Effect of Specimen Sizes on ACI Rectangular Stress Block for Concrete Flexural Members

by Seong-Tae Yi, Jang-Ho Jay Kim, and Jin-Keun Kim

It is important to consider the effect of concrete member sizes when estimating the ACI rectangular stress block of a concrete flexural member. The experimental data and analytical results, however, are still insufficient for a proper evaluation. For all types of loading conditions, the trend is that the size of the ACI rectangular stress block tends to change when the member size changes.

In this paper, the size variations of strength coefficients for ACI rectangular stress block ($\alpha_1$ and $\beta_1$) and the stress block parameters ($k_3$, $k_1$, $k_2$, and $k_3$) have been studied. The results from a series of C-shaped specimens subjected to axial compressive load and bending moment were adopted from other references.

The analysis results show the effect of specimen sizes on the strength coefficients for ACI rectangular stress block and the stress block parameters of a concrete member to be apparent. They also show that the current strength criteria-based design practice should be reviewed to include member size effect. More studies are needed in the future to verify that.

Keywords: flexural strength; specimen; stress.

INTRODUCTION

The actual compressive stress distribution in the compressive zone of concrete flexural members is extremely difficult to measure and to adequately model. Ignoring the tension stress carried by concrete, the stress and strain distributions in a reinforced concrete (RC) beam section when the compressive concrete strain has reached 0.003 are shown in Fig. 1. Koenen was the first to propose the theory of ultimate failure capacity of flexural members. He assumed that the stress distribution in the cross section of RC beam is linear and is uniform across the width of the cross section. Following Koenen’s proposal, various stress distribution shapes in the compressive zone of RC beams have been suggested. Among the suggested stress distributions, the equivalent rectangular stress block was found to be the most practical and simplest model, with a satisfactory accuracy for design purposes. The theory of equivalent rectangular stress distribution was first proposed by Emperger and modified by Whitney for application to Ultimate Strength Design (USD).

To obtain accurate and well-controlled data on flexure-compression-loaded members, a test procedure for a series of experiments on C-shaped concrete specimens (Fig. 2) subjected to axial load and bending moment was proposed by several researchers. The position of neutral axis depth $c$ was kept fixed by continuously monitoring strains on one surface of the C-shaped specimen and adjusting the eccentricity of the applied force so that the strains on the neutral surface remain zero. This test procedure was developed by the Portland Cement Association (PCA) and reported by several researchers. The results of tests carried out using this procedure formed the basis of the rectangular stress block used in ACI 318 code.

Several researchers experimentally studied the flexural stress distribution in the compressive zone of RC members and showed that the compressive stress during flexural loading increases until the maximum stress is reached and decreases afterwards in the USD approach. Several researchers further simplified their model by assigning the strength coefficients (that is, $\alpha_1$, $\beta_1$, $k_3$, $k_1$, $k_2$, and $k_3$) for application to design codes and practical usage.

Concrete as a quasibrittle material fails ultimately by the formation and propagation of cracks induced by stresses caused by external loads or environment changes resulting in the release of internal energy. Therefore, based on the energy concept, there is an effect of size on the nominal strength of specimens made with quasibrittle materials such as concrete, rock, ice, ceramic, and composite materials. In a
Kim and Eo, and Kim, Eo, and Park\(^{15}\) derived a size effect law (SEL) from dimensional analysis and similitude. This size effect law, which was also proposed by Baant\(^{16-17}\) and Baant and Xiang\(^{18}\), is assumed to have a constant value of 0.85. A concrete stress of 0.85 \(f'_{c}\) is assumed as uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance \(a = \beta_2 f'_{c}\) from the fiber of maximum compressive strain. The distance \(e\) from the fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis. In the ACI 318 Code, the factor \(\beta_2\) is taken as 0.85 for concrete strengths \(f'_{c}\) up to and including 4000 psi (27.6 MPa). For strengths above 4000 psi, \(\beta_2\) shall be reduced continuously at a rate 0.05 for each 1000 psi (6.9 MPa) of strength in excess of 4000 psi. However, \(\beta_2\) shall not be taken less than 0.65.

In Fig. 1, the softening branch of stress distribution in the compressive zone of the cross section (shown schematically) is used as a material property. Parameters \(k_3, k_1, k_2\), and \(k_3\) have been used in the strength-based design method to account for the shape of the compressive stress-strain diagram. However, experimental results by van Mier\(^{29}\) in uniaxially loaded specimens have indicated that the softening portion of the compressive stress-strain curves depends on the length of specimens. As the length of the compressive specimen increases, the slope of the decreasing or the softening branch of stress-strain curves becomes steeper.\(^{29}\) Alternatively, if stress is plotted against deformation for the postpeak behavior using the same test results, basically the same postpeak stress-displacement curves are obtained. Therefore, the stress-deformation curve, instead of the stress-strain curve, should be used to...
describe the softening stress-strain curve.\textsuperscript{23}

The ACI 318 Code states that an equivalent rectangular stress block coefficient $\alpha_1$ is a constant value, but the coefficient $\beta_1$ changes based on concrete compressive strength. Tests on C-shaped specimen under linear strain distribution\textsuperscript{23,24} show that it is reasonable to assume that the coefficient $\alpha_1$ will change when the specimen size changes. In particular, when the length-to-depth ratio $h/c$ (Fig. 2) is greater than or equal to 3.0 ($h/c \geq 3.0$),\textsuperscript{24} even though the failure strength does not change, the coefficient $\alpha_1$ will change, indicating a modification in equivalent stress block size.

Figure 3 shows representative compression stress-strain relationships that consider specimen size difference where the thin and the thick solid lines are the flexural compressive stress-strain curves from C-shaped specimens, and the uniaxial compressive stress-strain curve obtained from standard concrete cylinder tests, respectively. The specimens used for the size, length, and depth effect experiments are shown in Fig. 4. For size effect, the main test variable was a size ratio of 1:2:4 of the specimen. The specimen thickness $b$ was chosen to allow stable failure.

**DISCUSSIONS OF EFFECT OF SIZE DIFFERENCES ON ACI STRENGTH COEFFICIENTS**

The relationship between the effect of size differences in compression tests to ACI equivalent stress block size and actual stress distribution are discussed. The discussion is based on the experimental data of size effect (size, length, and depth variations) of C-shaped flexure compression test reported by Kim, Yi, and Yang,\textsuperscript{23} and Kim, Yi, and Kim.\textsuperscript{24} The specimens used for the size, length, and depth effect experiments are shown in Fig. 4. For size effect, the main test variable was a size ratio of 1:2:4 of the specimen. The height $h$ and the depth $c$ were changed proportionally (Fig. 4(a)). Specimen length/depth ratios of 1:1, 2:1, 3:1, and 4:1 were used to study the effect of length where a constant depth ($c = 10$ cm) was maintained and specimen lengths were varied from 10 to 20 to 30 to 40 cm (Fig. 4(b)). Specimen length/depth ratios of 1:1, 2:1, and 4:1 were used to study the effect of depth where a constant length ($h = 20$ cm) was maintained and specimen depths were varied from 5 to 10 to 20 cm (Fig. 4(c)). The thickness of all specimens was kept constant ($b = 12.5$ cm) to eliminate the out-of-plane size effect. The specimen thickness $b$ was chosen to allow stable failure.
The test results for size, length, and depth effect are shown in Table 1. Also, the numbering of the specimen (that is, L-I-1) and obtained data are tabulated in Table 1. The specimens for size, length, and depth effect are assigned with S, L, and D in the specimen names, respectively. Also, the roman numerals I, II, III, and IV represent the size of the specimens, with I being the smallest and increasing accordingly. The arabic numbers 1, 2, 3 are the three specimens tested for each specimen size.

The data are compared with the experimental data reported by Nilson and Slate to verify the accuracy. Also, the data are used to check with ACI 318 code and Ibrahim and MacGregor model equations of $\beta_1$ values. By comparing $\beta_1$ values, the effect of specimen size dependence on $\beta_1$ values is indicated. The other coefficient used to define the stress block was $\alpha_1$, which the ACI code takes constant at 0.85. The following sections will describe the specimen size effect on the strength coefficients $\alpha_1$, $k_3$, $k_1k_3$, and $k_2$.

### Table 1—Test results for size, length, and depth effect

<table>
<thead>
<tr>
<th>No. of specimen</th>
<th>$h$, cm</th>
<th>$c$, cm</th>
<th>$f'_c$, MPa</th>
<th>$P_{\text{iv}}, \text{kN}$</th>
<th>$\beta_1$, ACI</th>
<th>$e_{c_{\text{max}}}$, cm</th>
<th>$\alpha_1$</th>
<th>$k_1$</th>
<th>$k_1k_3$</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-I-1</td>
<td>10</td>
<td>5</td>
<td>51.80</td>
<td>272</td>
<td>16</td>
<td>1.0478</td>
<td>0.5618</td>
<td>1.1488</td>
<td>0.7753</td>
<td>1.2227</td>
</tr>
<tr>
<td>S-I-2</td>
<td>10</td>
<td>5</td>
<td>51.80</td>
<td>264</td>
<td>13</td>
<td>1.0061</td>
<td>0.5715</td>
<td>1.1086</td>
<td>0.7714</td>
<td>1.1920</td>
</tr>
<tr>
<td>S-I-3</td>
<td>20</td>
<td>10</td>
<td>57.88</td>
<td>286</td>
<td>15</td>
<td>1.0923</td>
<td>0.5304</td>
<td>1.1785</td>
<td>0.7878</td>
<td>1.2570</td>
</tr>
<tr>
<td>S-I-4</td>
<td>20</td>
<td>10</td>
<td>57.88</td>
<td>266</td>
<td>20</td>
<td>1.0403</td>
<td>0.6122</td>
<td>1.1710</td>
<td>0.7551</td>
<td>1.2906</td>
</tr>
<tr>
<td>S-I-5</td>
<td>20</td>
<td>10</td>
<td>57.88</td>
<td>265</td>
<td>14</td>
<td>1.0128</td>
<td>0.6335</td>
<td>1.1531</td>
<td>0.7466</td>
<td>†</td>
</tr>
<tr>
<td>S-I-6</td>
<td>20</td>
<td>10</td>
<td>57.88</td>
<td>270</td>
<td>17</td>
<td>1.0414</td>
<td>0.5725</td>
<td>1.1481</td>
<td>0.7710</td>
<td>1.2205</td>
</tr>
<tr>
<td>S-I-7$^7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-I-8</td>
<td>20</td>
<td>10</td>
<td>57.88</td>
<td>260</td>
<td>16</td>
<td>1.0046</td>
<td>0.5171</td>
<td>1.0766</td>
<td>0.7932</td>
<td>1.1476</td>
</tr>
</tbody>
</table>

$^7$S-I-7 and L-I-3 failed during testing due to failure of two end sections; D-III-1 failed during testing due to failure of rod that applies load $P_2$.

Note: All specimens had $h = 12.5$ mm.
strength $f'_c$ is 51.8 MPa, $\beta_1$ is equal to approximately 0.68. The values recommended for design are based on a lower-bound limit, which is not always safe. However, Reference 31 suggests that the ACI value of $\beta_1$ is extremely conservative. When the experimental data of Size III specimens are used to calculate the $\beta_1$ value, $\beta_1$ is equal to approximately 0.83. It should be emphasized that this calculated value is obtained using the polynomial equation, which is not recommended for design. Also, if the ultimate load values $P_{u}$ are substituted into Eq. (1) for simple calculations, the $\beta_1$ values are calculated as 0.82, 0.86, 0.84, 0.87, and 0.83. Comparing these values with Reference 31’s suggested value of 0.83, it can be seen that the difference is minute and insignificant. Therefore, the result shows that the accuracy of the experimental data is nearly equivalent to the data used by the ACI 318 Code.

Ratio of average compressive stress to maximum stress $\beta_1$ based on $\alpha_1$ value of 0.85

Figure 5 shows the relationship between specimen sizes and $\beta_1$ obtained using Eq. (1) and (2), which are based on a $\alpha_1$ value of 0.85 and suggested by Ibrahim and MacGregor, respectively.

$$\beta_{1,\text{test}} = \frac{P_{u}}{0.85f'_c \times b \times d}$$ (1)

$$\beta_{1,\text{ref.30}} = 0.95 - \frac{f'_c}{400} \geq 0.70 \text{ where } f'_c \text{ in MPa}$$ (2)

In Eq. (1), the thickness $b$ represents the value 12.5 of the specimen, in cm. Equation (1) is based on the calculation methodology to obtain a $\beta_1$ value following the method suggested by the ACI 318 Code. In Fig. 5, the hollow circular data points, the thick dashed line, and the thin solid line represent the experimental data, the results from Eq. (1), and results from Eq. (2), respectively. It is important to note that $\beta_{1,\text{test}}$ values in Eq. (1) are calculated using a constant $\alpha_1$ value of 0.85. The results show that $\beta_1$ values decrease as specimen sizes increase. The calculated values, however, are still higher than the test values based on a $\alpha_1$ value of 0.85. The ACI approach of using an equivalent rectangular stress block width of $0.85f'_c$ for concrete compressive strengths of 51.8, 57.88, and 55.43 MPa gives constant $\beta_1$ values of 0.68, 0.65, and 0.65, respectively.

The equation suggested by Ibrahim and MacGregor predicts a more accurate value to the experimental data, especially when the specimen size is increased. Figure 5 shows that the calculated $\beta_1$ values indicate an effect of member size rather than the constant value assumed in the ACI method. This clear size-effect trend is probably due to the member size difference as well as the aggregate size relative difference to the specimen size used to obtain the experimental data. Therefore, a more detailed analysis of calculating $\beta_1$ values should be performed.

Strength coefficients $\alpha_1$ and $\beta_1$ for rectangular stress block

In this section, the effect of member size difference on the calculated values of the equivalent rectangular stress block coefficients $\alpha_1$ and $\beta_1$ is presented. $e_{cu}$ is the eccentricity from the centroid of the cross section above the neutral axis due to the resultant load parallel to the member axis. $e_{cu}$ is obtained using the Levenberg-Marquardt’s Least Square Method (LSM) regression analyses based on a cubic equation

$$f_c = A_1 + A_2 \epsilon_c + A_3 \epsilon_c^2 + A_4 \epsilon_c^3$$

to obtain the stress $f_c$ and strain $\epsilon_c$ relationship on the compression zone for size, length, and depth effects of C-shaped specimens. The coefficients $\alpha_1$ and $\beta_1$ are calculated based on the previously obtained values of $e_{cu}$ using Eq. (3) and (4), respectively. Equation (3) is formulated based on the assumption that the external load $P_u$ is equal to the internal force of the equivalent.
rectangular stress block. Equation (4) is formulated based on the depth of the equivalent stress block. The calculated values of $\alpha_1$ and $\beta_1$ for various member sizes (that is, size, length, and depth) are shown in Fig. 6 and 7, respectively. In Fig. 6 and 7, the hollow circular data points represent the values calculated using the experimental data with Eq. (3) and (4); the thick dashed lines represent $\alpha_1$ value of 0.85 and $\beta_1$ values of 0.68, 0.65, and 0.65, calculated using the ACI 318 Code with $\alpha_1$ equal to 0.85 for size, length, and depth effect specimens, respectively.

$$\alpha_1 = \frac{P_u}{(c/2 - e_{cu}) \times 2 \times f'_c \times b} \quad (3)$$

$$\beta_1 = \frac{(c/2 - e_{cu}) \times 2}{c} \quad (4)$$

The change of $\alpha_1$ and $\beta_1$ values with respect to three types of size effect (size, length, and depth) are as follows. In Fig. 6, the $\alpha_1$ values, calculated using the experimental data, decrease as member size and depth increase. However, the $\alpha_1$ values increase when the member lengths increase. Also, Fig. 7 shows that the calculated $\beta_1$ values are not sensitive to the size and depth increase, but decreases when the member length increases. Based on the characteristic of $\alpha_1$ and $\beta_1$ values from the analysis results, it can be safely accepted that the trend that $\alpha_1$ and $\beta_1$ values increase and decrease, respectively, as the length of members increases. The reason behind these trends is due to the concentration of stress as the member length increases. More specifically, the flexure-compression failure behaviors of longer and shorter specimens are different. For shorter specimens, the flexure-compression failure occurred throughout the cross section of the specimen, whereas the flexure-compression failure of longer specimens initiated at a more localized zone of the cross section above the neutral axis in the compressive zone. This means that the flexure-compressive stress at the compressive zone of the specimens became more localized in the case of a longer specimen than in a shorter specimen. Therefore, the trend is represented by the width and depth of the equivalent rectangular stress block’s strength coefficients $\alpha_1$ and $\beta_1$ increasing and decreasing, respectively, as the specimen length increases. This trend is reasonable where the dependence of $\beta_1$ on $\alpha_1$ is intimate; therefore they would show inversely proportional characteristics.

**Stress block parameters $k_3$, $k_1k_3$, and $k_2$ for actual stress distribution**

In this section, the effect of member size on the effective stress distribution’s parameters $k_3$, $k_1k_3$, and $k_2$ is presented. The $k_3$, $k_1k_3$, and $k_2$ parameters are calculated using Eq. (5), (6), and (7), respectively. The calculated values for $k_3$, $k_1k_3$, and $k_2$ parameters for various specimen sizes are shown in Fig. 8, 9, and 10, respectively. In these figures, the hollow circular data points represent the values calculated from the experimental data with Eq. (5), (6), and (7); the thick dashed lines represent $k_3$, $k_1k_3$, and $k_2$ equal to 0.85, 0.85$\beta_1$, and $\beta_1/2$ calculated using the ACI 318 Code, respectively.

The parameter $k_3$, the ratio of maximum compression in the beam to the cylinder strength of the concrete, is found by evaluating the following equation

$$k_3 = \frac{f_{max}}{f'_c} \quad (5)$$

where $f_{max}$ is the maximum compressive stress obtained from the stress-strain relationship, and $f'_c$ is the uniaxial compressive strength of a standard concrete cylinder. Figure 8 shows the variation of $k_3$ depending on the specimen sizes. Figure 8 shows that $k_3$ decreases as the member size increases. Herein, $k_3$ characterizes the size effect$^{23,24}$ and is equal to $\sigma_{N(c)}f'_c$. If $k_3$ has a value of unity, then $k_1$ becomes the ratio of average compressive stress in the beam to the cylinder strength $f'_c$.

To calculate $k_1$, which relates the average to the maximum stress in the beam, one must first evaluate the product of $k_1$ and $k_3$. By the equilibrium of forces

$$k_1k_3 = \frac{P_u}{bcf'_c} = \frac{P_1 + P_2}{bcf'_c} \quad (6)$$

In Fig. 9, $k_1k_3$ decreases as member size increases. The obvious reason is that the $k_1k_3$ value is strongly influenced by the value $k_3$, a parameter, which characterizes the size effect. From Fig. 9, however, the $k_1k_3$ values are still larger than the ACI suggested values of 0.85$\beta_1$. To observe the independent characteristic of the $k_1$ value, the experimentally obtained $k_1k_3$ value can be divided by $k_3$. The calculated value of $k_1$ using the experimental data shows that $k_3$ is a dominating parameter when compared to $k_1$. Therefore, it is safe to assume that $k_1$ plays a minor role in the calculated value of $k_1k_3$.

The parameter $k_2$ establishes the depth of the compressive resultant relative to the neutral axis depth. Again by equilibrium of moments

$$k_2 = 1 - \frac{P_1\alpha_1 + P_2\alpha_2}{(P_1 + P_2)c} \quad (7)$$
where the values $a_1$ and $a_2$ include both the initial values and those due to deflection of the specimen. As shown in Fig. 10, the value of $k_2$ is not influenced greatly by the specimen size differences. When the specimen size increases, the specimen tends to show a brittle type of stress-strain characteristic. In addition, the brittle characteristic is dictated by the postpeak behavior of the stress-strain (Fig. 3). However, $k_2$ is a parameter that is significantly affected by concrete strength and the prepeak characteristic of the stress-strain curve. Therefore, $k_2$ is not greatly effected by specimen size differences because the strengths of the concrete used for the experiments are similar. The $k_2$ value, however, is still larger than the ACI-used $k_2$ value of $\beta_1/2$. The ACI strength coefficients are still conservative even though they do not consider the size effect.

CONCLUSIONS

The influence of member size differences on the strength coefficients for ACI rectangular stress block and the stress block parameters presented in this paper is based on the flexural compressive strength experimental data (specimen size, length, and depth effect) published previously in the *ACI Structural Journal*.\textsuperscript{23,24} When the stress block width of $0.85f_{c}^*$ is used as suggested by ACI, a distinct change in the $\beta_1$ coefficient has been observed. If the strength coefficients $\alpha_1$, $\beta_1$, $k_3$, $k_4$, and $k_2$ are calculated based on member size differences, the strength coefficients sometimes showed a significant influence of size effect, and at other times, showed no influence. The coefficient values, however, are always larger than the values obtained without considering size effect. From the analyses, the following conclusions are drawn:

1. The sizes of the ACI rectangular stress block and the actual stress distribution at failure should change as the sizes of the specimens change. The effect of specimen size on the strength coefficients is significant in the case of small-size specimens (Size I). In the case of large-size specimens (Size III), however, the effect of specimen size on the strength coefficients is minor and similar to ACI stress block. Because a design code must be able to consider all types of specimen sizes, the size effect must be introduced into the calculation of ACI rectangular stress block coefficient;

2. The stress block parameters that are influenced by the stress-strain curve characteristic and the postpeak behavior are strongly dependent on member size differences. However, the parameter $k_2$, which is influenced by the prepeak behavior
and the strength (peak) values are not dependent on specimen size differences; and

3. The results suggest that further studies are required to determine the effect of member size differences on ACI rectangular stress block. The current strength-based criteria should be reviewed.

ACKNOWLEDGMENTS

The authors would like to thank the Korea Institute of Science and Technology Evaluation and Planning (KISTEP) for the financial support of the National Research Laboratory (NRL). The second author also wishes to acknowledge the partial financial support of the Korea Science and Engineering Foundation (RO1-2000-00365).

CONVERSION FACTORS

\[
\begin{align*}
100 \text{ mm} & = 3.94 \text{ in.} \\
1 \text{ kN} & = 0.225 \text{ kips} \\
1 \text{ MPa} & = 145 \text{ psi} \\
1 \text{ kN-m} & = 0.738 \text{ kip-ft}
\end{align*}
\]

NOTATION

\[
\begin{align*}
a_1 & = \text{distance from neutral surface to center of member} \\
a_2 & = \text{distance from neutral surface to center of rod} \\
A_0 & = \text{area of steel} \\
b & = \text{thickness of specimen, width of section} \\
C & = \text{compressive force} \\
c & = \text{depth to neutral axis of critical section of C-shaped specimen} \\
d & = \text{effective depth, distance from compression face to centroid of tension steel} \\
d_0 & = \text{maximum aggregate size} \\
e_{cu} & = \text{eccentricity of resultant load parallel to member axis measured from centroid of cross section above neutral axis} \\
f_c & = \text{stress in concrete} \\
f' \text{c} & = \text{uniaxial compressive strength of standard concrete cylinder} \\
f_{\text{max}} & = \text{maximum compressive stress in beam} \\
f' \text{f} & = \text{direct tensile strength} \\
h & = \text{length of critical section of C-shaped specimen} \\
k_1 & = \text{ratio of average compression in beam to cylinder strength of concrete} \\
k_2 & = \text{depth of compressive resultant relative to neutral axis depth} \\
k_3 & = \text{ratio of maximum compression in beam to cylinder strength of concrete} \\
P_1 & = \text{major load} \\
P_2 & = \text{minor load} \\
P_u & = \text{ultimate axial load} = P_1 + P_2 \\
\alpha & = \text{empirical constant defining size-independent strength} \\
\alpha_1 & = \text{width of equivalent rectangular stress block} \\
\beta_1 & = \text{depth of equivalent rectangular stress block} \\
\epsilon_c & = \text{strain in concrete} \\
\epsilon_{cu} & = \text{ultimate strain in concrete} \\
\sigma_0 & = \text{size-independent strength} \\
\sigma_2(\epsilon) & = \text{nominal flexural compressive strength at failure in beam}
\end{align*}
\]

REFERENCES


