Size Effect on Flexural Compressive Strength of Concrete Specimens

by Jin-Keun Kim, Seong-Tae Yi, and Eun-Ik Yang

In estimation of the ultimate strength of a concrete flexural member, the effect of member size is usually not considered. For various types of loading, however, the strength always decreases with an increase of member size.

In this study, the size effect of a flexural compression member was investigated by experiments. For this purpose, a series of C-shaped specimens subjected to axial compressive load and bending moment was tested using three different sizes of concrete specimens with a compressive strength of 52 MPa. The three different size specimens were varied in height and width of their cross sections, which had a 1:2:4 ratio. The thickness of the specimens were constant where the size effect in outplan direction is not considered.

The test results are curve fitted using least square method (LSM) to obtain the new parameters for the modified size effect law (MSEL). The MSEL curve, graphed with new parameters, is compared with the uniaxial compressive strength of concrete cylinder test results; the results show a much stronger size effect in C-shaped member compared to the cylinder members.

Keywords: compressive strength; flexural strength; stress concentration.

INTRODUCTION

The actual stress distribution in the compression zone of concrete flexural members is extremely difficult to measure and to adequately model. In 1955, Hognestad et al.\(^1\) experimentally presented concrete stress distribution in an ultimate strength design by a rising curve from zero to maximum stress, and a descending curve beyond the maximum stress. Hognestad et al.\(^1\) also developed a test method using an eccentrically loaded C-shaped specimen to study the compressive stress and strain relationship of concrete specimens. Kaar et al.\(^2\) and Swartz et al.\(^3\) used a C-shaped specimen testing method to further extend their research. An application of ultimate strength design theory developed by Mattock et al.\(^4\) was based on an equivalent rectangular stress distribution that was adopted in the ACI code. To date, results of uniaxial cylinder tests are used to model stress distributions of beams because of similarities between flexural and uniaxial compressive stress and strain curves. In reality, however, the similarities will be changed with the member size of C-shaped specimens.

In 1925, Gonnerman\(^5\) experimentally showed that the ratio of the compressive failure stress to the compressive strength decreases as the specimen size increases. In 1984, Baúant\(^6\) derived the size effect law (SEL) from the dimensional analysis and similarity arguments for geometrically similar structures of different sizes with initial crack considering the energy balance at crack propagation in concrete.

Using this theory, much research has verified the fracture mechanics-type size effect for various types of failure of concrete structures, that is, diagonal shear failure of beams,\(^7\) punching shear failure of slabs,\(^8\) pullout failure of bars,\(^9\) and failure of other structures.\(^10,12\) Due to the fracture-based failure theory of the SEL, research has focused more toward pure tension and shear loading condition than flexural compression condition.

Therefore, only recently, the studies on compressive loading-based size effect became a focus of interest among the researchers. The phenomena of the concrete compressive failure stress of C-shaped specimens decreasing as the size of the specimen increases (reduction phenomena) is very interesting one. Both experimental and analytical studies on this topic, however, are scarce to the point of nonexistence.

The SEL of Baúant was basically derived under tension stress condition. Compressive failure, however, is also related to splitting cracks of localized tension effect that causes the ultimate failure of the concrete specimens. Thus, it can be concluded that the SEL of Baúant can also be used for compressive loading cases. Concrete, which is a quasibrittle material, will ultimately fail due to tensile cracking for any condition of loading, which will be reflected in the presence of size effect.

The design methodology and construction technologies of concrete structures have advanced to where the construction industries can build larger and more sophisticated structures. Due to the cost and testing facility restrictions, the testing of full-scale structural members is impossible. The concrete compressive strength used in design codes, however, is based on small size specimens, that is, \(\phi 10 \times 20\) cm or \(\phi 15 \times 30\) cm cylinders. Therefore, the strength of the actual structural members will be different than the laboratory-size specimens. Because of this deviation of the strengths, the small-size laboratory specimen strength must be extrapolated to the large-size structural member strength. Many efforts have focused on reflecting the size effect in the design understanding of the size effect mechanism. The final consensus of all the researchers in the field has agreed that the size effect is significant.
The purpose of this study is to experimentally investigate the size dependency on flexural compressive strength, which is flexural strength of reinforced concrete beams. Additionally, the results of C-shaped specimens were compared with those for uniaxial compressive strength of cylinders. Also, the size effect for ultimate strain was experimentally investigated. For plain concrete and very light reinforced concrete, the beam failed catastrophically with the appearance of cracking, and the size effect of these cases was different from the beam that had more than the minimum flexural reinforcement required in the code. Thus, this study can only be applied to a beam that contained a reinforcement amount that was more than the minimum flexural reinforcement defined in the code.

The reduction phenomena of flexural compressive strength with the size for the reinforced concrete beams is an important topic. Presently, however, appropriate analytical or experimental techniques for understanding this condition have still not been found. The research described herein was conducted to find out the size effect on the flexural compressive strength of flexural members. Analytical equations that predict the flexural compressive strength and the ultimate strains are proposed based on the size effect experiments of C-shaped specimens, and the equation for the compressive strength is compared with that of cylinder compressive strength.

**Table 1—Concrete mixture proportions and physical properties of concrete**

<table>
<thead>
<tr>
<th>Unit weight, kg/m³</th>
<th>W</th>
<th>C</th>
<th>S</th>
<th>G¹</th>
<th>S.P. ¹</th>
<th>fₑ</th>
<th>fₐ</th>
<th>Eₑ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>37</td>
<td>40</td>
<td>178</td>
<td>480</td>
<td>676</td>
<td>1014</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

¹Superplasticizer (ratio of cement weight).

Max aggregate size of 13 mm. Note: 1 MPa = 145 psi.

**TEST SPECIMENS AND EXPERIMENTAL PROCEDURE**

**Main test variable**

The shape of specimen and the test procedure employed were similar to those of Hognestad et al.¹ The main test variable was a size ratio of 1:2:4 of the specimen, with the same concrete compressive strength of 52 MPa. The height b and width c were changed proportionally, and the thickness was kept constant (b = 12.5 cm) because the effect of thickness of specimen on the size effect is not considered. The value of b is chosen for the stable mode of failure rather than the size effect parameter.

**Mixture proportioning**

The concrete mixture proportions selected for the C-shaped specimens and cylinders are listed in Table 1. Specimens were cast horizontally and cylinders were cast vertically on a level surface. Maximum aggregate size dₐ was 13 mm, and superplasticizers and a vibrator were used to consolidate the concrete and improve the workability of the concrete.

All beam specimens and cylinders were removed from the mold after 24 h and dry-cured under a wet burlap/towel until testing. As listed in Table 1, concrete compressive strength fₑ, splitting tensile strength fₑ, and elastic modulus Eₑ are an average of three identical φ 10 x 20 cm cylinders in the series. The cylinders were tested at an age similar to the concrete of C-shaped specimens.

**Details of test specimens**

The size, shape, and loading points of C-shaped specimens used to check the flexural compressive behavior of beams are shown in Fig. 2. The inner vertical, thick solid lines of the hollow circle in Fig. 3 represent the strain gages attached to each side of the specimen. More than three specimens per specimen size were prepared because it was a minimum data point required for the data curve fitting. In the case of Size III, which is the smallest size specimen, nine specimens, instead of six specimens, were selected because the data scattering is more likely to occur. The central test section, which is the critical section of C-shaped specimen under compression, was not reinforced. Flexural and shear reinforcement, as shown in Fig. 4, were used in the two ends of the specimen to eliminate the failure at the two end sections, and therefore forced the specimen to fail in the middle section.
This was achieved using displacement-controlled testing; the strain increment on the neutral face was $50 \times 10^{-6}$ at all sizes on the elastic region. Near the peak stress and postpeak regions, however, the strain increments were gradually reduced to allow a stable failure. During testing, strains were measured up to failure at midheight of the specimen by 12 strain gages. Two linear variable displacement transducers (LVDTs) were used to monitor the horizontal displacement at midheight of the specimen, from which information was used to adjust the load lever arm distances $a_1$ and $a_2$ for calculation of bending moment.

**Test procedure**

The major load $P_1$ in Fig. 2 was supplied by a universal testing machine (UTM) with a capacity of 2500 kN using a displacement control method. The minor load $P_2$ in Fig. 5 was applied using a hand-operated hydraulic jack with a 200 kN capacity. For the case of Size III, $P_2$ in Fig. 5 was applied by using a rotating assembly consisting of bolts, nuts, and rod to reduce the self-weight effect of hydraulic jack.

The testing procedure was as follows: an increment of load $P_1$ was applied and maintained while applying load $P_2$ and monitoring the value of the strain gages on the tension face. On reaching zero strain value (on the average), the load $P_2$ was maintained, while $P_1$ was increased further. This procedure was repeated until failure of the members.

**ANALYSIS OF TEST RESULTS**

**Test results**

In the specimen numbers (for example, I-1) of Table 2, Roman and Arabic numerals represent the size and the serial number of specimens, respectively. $P_u$, $\varepsilon_{cu}$, and $\delta$ represent the summation of $P_1$ and $P_2$, ultimate strain at failure, and displacement at the center of specimens at failure, respectively.

Twenty specimens were tested successfully; the stable failure occurred in the middle section of the specimens in a compressive mode in which spalling preceded failure. The failure shapes were similar, with the flexural compression failure of beams under two points loading regardless of size. Note that specimen No. III-7 failed at its low end section before it reached the usual failure load, and therefore, the data point was excluded from the analysis.

**Table 2—Experimental results of C-shaped specimens**

<table>
<thead>
<tr>
<th>No. of specimen</th>
<th>$P_u$, kN</th>
<th>$\varepsilon_{cu}$ ($\times 10^{-3}$)</th>
<th>$\delta$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>861</td>
<td>3.31</td>
<td>2.83</td>
</tr>
<tr>
<td>I-2</td>
<td>907</td>
<td>4.22</td>
<td>2.99</td>
</tr>
<tr>
<td>I-3</td>
<td>867</td>
<td>3.08</td>
<td>2.92</td>
</tr>
<tr>
<td>I-4</td>
<td>891</td>
<td>3.35</td>
<td>3.03</td>
</tr>
<tr>
<td>I-5</td>
<td>913</td>
<td>3.38</td>
<td>3.17</td>
</tr>
<tr>
<td>I-6</td>
<td>868</td>
<td>2.83</td>
<td>2.66</td>
</tr>
<tr>
<td>II-1</td>
<td>454</td>
<td>4.37</td>
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</tr>
<tr>
<td>II-2</td>
<td>469</td>
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<td>II-3</td>
<td>482</td>
<td>3.94</td>
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<tr>
<td>II-6</td>
<td>460</td>
<td>3.73</td>
<td>1.47</td>
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<tr>
<td>III-1</td>
<td>272</td>
<td>4.47</td>
<td>0.75</td>
</tr>
<tr>
<td>III-2</td>
<td>264</td>
<td>4.40</td>
<td>0.55</td>
</tr>
<tr>
<td>III-3</td>
<td>286</td>
<td>5.03</td>
<td>0.78</td>
</tr>
<tr>
<td>III-4</td>
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<td>0.77</td>
</tr>
<tr>
<td>III-6</td>
<td>270</td>
<td>4.68</td>
<td>0.65</td>
</tr>
<tr>
<td>III-7*</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>III-8</td>
<td>260</td>
<td>4.48</td>
<td>0.71</td>
</tr>
<tr>
<td>III-9</td>
<td>278</td>
<td>4.38</td>
<td>0.77</td>
</tr>
</tbody>
</table>

*III-7 failed during testing due to failure of bearing end.
Note: 1 kN = 0.225 kips; and 1 mm = 0.0394 in.
Size effect of flexural compressive strength

Kim et al.\textsuperscript{14,15} proposed the modified size effect law (MSEL) by adding to the SEL the size-independent strength $\sigma_n = \alpha f'_c$, which was also proposed by Ba\textsuperscript{a}nant\textsuperscript{16-18} in a different approach.

To obtain an analytical equation that can predict the flexural compressive strength of C-shaped specimens at failure, MSEL is used, and LSM regression analyses\textsuperscript{19,20} are carried out with 20 test data points. The width of crack band $l_o$ is empirically known that it is related to the maximum aggregate size; for example, $l_o = nd_a$ in which $n$ is approximately constant (2~3).\textsuperscript{6,13}

In this regression analysis, this constant was chosen as $2.0 \times d_a$ (2.6 cm). Eq. (1) is obtained from the analysis. The results are given in Fig. 6.

\begin{equation}
\sigma_n = \frac{0.70f'_c}{\sqrt{1 + \frac{c}{2.60}}} + 0.47f'_c
\end{equation}

where nominal flexural compressive strength $\sigma_n$ and uniaxial compressive strength $f'_c$ are in MPa, and the depth of C-shaped Specimen $c$ is in cm.

From the SEL of Ba\textsuperscript{a}nant and nonlinear regression analyses with the same data, Eq. (2) is obtained

\begin{equation}
\sigma_n = \frac{0.96f'_c}{\sqrt{1 + \frac{c}{22.27}}}
\end{equation}

In the previous study,\textsuperscript{13} Eq. (3) was proposed to obtain the compressive strength of cylindrical concrete specimens with various diameters and height-diameter ratios. For this purpose, the effects of the maximum aggregate size on the fracture process zone (FPZ) were considered, and the concept of characteristic length was newly introduced

\begin{equation}
\sigma_n = \frac{0.4f'_c}{\sqrt{1 + \frac{(h_c - d_c)}{5}}} + 0.8f'_c
\end{equation}

where the height of cylinder Specimen $h_c$ and the diameter of cylinder Specimen $d_c$ are in cm.

Figure 6 shows the value $\sigma_n f'_c$ as a function of the depth $c$, which is measured from the neutral axis to the compressive edge of member. In this figure, the hollow circular data points, the thick solid line, the thin solid line, and the dashed line represent experimental data, and the results from Eq. (1), (2), and (3), respectively.

As shown in Fig. 6, the results indicate a strong size effect condition. The new equation (Eq. (1)) shows the best agreement with the experimental results. The reduction of flexural compressive strength at failure as the specimen size increases is stronger than that for uniaxial compressive strength. This is because the FPZ for the uniaxial compressive strength of cylinders is larger than that for the flexural compressive strength of C-shaped specimens. By comparing Eq. (1) and (2), it can be seen that the size effect by Eq. (1) agrees better with the experimental result than Eq. (2).

It is observed that for specimens having no initial crack or notch, the use of the MSEL to predict their behavior is appropriate.

Size effect on ultimate strain $\varepsilon_{cu}$ for flexural compression

Because ultimate strain on the compressive outer layer of concrete beams subjected to flexural load is also affected by cracking, the size effect model for the ultimate strain can also be applied to the SEL similar to the stress (strength). To obtain a size effect equation that predicts the ultimate strain of C-shaped specimens at failure, LSM regression analyses were carried out. Eq. (4) is obtained from the analyses. The relationship between $\varepsilon_{cu}/\varepsilon_{co}$ and $c/20$ is given in Fig. 3.

\begin{equation}
\varepsilon_{cu} = \frac{1.70\varepsilon_{co}}{\sqrt{1 + 17\left(\frac{c}{20}\right)}} + 0.60\varepsilon_{co}
\end{equation}

where $\varepsilon_{co}$ is an average ultimate strain for the specimen of $c = 20$ cm.

In this figure, the hollow circular data points, the thin solid line, and the thick dashed line represent experimental data and results from Eq. (1) and (4), respectively. This shows that ultimate strain decreases as the specimen size increases. The pattern is similar with flexural compressive strength. MSEL can be also used for the size effect of the ultimate strain of the C-shaped specimen.

Other observations

Distribution of strain $\varepsilon$—Strain distributions across the test section of three C-shaped specimens chosen from each (one specimen from each size) at each loading stage are given in Fig. 7, where the hollow circular data points represent strains at the location of strain gage.

In most of the loading stages, the distribution of strains is found to be linear, but there is an insignificant nonlinear distribution at the ultimate loading stage. This means that the assumption, that is, “the plane cross sections before loading remain plane after the deformation,” which is a basis for the bending stress analysis of reinforced concrete structures, is reasonable without large error to the ultimate loading stage. In Fig. 7, it also can be seen that the ultimate strain increases as the specimen size decreases, as discussed in Fig. 3.

Load-displacement curve—Figure 8 shows the relationship between load ($P_1 + P_2$) and horizontal displacement at the middle test section of the C-shaped specimens obtained from the LVDT. In this figure, it was found that the relationship between applied load and horizontal displacement shows a little nonlinearity at the larger loading stage because stress distribution of
the section of the C-shaped specimen varies from linear to non-linear.

**Stress-strain relationship**—Stress values of compressed face $f_c$ obtained from LSM regression analyses using a cubic equation for C-shaped specimens are plotted against strain values of compressed face $\varepsilon_c$ in Fig. 9. It is assumed that the $f_c$ and $\varepsilon_c$ relationship established is valid for all layers in the section. Thus, a compressive stress can be determined from this relationship and the measured strain.

Also plotted as solid lines in this figure are the uniaxial compressive stress-strain curves for corresponding cylinders. Strains corresponding to the maximum stress of the C-shaped specimens remain relatively similar for Size I and II. The strain of the C-shaped specimen, however, increases significantly for Size III. In this case, the difference is approximately 0.0015.

Also, the significant variation of results in Size III compared with Size I and II cause a significant deviation in its size effect relationship. It also can be seen that stress and strain relationship for concrete in flexural compression may be different from that of a uniaxial compressive strength of cylinders, especially in the descending branch.

**CONCLUSIONS**

A series of compression tests for 21 C-shaped concrete specimens and the cylinder, cast from the same batch with the concrete compressive strength of 52 MPa, were carried out to evaluate the size effect on the flexural compressive strength and ultimate flexural compressive strain of flexural members. From analysis and test results, the following conclusions are drawn:
1. Size effect is apparent, that is, the flexural compressive strength at failure decreases as the specimen size increases. New parameter values of MSEL are suggested that better predict the reduction phenomena of the strength;
2. Size effect for the flexural compressive strength in C-shaped specimens is more distinct than that for the uniaxial compressive strength of cylinders;
3. Size effect for the ultimate flexural compressive strain (curvature, ductility) and stress-strain relationship is also observed; and
4. Size effect on flexural compressive strength can be expressed by the MSEL, as well as that for ultimate flexural compressive strain.

ACKNOWLEDGMENTS

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CONVERSION FACTORS

<table>
<thead>
<tr>
<th>Unit (SI)</th>
<th>Unit (US)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mm</td>
<td>3.94 in.</td>
</tr>
<tr>
<td>1 kN</td>
<td>0.225 kips</td>
</tr>
<tr>
<td>1 MPa</td>
<td>145 psi</td>
</tr>
<tr>
<td>1 kN·m</td>
<td>0.738 kip·ft</td>
</tr>
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</table>

NOTATIONS

- $a_1$ = distance from neutral axis to center of member
- $a_2$ = distance from neutral axis to center of rod
- $b$ = thickness of specimen
- $c$ = distance from neutral face to compressed face of member
- $d_a$ = maximum aggregate size
- $d_c$ = diameter of cylinder specimen
- $E$ = elastic modulus of concrete
- $f_0$ = stress in concrete
- $f'_c$ = uniaxial compressive strength of standard concrete cylinder
- $f_{ct}$ = splitting tensile strength of concrete cylinder
- $f'_t$ = direct tensile strength of concrete cylinder
- $h$ = height of C-shaped specimen
- $h_c$ = height of cylinder specimen
- $l_0$ = width of crack band
- $P_1$ = major load
- $P_2$ = minor load
- $P_u$ = ultimate axial load = $P_1 + P_2$
- $\sigma_0$ = size independent stress ($\sigma_0f'_c$)
- $\sigma_u$ = nominal flexural compressive strength at failure = $P_u/\beta bc$
- $\varepsilon_c$ = strain in concrete
- $\varepsilon_{uc}$ = average ultimate strain for specimen of $d = 20$ cm
- $\varepsilon_{uc}$ = ultimate strain in concrete

REFERENCES