Development of performance measures for dynamic parameter design problems

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Abstract: The objective of the Taguchi parameter design is to improve the performance of a product or a process by determining the levels of its design parameters such that the performance characteristic is robust against various causes of variation. Parameter design problems are broadly classified into static and dynamic ones. Static problems are further classified into the-nominal-the-best, the-larger-the-better and the-smaller-the-better ones, and for each problem type a unique performance measure (or an SN ratio) is appropriately defined. However, for the dynamic parameter design problem, such a classification has not been explicitly made. The only case that has been extensively discussed in the literature is the one where the slope $\beta$ of the relationship between the performance characteristic and the signal parameter has a finite ideal value. In this article, we consider the cases in which it is desired to have $\beta$ as large as and as small as possible, and develop a performance measure for each case. In addition, the existing performance measure for the case where $\beta$ has a finite ideal value is modified according to the framework proposed in this article. The developed parameter design procedures are illustrated with an example.
Keywords: parameter design; Taguchi method; dynamic characteristics; performance measures.

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1 Introduction

A variable which represents the performance of a product or a process is called a performance characteristic (or briefly, a characteristic). A characteristic deviates from its target value due to such noises as manufacturing variation, environmental noise or deterioration. The objective of the Taguchi parameter design is to determine a setting of design parameters at which the characteristic is robust against noise. For this purpose, Taguchi proposed an experimental method in which orthogonal arrays are used as experimental designs and a performance measure called the SN (signal-to-noise) ratio is employed for analyzing the experimental data [1,2].

Parameter design problems are broadly classified into static and dynamic ones. A characteristic with a fixed target value $t$ is called a static characteristic. The principal goal of parameter design
for these static problems is to achieve the smallest variation of the characteristic around $t$. Static problems are further classified into the-nominal-the-best (NB), the-larger-the-better (LB), and the-smaller-the-better (SB) problems with the target values being $0 < t < \infty$, $\infty$, and 0, respectively. For each problem type a unique performance measure (or an SN ratio) is appropriately defined. On the other hand, the target values of a characteristic could vary from time to time according to the requirements of customers. Such a characteristic is called a dynamic characteristic, and the input variable used to attain the required target values is called a signal parameter. The goal of parameter design for a dynamic characteristic problem is to achieve the smallest variation of the characteristic around its target values over the range of the signal parameter under various noise conditions.

For dynamic parameter design problems, most authors are concerned with the case where the slope $\beta$ of the relationship between the performance characteristic and the signal parameter has a finite ideal value [1,2,3]. In this article, we classify dynamic parameter design problems into three types as for static ones, and develop performance measures for the cases where $\beta$ is desired to be as large as and as small as possible. In addition, the existing performance measure for the case where $\beta$ has a finite ideal value is modified according to the framework proposed in this article. The developed parameter design procedures are illustrated with an example.

2 Development of performance measures

2.1 Taguchi methods

Let $M$ denote the signal parameter and $t(M)$ be the target value of the characteristic $y$ as a function of $M$. Then, Taguchi considered the case where the ideal target function is given by

$$t(M) = \beta_* M$$

(1)
where $\beta_i$ is the ideal slope of the relationship and $0 < \beta_i < \infty$ [2]. Consider a model of the form

$$y(M, N) = \beta M + e(M, N)$$

$$E[\{e(M, N)\}^2] = \sigma^2$$

where $N$ represents a collection of noise variables, $e$ is an error term, and $\sigma^2$ is a mean squared deviation of $y$ with respect to $M$ and $N$. Then, Taguchi approximated the loss caused by the departure of $y$ from the target function $t(M)$ as follows [1].

$$L(y) = k(y - t(M))^2$$

where $k$ is a constant. Taguchi assumed the existence of an adjustment parameter which can be used to adjust $\beta$ in (2) to $\beta_i$ in (1), and also assumed that the effect of such an adjustment on $y$ can be mathematically described by

$$y' = (\beta_i / \beta)y$$

where $y'$ is the characteristic value after adjustment [1]. The expected loss after adjustment is then given by

$$E[L(y')] = kE[\{y' - t(M)\}^2] = kE[\{(\beta_i / \beta)e(M, N)\}^2] = k(\beta_i^2 / \beta^2)\sigma^2.$$  

(3)

Since $\beta_i$ and $k$ in (3) are the same for all the design parameter settings, $\sigma^2 / \beta^2$ can be used to compare their performances. Taguchi proposed the following SN ratio as a performance measure (PM) to be maximized [1].

$$SN = 10\log(\beta^2 / \sigma^2).$$  

(4)

SN in (4) is then estimated based on the experimental data at each experimental point (i.e., at each design setting included in the experiment).

2.2 Classification of dynamic parameter design problems

Since dynamic characteristics are similar in nature to static ones, they can also be classified in a
similar manner. Figure 1 shows such a classification in which ‘D’ stands for ‘Dynamic’.

The reason why Taguchi’s SN ratio in (4) is inadequate for DLB and DSB problems is that the SN ratio in (4) is based on the assumption that the slope at any design parameter setting can be adjusted to the ideal slope, which may not be possible for DLB and DSB problems. In this article, a new PM for each of DLB and DSB cases is developed. In addition, a modified version of the Taguchi SN ratio in (4) for a DNB characteristic is presented following the framework proposed in this article.

2.3 PM for DLB characteristics

The following model is assumed for a DLB characteristic at a setting of design parameters.

\[ y(M, N) = \beta M + e(M, N) \]  

(5)

where \( \beta > 0 \) and random error term \( e(M, N) \) is distributed with mean 0 and variance \( \sigma^2(M) \). It is further assumed that for a given product there exists a lower tolerance limit \( \lambda \) on \( y \). \( \lambda \) may change from product to product over a known interval \([a,b]\).

In choosing an appropriate value of \( M \) for a given product, two types of cost (or loss) need to be considered. One is the cost related to the value of \( M \) chosen. In a welding process, for instance, the cost for the welding length (\( M \)) of 20mm is larger than that of 10mm due to additional material and processing time required. The other is the loss incurred by the variation of \( y \) around its mean value as shown in Figure 2. In summary, for a given product at a design setting, it is desired that \( M \) is chosen to minimize the following total cost (or loss).

\[ C = \text{cost for choosing } M + \text{(loss due to the variation of } y). \]  

(6)

The first component in the right hand side of (6) is assumed to be as follows.

\[ \text{cost for choosing } M = C_v M \]  

(7)

where \( C_v \) is a positive constant. The loss due to the variation of \( y \) can be described using the
loss function of an LB characteristic, i.e., \( L(y) = k / y^2 \), since it is desirable to have \( \beta \) as large as possible for a DLB characteristic. Thus, the expected loss is given by [2]

\[
E[L(y)] = E \left[ \frac{k}{y^2} \right] \approx \frac{k}{E(y)} \left[ \frac{3 \text{var}(y)}{E(y)^2} + 1 \right] = \frac{k}{(\beta M)^2} \left[ \frac{3\sigma^2(M)}{(\beta M)^2} + 1 \right].
\] (8)

Let \( A \) be the loss when \( y \) equals the tolerance limit \( \lambda \) (see Figure 3). Then, \( k = A\lambda^2 \), and the expected loss in (8) can be rewritten as

\[
E[L(y)] \approx \frac{A\lambda^2}{(\beta M)^2} \left[ \frac{3\sigma^2(M)}{(\beta M)^2} + 1 \right].
\]

Therefore, \( C \) in (6) can be rewritten as

\[
C = C_vM + E[L(y)] \approx C_vM + \frac{A\lambda^2}{(\beta M)^2} \left[ \frac{3\sigma^2(M)}{(\beta M)^2} + 1 \right].
\] (9)

Note that not only the magnitude of the slope (i.e., \( \beta \)), but also the stability of \( y \) (i.e., \( \sigma^2(M) \)) is reflected in (9).

Since the total cost in (9) is defined for a tolerance limit \( \lambda \), which may change from product to product at a given design setting, it is desired that a performance measure for a design setting should be computed as an ‘average’ of the total costs over \( \lambda \). Therefore, we define the PM as follows.

\[
PM = \int_{a}^{b} w(\lambda) \times \left[ C_vM + \frac{A\lambda^2}{(\beta M)^2} \left[ \frac{3\sigma^2(M)}{(\beta M)^2} + 1 \right] \right] d\lambda.
\] (10)

where \([a, b]\) is the range of \( \lambda \) and \( w(\lambda) \) is a weight function.

In (9), \( A \) may take various forms. In this article, we consider a case where \( A \) can be approximated by

\[
A \approx C_s + C_vM
\] (11)

where \( C_s \) is a nonnegative constant and \( C_vM \) is defined in (7). In addition, for \( \sigma(M) \) in (9),
we assume the most frequently encountered cases where $\sigma(M)$ is proportional to the mean of $y$, namely, $\sigma(M) = \alpha(\beta M)$. Under these assumptions, the total cost in (9) can be rewritten as

$$C \approx C_v M + \frac{(C_s + C_v M) \lambda^2}{(\beta M)^2} \left\{ \frac{3(\alpha \beta M)^2}{(\beta M)^2} + 1 \right\} = C_v M + \frac{(C_s + C_v M) \lambda^2}{(\beta M)^2} (3\alpha^2 + 1).$$

(12)

Since

$$\frac{d^2 C}{dM^2} = \frac{\lambda^2 (3\alpha^2 + 1) 2C_v M + 6C_s}{\beta^2 M^4} > 0 \quad \text{for } M > 0$$

and

$$\lim_{M \to 0} C = \infty, \quad \lim_{M \to \infty} C = \infty,$$

$C$ is a convex function of $M$ over the region $M > 0$, and there exists a unique positive value of $M$ (i.e., $M^*$) which minimizes $C$. $M^*$ can be determined by solving $dC/dM = 0$ and taking the positive-valued solution for given $C_s$, $C_v$, $\lambda$, $\alpha$ and $\beta$. Then, inserting $A \approx C_s + C_v M$, $\sigma(M) = \alpha \beta M$, and $M = M^*$ into the expression in (10) yields

$$PM = \int_a^b w(\lambda) \times \left\{ C_v M^* + \frac{(C_s + C_v M^*) \lambda^2}{(\beta M^*)^2} (3\alpha^2 + 1) \right\} d\lambda.$$  

(13)

In general, a closed form expression for $PM$ in (13) is difficult to obtain, if not impossible, and therefore, a numerical evaluation is necessary for a given situation. The following describes a special case for which a closed form expression for $PM$ can be obtained.

Consider a special case where a nonconforming product is reworked, for which $A$ in (11) may be approximated by

$$A \approx C_v M.$$

Then, from (12), the value of $M$ which minimizes $C$ is uniquely given by

$$M^* = \frac{\lambda \sqrt{3\alpha^2 + 1}}{\beta}.$$

Subsequently, $PM$ in (13) is simplified as
Finally, if the weights of all \( \lambda \)'s are assumed to be the same, that is, \( w(\lambda) = 1/(b - a) \), then, PM in (14) is further simplified as

\[
PM = (a + b)C_v \frac{\sqrt{3\alpha^2 + 1}}{\beta} .
\]  

(15)

To estimate the PM for each design parameter setting, \( \alpha \) and \( \beta \) need to be estimated from the experimental data. At an experimental setting of design parameters, several values of \( M \) are chosen and replicated observations are usually made under various noise conditions at each value of \( M \) (e.g., see Table 1 in Section 3). In such an experimental situation the characteristic \( y \) at a design setting can be represented as

\[
y_{ij} = \beta M_i + e_{ij}
\]

\[
e_{ij} \sim (0, \alpha^2 \beta^2 M_i^2)
\]

where \( i \) and \( j \) denote a level of the signal parameter \( (i = 1, 2, \Lambda, m) \) and a replicated observation at \( i \) \( (j = 1, 2, \Lambda, n) \), respectively. The weighted least squares estimators for \( \beta \) and \( (\alpha \beta)^2 \) are respectively given by

\[
\hat{\beta} = \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} / M_i) / (mn)
\]  

(16)

\[
(\hat{\alpha} \hat{\beta})^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} / M_i - \hat{\beta})^2 / (mn - 1).
\]  

(17)

In addition, an estimator for \( \alpha^2 \) is given by

\[
\hat{\alpha}^2 = \frac{(\hat{\alpha} \hat{\beta})^2}{(\hat{\beta})^2}.
\]  

(18)

Then, PM in (15), for example, can be estimated as

\[
\hat{PM} = (a + b)C_v \frac{\sqrt{3\hat{\alpha}^2 + 1}}{\hat{\beta}} .
\]  

(19)
2.4 PM for DSB characteristics

A PM for DSB characteristics can be developed in a similar manner as for DLB characteristics. That is, the model in (5) is assumed and, for a given product, an upper tolerance limit \( u \) on \( y \) is assumed to exist. \( u \) may change from product to product over a known interval \([a,b]\). The total cost (or loss) in (6) is also assumed with the first component in the right hand side being as follows.

\[
\text{cost for choosing } M = C_y/M . \tag{20}
\]

For instance, the cost function in (20) can be justified for a polishing process in which the polishing material of a smaller grain size (\( M \)) is more expensive and requires more processing time than that of a larger grain size.

The loss due to the variation of \( y \) can be described using the loss function of an SB characteristic, i.e., \( L(y) = ky^2 \), since it is desirable to have \( \beta \) as small as possible for a DSB characteristic. Thus, the expected loss is given by [2]

\[
E[L(y)] = E[ky^2] = k\{E(y)^2 + \text{var}(y)\} = k\{(\beta M)^2 + \sigma^2(M)\} . \tag{21}
\]

Let \( A \) be the loss when \( y \) equals the tolerance limit \( u \). Since \( k = A/u^2 \), the expected loss in (21) can be rewritten as

\[
E[L(y)] = \frac{A}{u^2}\{(\beta M)^2 + \sigma^2(M)\} .
\]

Therefore, \( C \) in (6) can be rewritten as

\[
C = C_y/M + E[L(y)] = \frac{C_y}{M} + \frac{A}{u^2}\{(\beta M)^2 + \sigma^2(M)\} . \tag{22}
\]

Integrating \( C \) over the range of \( u \), we obtain a PM as follows.
\[ PM = \int_a^b w(u) \left[ \frac{C_s}{M} + \frac{A}{u^2} \left( (\beta M)^2 + \sigma^2(M) \right) \right] du \]  

(23)

where \( w(u) \) is a weight function.

We assume that \( A \) can be approximated by

\[ A \approx C_s + C_v / M \]  

(24)

where \( C_s \) is a nonnegative constant and \( C_v / M \) is defined in (20). In addition, we assume that \( \sigma(M) = \alpha(\beta M) \) as in the case of DLB characteristics. Under these assumptions, the total cost in (22) can be rewritten as

\[ C = \frac{C_s}{M} + \frac{(C_s + C_v / M)}{u^2} \left( (\beta M)^2 + (\alpha \beta M)^2 \right). \]  

(25)

Since

\[ \frac{d^2 C}{dM^2} = \frac{2C_s}{M^3} + \frac{2C_s \beta^2 (1 + \alpha^2)}{u^2} > 0 \]  

for \( M > 0 \)

and

\[ \lim_{M \to 0} C = \infty, \quad \lim_{M \to \infty} C = \infty, \]

\( C \) is a convex function of \( M \) over the region \( M > 0 \), and there exists a unique positive value of \( M \) (i.e., \( M^* \)) which minimizes \( C \). \( M^* \) can be determined by solving \( dC / dM = 0 \) and taking the positive-valued solution for given \( C_s, C_v, u, \alpha \) and \( \beta \). Then, inserting \( A \approx C_s + C_v / M \), \( \sigma(M) = \alpha \beta M \), and \( M = M^* \) into the expression in (23) yields

\[ PM = \int_a^b w(u) \left\{ \frac{C_s}{M^*} + \frac{(C_s + C_v / M^*)}{u^2} \beta^2 (1 + \alpha^2) M^* \right\} du \]  

(26)

As for the case of DLB characteristics, \( PM \) in (26) requires a numerical evaluation in general. The following describes a special case for which \( PM \) in (26) can be expressed in a closed form.

Consider a special case where a nonconforming product is reworked, for which \( A \) in (24) may be approximated by
\[ A \approx C_v / M . \]

Then, from (25), the value of \( M \) which minimizes \( C \) is uniquely given by

\[ M^* = \frac{u}{\beta \sqrt{1 + \alpha^2}} . \]

Subsequently, \( PM \) is simplified as

\[ PM = 2 \int_a^b w(u) C_v \frac{\beta \sqrt{1 + \alpha^2}}{u} du . \]

If \( w(u) = 1/(b-a) \), \( PM \) is further simplified as

\[ PM = \frac{2C_v}{b-a} \beta \sqrt{1 + \alpha^2} \ln \frac{b}{a} . \]

Estimation of \( \beta \) and \( \alpha^2 \) follows the same procedure as for DLB characteristics.

2.5 \( PM \) for DNB characteristics

In this section, the Taguchi approach for a DNB characteristic is modified according to the framework proposed in this article. First, we assume the following model.

\[ y(M, N) = \beta M + e(M, N) \]

\[ e(M, N) \sim (0, \sigma^2(M)) . \]

For a DNB characteristic of a given product at any design parameter setting, the target value for the mean of \( y \) is given by \( t \), the slope \( \beta \) is eventually adjusted to the ideal slope \( \beta_t \), and, after adjustment, the value of the signal parameter is set to \( M' ( = t / \beta_t ) \) to achieve the target value \( t \). This implies that the cost for choosing \( M \) is the same for all design parameter settings, and therefore, needs not be considered in comparing them. Then, the total cost (or loss) consists of the expected loss after adjustment which is given by
for any $t$ where $y'$ is the characteristic after adjustment and is given by $y'=(\beta_1/\beta)y$.

Suppose that a cost $A$ is incurred when $y$ equals the tolerance limit $t+\Delta(t)$ or $t-\Delta(t)$. Then, $k(t)$ in (27) is calculated as

$$k(t) = \frac{A}{\Delta^2(t)}.$$  \hfill (28)

Note that, unlike Taguchi, $k(t)$ is allowed to depend on $t$ through $\Delta(t)$. This is more reasonable since, in practice, $\Delta(t)$ is usually given by

$$\Delta(t) = \delta t$$  \hfill (29)

for a DNB characteristic where $\delta$ is a small positive number. In this article, we assume that $A$ can be approximated by

$$A \approx C_3 + C_vM.$$  \hfill (30)

Since the expected loss in (27) is for a single target value, it is integrated over its range $[a,b]$ to obtain a PM as follows.

$$PM = \int_{a}^{b} w(t)k(t)\beta_i^2 \frac{\sigma^2(M')}{\beta^2} dt.$$  \hfill (31)

Assuming that $w(t) = 1/(b-a)$, (28), (29) and (30) hold for $k(t)$, and $\sigma(M) = \alpha(\beta M)$, we can simplify the PM in (31) as follows.

$$PM = \frac{1}{\delta^2} \left[ C_v + \frac{(a+b)C_v}{2\beta} \right] \alpha^2.$$  

In other words, design parameter settings can be essentially compared in terms of $\alpha^2$ (or $\alpha$), an estimate of which can be obtained using (18).
3 Example

The proposed procedures are illustrated with a constructed example of a DLB characteristic. Consider a laser welding process for steel boards in which the welding length is a signal parameter $M$ and the welding strength is a DLB characteristic $y$. Since the welding strength is proportional to the welding length, the relationship in (5) is assumed.

Hypothetical experimental data are obtained as follows. First, five design parameters $A, B, C, D, E$ are assumed and they are arranged in an $L_8(2^7)$ orthogonal array as shown in Table 1. At each experimental point (or design parameter setting), three values of $M$, namely, 10, 20, and 30mm, are chosen. Then, the experimental data at each experimental point and $M$ are generated according to the following steps.

1. The mean value of $\beta$ and $(\alpha\beta)$ are assumed to be 5 and 1, respectively.
2. The effects of design parameters on $\beta$ and $(\alpha\beta)$ are assumed as shown in Table 2.
3. Values of $\beta$ and $(\alpha\beta)$ are calculated. For instance, at the first experimental point $A_1B_1C_1D_1E_1$, $\beta$ and $(\alpha\beta)$ are calculated as follows.

\[
\beta = \text{(mean of } \beta\text{)} + \text{(effect of } A_1\text{)} + \text{(effect of } B_1\text{)} + \text{(effect of } C_1\text{)} + \text{(effect of } D_1\text{)} + \text{(effect of } E_1\text{)}
\]

\[
= 5 + (-0.1) + (0.1) + 0 + 0 + (0.07) = 5.07, 
\]

\[
(\alpha\beta) = \text{(mean of } (\alpha\beta)\text{)} + \text{(effect of } A_1\text{)} + \text{(effect of } B_1\text{)} + \text{(effect of } C_1\text{)} + \text{(effect of } D_1\text{)} + \text{(effect of } E_1\text{)}
\]

\[
= 1 + 0 + 0 + (0.1) + (-0.15) + (+0.25) = 1.2. 
\]

4. Error terms ($e_j$’s) are randomly generated from a normal distribution with mean 0 and standard deviation $\alpha\beta M$. 

13
5. Eight replicated observations are obtained as follows.

\[ y_j = \beta M + e_j, \quad j = 1,2,\Lambda,8. \]

At each experimental point, \( \hat{\beta} \) and \( \hat{\alpha}^2 \) are computed using (16), (17) and (18) and are shown in Table 1. The special case described in Section 2.3 is assumed (i.e., \( A \approx C,M \) and \( w(\lambda) = 1/(b-a) \)). Suppose that \( C_v \) is 10 and the lower tolerance limit \( \lambda \) ranges from 50 \((=a)\) to 100 \((=b)\). PMs computed by (19) are also shown in Table 1.

The analysis of variance results for the PM are given in Table 3, from which we may conclude that all the design parameters have reasonable effects on the PM (see the p-values).

The average PM values at each level of the five design parameters are given in Table 4, from which the optimal setting of design parameters is determined as \( A_2B_1C_2D_1E_2 \). According to the effects in Table 2, \( A_2 \) and \( B_1 \) are optimal since they make \( \beta \) larger, and \( C_2 \) and \( D_1 \) are optimal since they make \( \alpha \beta \) smaller. Therefore, the developed procedures correctly identify optimal levels of \( A, B, C \) and \( D \). Design parameter \( E \) has effects on both \( \beta \) and \( \alpha \beta \) as shown in Table 2. However, it can be shown that \( E_2 \) is better than \( E_1 \) in terms of PM in (15).

4 Conclusion

Taguchi suggests to determine the optimal parameter setting by first selecting the levels of design parameters which have significant effects on the slope such that the slope is maximized, and then, by determining the levels of the others such that the SN ratio is maximized [5]. However, it is more desirable to determine the optimal levels of design parameters using a combined performance measure such as the one proposed in this article. Note that the optimal parameter setting according
to Taguchi is given by $A_2B_1E_1$ for the design parameters which have significant effects on the slope, and by $C_2D_1$ for those that have significant effects on the SN ratio. Comparing this optimal setting with the one determined in this article, we find that the difference exists in the optimal level of $E$. Since $E$ has some significant effects on both $\beta$ and $(\alpha\beta)$, its optimal level needs to be determined using a combined performance measure.

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References


Table 1  Experimental data and estimates for $\beta$, $\alpha^2$ and $PM$

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Table 2  Main effects of design parameters on $\beta$ and $(\alpha\beta)$

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Table 4  Average PMs at each level of design parameters

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Figure Captions

**Figure 1** Classification of static and dynamic problems

**Figure 2** Mean and variation of $y$ at a selected value of $M$

**Figure 3** The loss function of $y$
### Static Problems

1. **NB**: 0 < t < ∞
   - ex) deposition process in semiconductor manufacturing.
     - y = thickness of layer, \( M \) = deposition time

2. **LB**: \( t = \infty \)
   - ex) welding [4].
     - y = welding strength, \( M \) = welding length

3. **SB**: \( t = 0 \)
   - ex) polishing process.
     - y = surface roughness, \( M \) = grain size of polishing material

### Dynamic Problems

1. **DNB**: 0 < \( \beta_t < \infty \)
   - ex) deposition process in semiconductor manufacturing.

2. **DLB**: \( \beta_t = \infty \)
   - ex) welding [4].

3. **DSB**: \( \beta_t = 0 \)
   - ex) polishing process.

---

*Figure 1*

Park & Yum
$E(y) = \beta M$

distribution of $y$ when $M'$ is used for $M$.

$\lambda$

$\beta M'$

$0$

$M'$

$M$

$y$
Figure 3
Park & Yum

$L(y) = k / y^2$