$ represents the deviation of $\mu$ from $M$, and $0 \leq k \leq 1$ if $LSL \leq \mu \leq USL$. Note that as $k$ increases the process capability represented by $C_{pk}$ is reduced.
For an SB and LB (the-larger-the-better) performance characteristics, the following PCI’s are respectively used.

- For an SB characteristic,
  \[ C_{pu} = \frac{USL - \mu}{3\sigma} . \]  
  (2)

- For an LB characteristic,
  \[ C_{pl} = \frac{\mu - LSL}{3\sigma} . \]  
  (3)

\( C_{pk} \) in (1) can also be represented using \( C_{pu} \) and \( C_{pl} \) as follows:

\[ C_{pk} = \text{Min}\{C_{pu}, C_{pl}\} . \]

Other investigators incorporate the concept of Taguchi’s quadratic loss function\(^2\) into PCI’s. The first of such indices, using the formulation of the expected loss for an NB characteristic, is proposed by Chan et al.\(^21\) as follows:

\[ C_{pm} = \frac{USL - LSL + (\mu - \mu)^2}{6\sqrt{\sigma^2 + (\mu - m)^2}} . \]  
(4)

where \( m \) is the target value of an NB characteristic and the expected loss is proportional to \( \{\sigma^2 + (\mu - m)^2\} \). Note that \( C_{pm} \) is independent of the closeness of \( \mu \) to the specification limits and considers only the deviation of \( \mu \) from the target. Pearn et al.\(^22\) propose the following \( C_{pk} \) combining \( C_{pu} \) and \( C_{pl} \) into a single index.

\[ C_{pk} = \frac{\min \{USL - \mu, \mu - LSL\}}{3\sqrt{\sigma^2 + (\mu - m)^2}} . \]  
(5)

\( C_{pk} \) is equal to zero when \( \mu \) is at the specification limits as is \( C_{pk} \). Kotz and Johnson\(^23\) modify \( C_{pu} \) by assigning a weight \( a \) to the difference between \( \mu \) and \( m \) as follows:
\[
C_{m}(a) = \frac{USL - LSL}{6\sqrt{\sigma^2 + a(\mu - m)^2}}. 
\] 

(6)

4. A PCI-based Approach to Multi-Characteristics Parameter Design

4.1. PCI’s for MCPD problems

We propose the following PCI for an NB performance characteristic.

\[
C_{x} = \frac{USL - LSL}{6(m/\mu)\sigma}. 
\]

According to Taguchi\(^1,2\), the unadjusted expected loss for an NB performance characteristic, which is proportional to \(\{\sigma^2 + (\mu - m)^2\}\), is not a proper measure since the deviation between the target and the mean of the characteristic can be eliminated by adjusting the mean to the target using an adjustment parameter. Taguchi\(^1\) classifies adjustments into multiplicative and additive ones depending on the underlying transfer function. Taguchi\(^1\) also states that most engineering problems usually belong to the multiplicative case. The performance characteristic after a multiplicative adjustment \(Y'\) can be mathematically represented as

\[
Y' = \frac{m}{\mu}Y 
\]

where \(Y\) is the performance characteristic before adjusting the mean to the target, and has mean \(\mu\) and variance \(\sigma^2\). Then, the mean and the variance of \(Y'\) are respectively given by \(\mu' = m\) and \(\sigma'^2 = (m/\mu)^2\sigma^2\). Therefore, the expected loss after adjustment is proportional to \(\{\sigma'^2 + (\mu' - m)^2\} = (m/\mu)^2\sigma^2\).

Taguchi\(^1\) proposes the expected loss after adjustment as a performance measure for an
NB performance characteristic. In this sense, \( C_s \) may be regarded as a PCI after the mean of an NB performance characteristic is adjusted to the target. Note that in the PCI’s in Eqs. (4) ~ (6) the concept of adjustment is not considered.

As a PCI for an SB performance characteristic, the traditional PCI in (2) is adopted. That is,

\[
C_s = \frac{USL - \mu}{3\sigma}.
\]

Maximizing \( C_s \) implies minimizing \( \mu \) and \( \sigma \), which is a desirable property for an SB performance characteristic.

For an LB performance characteristic, we adopt the following PCI.

\[
C_L = \frac{\mu - LSL}{3\sigma}
\]

\( C_L \) is equal to \( C_{\mu L} \) in Eq. (3). Maximizing \( C_L \) implies maximizing \( \mu \) and minimizing \( \sigma \), which is desirable for an LB performance characteristic.

### 4.2. Desirability function for PCI's

Existing approaches reviewed in Section 1 use the desirability functions proposed by Derrringer and Suich.\(^4\) When PCI’s are used, it is desirable that the rate of increase in desirability should be small in the small or the large PCI regions as shown in Figure 1. This suggests that an S-shaped desirability function is more appropriate than the one proposed by Derringer and Suich.\(^4\) Among such S-shaped functions, we propose the following logistic function.

\[
d = \frac{\exp(\alpha + \beta C)}{1 + \exp(\alpha + \beta C)}
\]

where \( d \) is a desirability value, \( C \) is \( C_{\gamma} \), \( C_{\mu} \) or \( C_{\xi} \), and \( \alpha \) and \( \beta \) are constants.
Eq. (7) can be rewritten as:

$$\ln\left(\frac{d}{1-d}\right) = \alpha + \beta C,$$

Therefore, if two points \((C_1, d_1)\) and \((C_2, d_2)\) are given, then \(\alpha\) and \(\beta\) can be obtained by solving the following equations.

$$\ln\left(\frac{d_1}{1-d_1}\right) = \alpha + \beta C_1,$$

$$\ln\left(\frac{d_2}{1-d_2}\right) = \alpha + \beta C_2.$$  \hfill (8)

![Fig. 1. Desirability function for the motivational example.](image)

As an aggregated, overall desirability measure, Derringer’s\(^5\) weighted geometric mean,
of the desirability values of all performance characteristics is adopted. That is,
\[ D_{\text{a}} = \left( d_1^{w_1} \times d_2^{w_2} \times \Lambda \times d_p^{w_p} \right)^{1/w} \]  
(9)

where \( p \) is the number of performance characteristics, \( W = \sum_{i=1}^{p} w_i \) and \( w_i (> 0) \) is the importance or weight of the \( i \)th performance characteristic. According to Derringer, the geometric mean is more useful than an arithmetic mean because in most product development situations, a single property at an unacceptable level renders the product useless. \( w_i \)'s may be determined through some process (e.g., a survey) to reach a consensus of process engineers, marketing personnel, quality control engineers, etc.

### 4.3. Estimation of overall desirability

In order to estimate \( D_{\text{a}} \) from the experimental data, let \( y_{ij} \) be the value of the performance characteristic \( Y_i \) at the \( j \)th experimental run and at the \( k \)th replications. In addition, let \( r \) be the number of replications. Then, the PCI of \( Y_i \) at the \( j \)th experimental run is estimated according to the type of characteristics as follows:

- \( \hat{C}_{\text{N},i} = \frac{\text{USL}_i - \text{LSL}_i}{6(m_i / \bar{y}_y)s_y} \) (for an NB performance characteristic)  
  \[ (10) \]
- \( \hat{C}_{\text{S},i} = \frac{\text{USL}_i - \bar{y}_y}{3s_y} \) (for an SB performance characteristic)  
  \[ (11) \]
- \( \hat{C}_{\text{L},i} = \frac{\bar{y}_y - \text{LSL}_i}{3s_y} \) (for an LB performance characteristic)

where
\[ \bar{y}_y = \frac{1}{r} \sum_{i=1}^{r} y_{ij} \]
The desirability of the $i$th performance characteristic at the $j$th experimental run is calculated as follows using Eq. (7).

$$d_q = \frac{\exp(\alpha + \beta \hat{C}_q)}{1 + \exp(\alpha + \beta \hat{C}_q)}$$

where $\hat{C}_q$ is $\hat{C}_{x,q}$, $\hat{C}_{y,q}$ or $\hat{C}_{z,q}$. Then, the overall desirability at the $j$th experimental run is estimated as:

$$D_{sj} = \left(d_{1j} \times d_{2j} \times \Lambda \times d_{pj}^{sp}\right)^{1/\pi}.$$  \hspace{1cm} (12)

5. Analysis of the Motivational Example

In this section, the approach developed in Section 4 is applied to the motivational example introduced in Section 2. For each performance characteristic, $C_y$ or $C_z$ is estimated using Eq. (10) or (11) and the values in Tables 1, 3 and 4. The target value of an NB performance characteristic corresponds to the midpoint of the interval ($LCL$, $UCL$).

Table 5 summarizes the estimated PCI’s at each experimental run.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$\hat{C}_{y,1,j}$</th>
<th>$\hat{C}_{y,2,j}$</th>
<th>$\hat{C}_{y,3,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1444</td>
<td>1.5442</td>
<td>1.5351</td>
</tr>
<tr>
<td>2</td>
<td>1.0076</td>
<td>0.9297</td>
<td>1.6870</td>
</tr>
<tr>
<td>3</td>
<td>0.8675</td>
<td>1.4730</td>
<td>1.7628</td>
</tr>
<tr>
<td>4</td>
<td>1.6567</td>
<td>0.9306</td>
<td>0.9775</td>
</tr>
<tr>
<td>5</td>
<td>1.4700</td>
<td>0.8029</td>
<td>0.9810</td>
</tr>
<tr>
<td>6</td>
<td>1.4577</td>
<td>1.0717</td>
<td>1.2979</td>
</tr>
<tr>
<td>7</td>
<td>1.3877</td>
<td>0.5863</td>
<td>1.2680</td>
</tr>
<tr>
<td>8</td>
<td>1.5043</td>
<td>1.1781</td>
<td>1.0773</td>
</tr>
<tr>
<td>9</td>
<td>1.2240</td>
<td>0.4241</td>
<td>0.9539</td>
</tr>
</tbody>
</table>

Table 5. Estimates of $C_y$ or $C_z$. 
To characterize the desirability function in Eq. (7), two reference points, \((C, d) = (0.5, 0.1)\) and \((2.0, 0.95)\) are selected based on the consensus of the engineers and marketing personnel concerned. Solving the simultaneous equations in (8) yields \(\alpha = -3.9111\) and \(\beta = 3.4278\). Figure 1 shows the resulting desirability function, and Table 6 summarizes desirability values of all performance characteristics at each experimental run.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>(d_{1j})</th>
<th>(d_{2j})</th>
<th>(d_{3j})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5029</td>
<td>0.7993</td>
<td>0.7943</td>
</tr>
<tr>
<td>2</td>
<td>0.3876</td>
<td>0.3265</td>
<td>0.8667</td>
</tr>
<tr>
<td>3</td>
<td>0.2814</td>
<td>0.7573</td>
<td>0.8939</td>
</tr>
<tr>
<td>4</td>
<td>0.8542</td>
<td>0.3271</td>
<td>0.3635</td>
</tr>
<tr>
<td>5</td>
<td>0.7554</td>
<td>0.2388</td>
<td>0.3662</td>
</tr>
<tr>
<td>6</td>
<td>0.7476</td>
<td>0.4409</td>
<td>0.6313</td>
</tr>
<tr>
<td>7</td>
<td>0.6996</td>
<td>0.1300</td>
<td>0.6071</td>
</tr>
<tr>
<td>8</td>
<td>0.7765</td>
<td>0.5318</td>
<td>0.4457</td>
</tr>
<tr>
<td>9</td>
<td>0.5707</td>
<td>0.0789</td>
<td>0.3450</td>
</tr>
</tbody>
</table>

To determine the weights in Eq. (9), engineers and marketing personnel are asked to assign importance values to each performance characteristic in the range of 1 to 5. Based on this survey results, we determine the importance values as follows.

\[ w_1 = 4.11, \quad w_2 = 3.78 \quad \text{and} \quad w_3 = 2.56. \]

The overall desirability is given by the following equation.

\[ D_{ij} = \left( d_{1j}^{w_1} \times d_{2j}^{w_2} \times d_{3j}^{w_3} \right)^{\frac{1}{\sum w}}. \]

Table 7 shows the resulting \(D_{ij}\) values at each experimental run, while Table 8 shows the corresponding sum and mean of \(D_{ij}\) for each level of design parameters. The analysis of variance results for \(D_{ij}\) are given in Table 9, which can be constructed either manually using the formulas in Taguchi or using a commercial statistical software. The
means of $D_A$ are depicted in Figure 2.

Table 7. Overall desirability at each run.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$D_{AD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.6651</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.4436</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0.5343</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.4896</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0.4171</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.5926</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0.3676</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.5910</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0.2466</td>
</tr>
</tbody>
</table>

Table 8. Sums and means of overall desirability.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Level</th>
<th>Sum of $D_{AD}$</th>
<th>Mean of $D_{AD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1</td>
<td>1.6431</td>
<td>0.5477</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.4993</td>
<td>0.4998</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.2052</td>
<td>0.4017</td>
</tr>
<tr>
<td>$B$</td>
<td>1</td>
<td>1.5223</td>
<td>0.5074</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.4518</td>
<td>0.4839</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.3735</td>
<td>0.4578</td>
</tr>
<tr>
<td>$C$</td>
<td>1</td>
<td>1.8487</td>
<td>0.6162</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.1799</td>
<td>0.3933</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.3190</td>
<td>0.4397</td>
</tr>
<tr>
<td>$D$</td>
<td>1</td>
<td>1.3289</td>
<td>0.4430</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.4038</td>
<td>0.4679</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.6150</td>
<td>0.5383</td>
</tr>
</tbody>
</table>

Overall Mean $= 0.4831$

Table 9. Analysis of variance for $D_{AD}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2</td>
<td>0.033201</td>
<td>0.016601</td>
</tr>
<tr>
<td>$B$</td>
<td>2</td>
<td>0.003693</td>
<td>0.001847</td>
</tr>
<tr>
<td>$C$</td>
<td>2</td>
<td>0.083043</td>
<td>0.041521</td>
</tr>
<tr>
<td>$D$</td>
<td>2</td>
<td>0.014671</td>
<td>0.007336</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>0.134609</td>
<td></td>
</tr>
</tbody>
</table>
Since the size of experiment is relatively small and the information on experimental error is not available as shown in Table 9, no formal statistical test of significance for each source is conducted. Instead, initial optimum settings of design parameters $A$, $B$, $C$, and $D$ are determined as $A_1$, $B_1$, $C_1$, and $D_1$, respectively, by examining Figure 2.

Later, the optimal level of $B$ was changed from $B_1$ to $B_2$ since the difference between the means of $D_1$ at $B_1$ and $B_2$ is small and engineers pointed out that $B_2$ is easier to implement than $B_1$.

A verification run is conducted to verify whether the optimum condition actually yields an improvement over the existing condition. Four preforms are processed at the optimum condition and several spools are obtained per preform. Each of the three performance characteristics is measured for each spool and the values of PCI’s are estimated using Eq. (10) or (11). Table 10 compares the improvement in the process capability and the desirability at the existing and the optimum conditions. Note that no verification run was made at the existing condition since it is already included in the initial experiment.
The verification results indicate that about a 66.6% improvement in the overall desirability is achieved, although the PCI for $Y_3$ is slightly reduced. A subsequent monitoring of the drawing process for one month shows that the overall stability of the drawing process is enhanced as expected.

6. Comparisons

The data are also analyzed using the Seo and Choi\(^6\) and Phillips and Kim\(^7\) approaches. For the Seo and Choi approach, three values of the shape parameter $t$ (i.e., 0.5, 1 and 2) are used (refer to Eq. (A.1) in Appendix), and all result in the same optimal design parameter combination, $A_1B_1C_1D_2$. In applying the Phillips and Kim approach to the experimental data, the following parameter values are used (refer to Appendix).

$$y^*_i = LSL, \quad y^\alpha_i = USL, \text{ for } Y_i \text{ and } Y_z$$

$$y^*_i = 0, \quad y^\alpha_i = USL, \text{ for } Y_s.$$  

In addition, the weighted overall desirability in Eq.(A. 2) is employed with the same set of weights as in Section 5, together with the unweighted MSE in Eq.(A. 3). The above process is repeated for three sets of shape parameter values, $(s, t, v) = (0.5, 0.5, 0.5), (1, 1, 1), \text{ and } (2, 2, 2)$. For the first and the third sets, $D_{\lambda}$'s and $S^2$ show some dependency, and therefore, $D_{\lambda}$’s are transformed with transformation parameter $\lambda = 3$ and 0.67.
respectively. Finally, for all three cases, \( A_1B_1C_1D_3 \) is found to be the optimal design parameter combination with respect to the MSE in Eq. (A. 3). Note that this optimal condition is the same as the original optimal condition obtained by the present PCI-based approach. In addition, the same optimal condition is obtained when only the bias term in Eq. (A. 3) is considered. In other words, the bias term dominates the variance term in Eq. (A. 3), which is a similar phenomenon also experienced by Phillips and Kim.\(^7\) Further investigations and experience are needed to determine if the present and the Phillips and Kim\(^7\) approaches yield similar results in practice.

A verification run is also made at \( A_1B_1C_1D_2 \), the optimal design parameter combination generated by the Seo and Choi approach, and the results are summarized in Table 11.

<table>
<thead>
<tr>
<th>Performance Characteristic</th>
<th>PCI-based</th>
<th>SN-based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>( Y_1 )</td>
<td>13.4631</td>
<td>0.0274</td>
</tr>
<tr>
<td>( Y_2 )</td>
<td>141.3053</td>
<td>0.1873</td>
</tr>
<tr>
<td>( Y_3 )</td>
<td>0.5942</td>
<td>0.0809</td>
</tr>
</tbody>
</table>

Note that the PCI values for the PCI-based optimum combination are larger than those for the SN-based one as expected. The SN ratios of \( Y_1 \) and \( Y_2 \) (the two most important characteristics) for the former are also larger than those for the latter. Based on these observations, the PCI-based optimum condition is selected and implemented.

7. Conclusions

A new PCI-based logistic desirability function approach is developed for solving multi-
characteristics parameter design problems. Using PCI’s and the logistic desirability function alleviate the difficulties frequently encountered in selecting appropriate shape parameter values in the traditional desirability function approach. In addition, the proposed approach yields dispersion-sensitive results regardless of the type of performance characteristics. This may be due to the fact that the standard deviation of a performance characteristic is placed in the denominator when calculating the corresponding PCI.

Although the present investigation is concerned with the desirability function approach, it is desired to compare various approaches mentioned in Section 1 with respect to the amount of information required from the user, complexity, solution quality, etc., especially when an actual MCPD problem is formulated and solved in practice.

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Appendix

The Seo and Choi\textsuperscript{a} desirability function approach proceeds as follows.

1. For each performance characteristic \( i \), calculate the corresponding SN ratio at each experimental run \( j \).

\[
SN_y = \begin{cases} 
-10 \log \left( \frac{1}{p} \sum_{k=1}^{p} y_{jk}^2 \right) & \text{for an SB characteristic} \\
-10 \log \left( \frac{1}{p} \sum_{k=1}^{p} \frac{1}{y_{jk}} \right) & \text{for an LB characteristic} \\
10 \log \frac{y_{ij}}{s_y} & \text{for an NB characteristic}
\end{cases}
\]

2. Convert SN ratios in Step 1 to individual desirabilities.

\[
d_y = \begin{cases} 
0, & SN_y \leq SN_i^L \\
\frac{SN_y - SN_i^L}{SN_i^U - SN_i^L}, & SN_i^L < SN_y < SN_i^U \\
1, & SN_y \geq SN_i^U
\end{cases}
\]

where

\[
SN_i^U = \max_{y_{ij} \leq 0} \left\{ SN_y \right\}, \quad i = 1, 2, \ldots, p
\]

\[
SN_i^L = \begin{cases} 
-10 \log(USL) & \text{for an SB characteristic} \\
10 \log \left( \frac{LSL^2_i}{s_i^2} \right) & \text{for an NB characteristic}
\end{cases}
\]

\[
s_i^2 = \max_{y_{ij} \leq 0} \left\{ s_y^2 \right\}
\]

For an LB characteristic, a reciprocal is taken and treated as an SB characteristic.

3. At each experimental run \( j \), calculate the overall desirability \( D_j \) using Eq.(12), and
analyze the overall desirabilities to determine optimal design parameter combination.

In a similar manner, the Phillips and Kim\(^7\) approach can be described as follows.

1. Convert each performance characteristic value to the individual desirability.
   - For an NB characteristic
     \[
     d_{ijk} = \begin{cases} 
     \left( \frac{y_{ik} - y_i^c}{y_i^H - y_i^c} \right)^\gamma, & y_i^c \leq y_{ik} \leq m_i \\
     \left( \frac{y_{ik} - y_i^m}{m_i - y_i^m} \right)^\gamma, & m_i \leq y_{ik} \leq y_i^m \\
     0, & y_{ik} \leq y_i^c \text{ or } y_{ik} \geq y_i^m
     \end{cases}
     \]
   - For an LB characteristic
     \[
     d_{ijk} = \begin{cases} 
     0, & y_{ik} \leq y_i^c \\
     \left( \frac{y_{ik} - y_i^l}{y_i^m - y_i^l} \right)^\gamma, & y_i^l \leq y_{ik} \leq y_i^m \\
     1, & y_{ik} \geq y_i^m
     \end{cases}
     \]
   - For an SB characteristic
     \[
     d_{ijk} = \begin{cases} 
     1, & y_{ik} \leq y_i^c \\
     \left( \frac{y_{ik} - y_i^v}{y_i^m - y_i^v} \right)^\gamma, & y_i^v \leq y_{ik} \leq y_i^m \\
     0, & y_{ik} \geq y_i^m
     \end{cases}
     \]
   where \(y_i^c\) and \(y_i^m\) are the minimum and maximum desirable values of performance characteristic \(i\), respectively.

2. At each experimental run \(j\) and noise condition \(k\), calculate the overall desirability using one of the following formulas.
\[ D_{\mu} = (d_{j_k} \times d_{z_\mu} \times \Lambda \times d_{m_\mu})^{1/p} \]
\[ D_{\omega} = (d_{j_k}^{1/n} \times d_{z_\mu}^{1/n} \times \Lambda \times d_{m_\mu}^{1/n})^{1/w} \]  \hspace{1cm} (A. 2)

where \( W = \sum_{j=1}^{k} w_j \) and \( w_i > 0 \).

3. At each experimental run \( j \), calculate the sample mean \( \bar{D}_j \) and sample variance \( S_j^2 \) of \( D_{\mu} \)’s.

4. Evaluate whether or not \( \bar{D} \) and \( S^2 \) in Step 3 are related. If needed, transform \( D_{\mu} \)’s using the method in Logothetis and Haigh, for instance.

5. At each experimental run \( j \), calculate either the unweighted or weighted MSE of (transformed) \( D_{\mu} \)’s as follows.
\[ \text{MSE}_j = (1 - \bar{D}_j)^2 + S_j^2 \]  \hspace{1cm} (A.3)
\[ \text{MSE}_j = \phi_1 (1 - \bar{D}_j)^2 + \phi_2 S_j^2 \]

Then, analyze the MSE to determine an optimal design parameter combination.
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