A rational and mechanics-based equation is proposed for the prediction of shear strength of reinforced concrete beams without web reinforcement. This prediction is based on basic shear transfer mechanisms, a modified Bazant's size effect law, and numerous published experimental data, including high-strength concrete beams with compressive strengths of concrete up to 100 MPa (14,500 psi). Comparisons with experimental data indicate that the proposed equation estimates properly the effects of primary factors, such as concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth. It is shown that the proposed equation is considered to be better than the other equations compared in this study with respect to accuracy and estimation of primary factors. A simplified design equation is also derived within the limited range of effective depth for practical purposes.

Keywords: aggregate interlock; beams; dowel action; failure mode; shear strength; size effect.

According to the shear span-to-depth ratio \((a/d)\), shear failure of a reinforced concrete beam without web reinforcement is divided into two modes, as shown in Fig. 1. For \(a/d\) greater than 2.0 ~ 3.0, the inclined cracking load exceeds the shear compression failure load. With the formation of the inclined crack, a beam without web reinforcement becomes unstable and fails. This type of failure is usually called “diagonal tension failure.” For \(a/d\) less than 2.0 ~ 3.0, however, the failure load exceeds the inclined cracking load. If sufficient anchorage length is provided, after the inclined crack develops, failure may occur by concrete crushing in the upper end, and this type of failure is called “shear compression failure.”

Shear force in reinforced concrete member is transferred in various ways. For slender beams where \(a/d\) is greater than 2.0 ~ 3.0, shear force is carried by the shear resistance of uncracked concrete in the compression zone, the interlocking action of aggregates along the rough concrete surfaces on each side of the crack, and the dowel action of the longitudinal reinforcement. For relatively short beams, however, after the breakdown of beam action, shear force is resisted mainly by arch action.

Test results have shown that shear strength of reinforced concrete beams without web reinforcement depends mainly on concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth. Of course, factors such as maximum aggregate size, diameter of the bars, and spacing of the flexural cracks show some minor contribution. A part or all of these primary factors are included in the existing shear strength prediction models, but the effects of these factors are estimated differently according to the models.

Recently, high-strength concrete has been increasingly used in practice. With the development of concrete technology and the introduction of superplasticizers and silica fume, the compressive strength of concrete in the field of ready-mixed concrete reached 100 MPa (14,300 psi) and higher. Since the mechanical properties of concrete are changed in high-strength concrete, a reevaluation of the prediction model...
is necessary to reliably estimate the shear strength of beams made with high-strength concrete. Moreover, because of the wider range of concrete strength used, more accurate predictions of shear strength of reinforced concrete members are required.

The shear failure of reinforced concrete beams without web reinforcement has been known to be a typical case of brittle failure and indicates significant size effect. In 1981, Reinhardt\(^1\) introduced fracture mechanics in the prediction of shear strength. He analyzed limited test data for shear failure based on linear elastic fracture mechanics. Subsequently, it was established that the size effect implied by linear elastic fracture mechanics is too strong in the case of concrete, and that brittle failures of concrete structures are better described by nonlinear fracture mechanics. Meanwhile, a simple and approximate size effect law on the basis of nonlinear fracture mechanics was proposed by Bazant.\(^2\) Several studies\(^3\)–\(^6\) have shown that Bazant's size effect law is in good agreement with test results. However, there is some discrepancy between the prediction by Bazant's law and the test data, particularly for large-sized specimens. Recently, Kim and Eo\(^7\) proposed a modified Bazant's size effect law to reduce the discrepancy.

In the present study, a simple and accurate equation predicting the shear strength of reinforced concrete beams without web reinforcement is proposed based on basic mechanisms of shear transfer and a modified Bazant's size effect law deduced by Kim and Eo, and it was verified by the published test data. In addition, a simplified equation is also proposed for practical design purposes. These equations that include the effects of all the factors previously mentioned are supported by test results and are compared with other prediction equations for the shear strength of beams without web reinforcement.

**RESEARCH SIGNIFICANCE**

Research reported in this paper provides a rational and accurate equation for the prediction of shear strength of reinforced concrete beams without web reinforcement. The results show that the proposed equation predicts the existing experimental data more accurately than the other equations in this study. A simplified design equation is also proposed for practical purposes without a significant accuracy reduction compared with the original equation.

**BASIC SHEAR TRANSFER MECHANISMS**

For slender beams where \(d/d\) is greater than 2.0 ~ 3.0, the shear force in a cracked section of a reinforced concrete beam is mainly resisted by the shear resistance of compression zone, interlocking action of aggregates, and dowel action, as shown in Fig. 2. For rectangular beams, after an inclined crack has formed, the proportion of the shear force transferred by the various mechanisms is as follows: 9 to 20 percent by the uncracked concrete of compression zone; 33 to 50 percent by interlocking action of aggregates; and 15 to 25 percent by dowel action. Meanwhile, in a relatively short beam, the load is transferred directly from the loading points to supports owing to arch action.

**Shear resistance of uncracked concrete**

In a reinforced concrete beam, after the development of flexural cracks, a certain amount of shear is carried by the concrete in the compression zone. It is clear that the shear failure in the uncracked concrete is recognized as the failure under combined compression and shear\(^9\) and the area of uncracked zone. The position of the neutral axis in a beam after flexural cracking is mainly dependent on the elastic modulus of concrete and the longitudinal steel ratio because elastic modulus of steel is nearly constant. Therefore, the shear force carried by the uncracked concrete in the compression zone can be represented by the compressive strength of concrete and the longitudinal steel ratio, since the concrete strength in the biaxial state and elastic modulus of concrete is the function of the compressive strength of concrete.

**Interlocking action of aggregates**

Previous experimental studies\(^8\)–\(^10\) have shown that a large portion of the total shear force on the beam without web reinforcement is carried across the cracks by aggregate interlocking. Among many variables, the width of the crack and the concrete strength are likely to be the most important factors.

Since the flexural crack width is approximately proportional to the strain of tension reinforcement, the crack width at failure becomes smaller as the longitudinal steel ratio is increased. Also, with increasing \(d/d\), the strain of tension reinforcement at failure is increased. Meanwhile, it is naturally expected that the interlocking force will be increased when the strength of concrete is high.

**Dowel action**

When shear displacement occurs along the cracks, a certain amount of shear force is transferred by means of dowel action of the longitudinal bars. Although there is some contribution in dowel action by the number and arrangement of longitudinal bars, spacing of flexural cracks, and the amount
of concrete cover, etc., the main factors influencing this action are flexural rigidity of longitudinal bars and the strength of the surrounding concrete.

**Arch action**

In relatively short beams, applied loads are transferred directly to the supports by arch action. The main factors influencing this action are the span-to-height ratio of the analogous arch and the strength of the compression strut. The span-to-height ratio of the analogous arch is approximately equal to the shear span-to-depth ratio. The strength of the compression strut is closely related to the compressive strength of concrete and the area of tension reinforcement.

**DEVELOPMENT OF EQUATION**

As previously discussed, shear resistance of uncracked concrete in the compression zone can be expressed as

\[ V_c = c_1(f'_c)^l(bkd) \]  \hspace{1cm} (1)

in which \( f'_c \) is the compressive strength of concrete, \( b \) is the width of the beam, \( kd \) is the neutral axis depth, and \( c_1 \) and \( l \) are empirical constants. The constant \( l \) depends on the ratio of shear stress to compressive stress in an uncracked zone, and it is in the range of 0.5 to 1.0. According to the classical bending theory of reinforced concrete beams with only tensile reinforcement and a negligible tensile capacity of concrete, we would have

\[ k = (n^2 \rho^2 + 2nP)^{1/2} - n\rho \]  \hspace{1cm} (2)

in which \( n = E_s/E_c \) = ratio of elastic moduli of steel and concrete. Eq. (2) is, however, unnecessarily complicated and may be replaced by the following simpler expression

\[ k = c_2(n\rho)^m \]  \hspace{1cm} (3)

in which \( c_2 \) and \( m \) are certain constants. Within the practical range, i.e., \( 5 \leq n \leq 10 \) and \( 0.005 \leq \rho \leq 0.05 \); consequently, \( 0.025 \leq n\rho \leq 0.5 \). These constants can be chosen so that the values given by Eq. (2) and (3) are almost indistinguishable. Fig. 3 shows the optimum values of \( c_2 \) and \( m \).

The elastic modulus of normal-weight concrete can be represented as

\[ E_c = c_3(f'_c)^{0.5} \]  \hspace{1cm} (4)

in which \( f'_c \) is the compressive strength of concrete and \( c_3 \) is a certain constant. Therefore, substituting with Eq. (3), where \( c_2 \) and \( m \) are the values shown in Fig. 3, and Eq. (4) into Eq. (1), we obtain

\[ V_c = c_5(f'_c)^{0.18} \rho^{0.36} bd \]  \hspace{1cm} (5)

in which \( c_5 \) is a constant.

According to Reineck,\(^{11} \) the maximum value of shear stress along the crack surface due to aggregate interlock depends on the tensile strength of concrete and is decreased linearly with increasing crack width \( \Delta n \) as follows

\[ \tau_{fu} = 0.45f'_t(1 - \frac{\Delta n}{\Delta n_o}) \]  \hspace{1cm} (6)

Crack width is resolved as \( \Delta n = 0.71\varepsilon_s s_{cr} \), in which \( \varepsilon_s \) is the strain of the longitudinal reinforcement and \( s_{cr} \) is the spacing of the primary cracks. From theoretical considerations and experimental comparisons, the spacing of the primary cracks was also derived by Reineck as \( s_{cr} = 0.7(d - kd) \). Substituting the values of \( \Delta n \) and \( s_{cr} \) into Eq. (6) and assuming that \( k = 0.3 \) for the usual simplification and \( d = 500 \text{ mm} \) as a reference size

\[ \tau_{fu} = 0.45f'_t(1 - 193.3\varepsilon_s) \]  \hspace{1cm} (7)

Eq. (7) can be replaced approximately by the following simpler expression within the practical range of \( \varepsilon_s \), i.e., \( 0.0008 \leq \varepsilon_s \leq 0.003 \).
\[ \tau_{fu} = 0.012 f'_c / \sqrt{E_s} \]  \hfill (8)

The comparison of Eq. (7) and Eq. (8) is shown in Fig. 4. In reinforced concrete beams, as long as the tension reinforcement has not yielded, the strain of the longitudinal reinforcement at the ultimate state can be approximately expressed as

\[ \varepsilon_s = \frac{V_s a_{eff}}{E_s A_f d} \]  \hfill (9)

in which \( V_s \) is ultimate shear force and \( a_{eff} \) is effective length of shear span. Substituting Eq. (9) into Eq. (8) and tentatively assuming that shear strength of reinforced concrete beams is approximately proportional to \( \frac{1}{\beta} \) power of \( (f'_c \times \rho \times d/a) \) suggested by Zsutty, the shear force carried by the aggregate interlock \( \approx \tau_{db}(d - kd) \) is expressed as

\[ V_a = c_6(f'_c \rho d / a)^{1/3} bd \]  \hfill (10)

in which \( c_6 \) is some constant.

According to Vintzeleou and Tassios,\(^{12}\) the dowel force is represented as

\[ V_d = c_9 b_c d_b f'_c \]  \hfill (11)

in which \( c_9 \) is a certain constant, \( d_b \) is diameter of the bar, and \( b_c \) is net width of the section \( (= b - q d_b, q \) being the number of bars). Eq. (11) can only be used when the number and diameter of the bar are known. Assuming the number of the bar is not changed, the dowel force can be expressed approximately by more general terms \( \rho \) as follows

\[ V_d = c_{10}(f'_c \rho t)^{0.5} \rho t bd \]  \hfill (12)

in which \( c_{10} \) is some constant and \( t \), varying from 0.3 to 0.5 in a practical range, is a parameter that is dependent on spacing of the reinforcement.

As previously discussed, the shear capacity of slender beams may be represented as Eq. (13), although the primary shear transfer mechanisms are coupled

\[ V_u = V_c + V_a + V_d \]  \hfill (13)

Substituting Eq. (5), (10), and (12) into Eq. (13), we obtain

\[ V_u = [c_5(f'_c)^{(\beta - 0.18)} \rho^{0.36} \]

\[ + c_4(f'_c \rho d / a)^{0.33} + c_{10}(f'_c \rho^t)^{0.5} \rho t bd \]

It is noticed in Eq. (14) that the powers of \( f'_c \) have similar values. In practice, uncracked parts of reinforced concrete beams are stressed in two directions (biaxial stress condition). The power of \( f'_c \) in the first term of Eq. (14), therefore, may be varied in some range. Considering this fact and the minor contribution of dowel action (approximately 20 percent of total shear resistance), it can be assumed for simplicity that the power of \( f'_c \) is identical for all three terms in Eq. (14). In addition, the values of power of \( \rho \) in all three terms of Eq. (14) are very similar to each other. Consequently, Eq. (14) is then expressed as

\[ V_u = c_{11}(f'_c \rho^3 [c_{12} + (d/a)^{1/3}] bd \]  \hfill (15)

in which \( c_{11}, c_{12}, s, \) and \( t \) are certain constants. Because the term \( [c_{12} + (d/a)^{1/3}] \) can be replaced by \( [c_{13} + c_{14}(d/a)] \) without a large difference (in which \( c_{13} \) and \( c_{14} \) are certain constants), as shown in Fig. 5, the shear capacity of reinforced concrete beams with constant effective depth can be expressed as

\[ V_u = c_{15}(f'_c \rho^4 (c_{16} + d/a) bd \]  \hfill (16)

in which \( c_{15} \) and \( c_{16} \) are certain constants.

To reduce the discrepancy between the prediction by Bazant’s size effect law and the experimental data, especially for very large-sized specimens, Kim and Eo\(^{7}\) recently suggested a modified Bazant’s size effect law based on the concept of dissimilar initial cracks. The formula is as follows

\[ \sigma_N = \frac{k_1 \sigma_r}{(1 + k_2 \beta)^{1/2}} + k_2 \sigma_r \]  \hfill (17)

in which \( \sigma_N \) is the nominal strength of a specimen at failure, \( \sigma_r \) is the nominal strength of the specimen with reference size, and \( k_1, k_2, \) and \( k_3 \) are empirical constants. \( \beta \) is brittleness number, expressed as

\[ \beta = \frac{(f'_c)^2 D}{E_s G_f} \]  \hfill (18)

in which \( f'_c \) is tensile strength, \( E_s \) is modulus of elasticity, \( G_f \) is fracture energy, and \( D \) is the characteristic size of the

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Fig. 5—Comparison of two equations on \( \xi \)
structure. $f'_c$ and $E_c$ are mainly dependent on compressive strength, and $G_f$ is dependent on maximum aggregate size ($d_a$) and compressive strength. $G_f$ is increased with $d_a$ and compressive strength, although the increase is minimal in the higher strength range. For reinforced concrete beams, $D$ is effective depth. Consequently, $\beta$ can be expressed as a function of $f'_c$, $d_a$, and $d$. If stirrups are used, $G_f$ is increased because crack localization is prevented, to some extent, by stirrups. Thus, the effect of stirrups in the size effect of reinforced concrete beams in shear can be considered. Meanwhile, since the effects of factors influencing $G_f$ are not clear and the ranges of $f'_c$ and $d_a$ are limited in the construction field, experimental data have shown that the capacity of beams is not significantly influenced by $d_a$; therefore, it is rational to assume that $(f'_c)^2/E_c G_f$ is a constant. Therefore, Eq. (17) can be expressed as

$$\sigma_N = \frac{k_1 \sigma_r}{(1 + k_4 d_a)^{1/2}} + k_2 \sigma_r$$  (19)$$

in which $k_a$ is an empirical constant. Eq. (16) does not take into account the size effect. According to Eq. (19), we finally obtain

$$v_u = c_{17} (f'_c)^{1/3} \rho^{3/8} (0.4 + d/a) \lambda(d)$$  (20)$$

in which $c_{17} = c_{14} k_1$ and $k_5 = k_2/k_1$. Eq. (20) has been compared to all other important experimental data, with regard to not only the effects of concrete strength, longitudinal steel ratio, and shear span-to-depth ratio, but also to the effect of effective depth.

From the statistical analysis of existing experimental data, the values of all six parameters in Eq. (20) have been determined. The experimental data used in the analysis are only three or four point test results of simple beams with rectangular sections except for Shioya’s test results, which applied uniform load. Moreover, test results of beams with small $d$ [less than 100 mm (3.94 in.)] or small maximum aggregate size [less than 5 mm (0.2 in.)] were excluded for practical purposes. Cubic compressive strength of concrete reported in some tests was converted to the cylindrical compressive strength according to Neville’s empirical relation. Finally, the following equation is proposed for the mean nominal shear strength of reinforced concrete beams with $a/d$ equal to or greater than 3.0

$$v_u = 3.5 f'_c \rho^{3/8} (0.4 + d/a) \lambda(d)$$  (21)$$

in which

$$\lambda(d) = \frac{1}{\sqrt{1 + 0.008d}} + 0.18$$

$f'_c$ and $v_u$ are in MPa and $d$ is in mm. Fig. 6 shows the ratio of shear strength tested to predicted with $a/d \geq 3.0$ for each

![Fig. 6—Ratio of shear strength for each primary factor](image-url)
factor by the proposed equation. As shown in the figure, it can be concluded that the proposed equation estimates properly the effects of primary factors.

For relatively short beams, the failure mode is changed gradually from diagonal tension to extreme shear compression. Consequently, the effect of concrete strength on the shear strength is also gradually increased with decreasing \(a/d\). Previous experimental work\(^{32}\) has shown that the effect of concrete strength on the shear strength of reinforced concrete beams becomes more significant with decreasing \(a/d\). If we roughly assume that the effects of \(\rho\) and \(d\) are not changed with a change of failure mode, the shear strength of relatively short beams can be expressed as

\[
v_u = 3.5f_c^{\alpha/3} \left(0.4 + d/a\right) \lambda(d)
\]

(22)

where \(\alpha\) is the failure mode index that is dependent on the \(a/d\) ratio. From the compatibility condition, which is \(\alpha = 1\) at \(a/d = 3.0\), and the statistical analysis of existing experimental data, \(\alpha\) is determined as

\[
\alpha = 2 - \frac{a/d}{3} \quad \text{for} \quad 1 \leq a/d < 3
\]

(23)

**EVALUATION OF PROPOSED EQUATION**

Many equations have already been proposed to estimate the shear strength of reinforced concrete beams. To evaluate the proposed model, three well-known equations are selected for comparison: 1) Zsutty’s equation\(^{46}\) deduced by multiple regression analysis; 2) Bazant’s equation\(^3\) derived based on Bazant’s size effect law; and 3) the ACI Code equation.\(^{47}\) These equations are as follows:

**ACI Code equation**

\[
v_c = 0.1578 \sqrt{f_c} + 17.25\rho_u \frac{V_d}{M_u} \quad \text{(MPa)} \quad (a/d \geq 2.5)
\]

(24)

\[
v_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right) \times [\text{Eq. (24)}] \quad (a/d < 2.5)
\]

(25)

**Zsutty’s equation**

\[
v_u = 2.1746 \left(\rho \frac{d}{a}\right)^{1/3} \quad \text{(MPa)} \quad (a/d \geq 2.5)
\]

(26)

\[
v_u = \left(2.5\frac{d}{a}\right) \times [\text{Eq. (26)}] \quad (a/d < 2.5)
\]

(27)

**Bazant’s equation**

\[
v_u = 0.54\sqrt{f_c} \left(\sqrt{f_c} + 249 \frac{\rho}{a/d} \right) \times \frac{1 + \sqrt{5.08/d}}{\sqrt{1 + a/(25d_a)}}
\]

(28)

The ratios of shear strength (tested to predicted) with the bulk of existing experimental data determined by three other equations and the proposed equation are shown in Fig. 7. For the ACI Code equation, the scatter is much larger than those of the other equations. It is shown that the ratio of shear strength obtained by Zsutty’s equation increases gradually with increasing shear strength. This phenomenon is mainly due to the fact that Zsutty’s equation ignores size effect and slightly underestimates the effect of \(a/d\) when \(a/d\) is less than 2.5. Bazant’s equation predicts the shear strength of experimental data comparatively well, although the scatter is somewhat larger than that of the proposed equation. As shown in the figure, the proposed equation, which has a standard deviation of 0.162 with respect to average ratio of shear strength and a correlation coefficient of 0.926, predicts more accurately than the other equations.

Fig. 8 shows the ratios of tested-to-calculated shear strength of existing test data failed by diagonal tension (\(a/d \geq 3.0\)) with variation of \(d\). The ratios of shear strength obtained from the ACI Code equation and Zsutty’s equation are decreased with increasing \(d\) due to the disregard of size effect, whereas Bazant’s equation and the proposed equation have nearly constant value. Although Bazant’s equation reflects not only the effect of \(d\) but also the effect of maximum aggregate size, the scatter is somewhat larger than that of the proposed equation.

For relatively short beams failing in shear compression, the scatter of test data is larger than that of slender beams failing in diagonal tension. As shown in Fig. 9, the standard deviation of the ratio of shear strength for the ACI Code equation is more than twice that of the proposed equation. Although scatters of the ratio of shear strength for Zsutty’s equation and Bazant’s equation are lower than that of the ACI Code equation, they are larger than that of the proposed equation, to some extent. Similar to the case of slender beams, the proposed equation with a standard deviation of 0.230 and a correlation coefficient of 0.901 is considered to be better than the other equations in terms of accuracy and estimation of primary factors.

**PROPOSED SIMPLE DESIGN EQUATION**

From a practical point of view, design equations should be as simple as possible within a range of acceptable accuracy. Although the proposed equation is relatively simple and accurate, and estimates the effects of the primary factors well, it is somewhat complicated to use in practical design, especially regarding the size factor \(\lambda(d)\). The effective depth of reinforced concrete beams in construction is usually limited within the range of \(d \geq 250\) mm (9.84 in.). Since small-sized structures that conform to the strength criterion are not presented, the term \(\lambda(d)\), which indicates size effect, can be approximately represented using a simpler expression as follows

\[
\lambda(d) = \frac{k_6}{\sqrt{d}} + k_7
\]

(29)

in which \(k_6\) and \(k_7\) are some constants. From the regression analysis of existing experimental data, the values of \(k_6\) and \(k_7\) have been determined. As a result, the simplified size factor
Fig. 7—Comparison of various equations with bulk of existing data

Fig. 8—Comparison of various equations with increasing d (a/d ≥ 3.0)
is nearly identical with the one in the CEB-FIP model code. Consequently, the following simplified equation is derived for the mean nominal shear strength

$$v_u = 19.4 f'_c \alpha^{3/8} (0.4 + d/a) \left( \frac{1}{\sqrt{d}} + 0.07 \right)$$

for $d \geq 250$ mm (9.84 in.)

in which $\alpha = 1$ for $ald \geq 3.0$ and $2 - (ald)/3$ for $1.0 \leq ald < 3.0$.

Fig. 10 shows the comparison of results obtained from the original equation and the simplified equation. As shown in the figure, the simplified equation has nearly identical accuracy compared with the original equation, although some approximation has been made.

Assuming a normal distribution of the ratios of shear strength, the design equation for the shear strength of reinforced concrete beams with 90 percent reliability is represented as

$$v_{u,d} = 15.5 f'_c \alpha^{3/8} (0.4 + d/a) \left( \frac{1}{\sqrt{d}} + 0.07 \right)$$

This equation is shown by the straight line in Fig. 11. As shown, only a few data points lie below the equation, and no data points fall significantly below this equation.
CONCLUSIONS

On the basis of the results obtained in this study, the following conclusions may be drawn.

1. Based on shear transfer mechanisms, modified size effect law, and numerous published experimental data, the following equation is derived for the nominal shear strength of reinforced concrete beams without web reinforcement

\[ v_u = 3.5f'_c^{\alpha/3} \rho^{3/8} \left(0.4 + d/a\right) \left(\frac{1}{\sqrt[1.7]{1 + 0.008d}} + 0.18\right) \]

in which \( \alpha = 1 \) for \( ald \geq 3 \), \( 2 - (ald)/3 \) for \( 1 \leq ald < 3 \).

2. Comparison with published experimental data indicates that the proposed equation estimates properly the effects of primary factors, such as concrete strength, longitudinal steel ratio, shear span-to-depth ratio, and effective depth.

3. As a result of the comparison with the ACI Code equation, Zsutty’s equation, and Bazant’s equation, the accuracy of the proposed equation is better than that of any other equation.

4. For practical design, a simple and accurate equation deduced from the simplification of the originally proposed equation is derived as

\[ v_{u,d} = 15.5f'_c^{\alpha/3} \rho^{3/8} \left(0.4 + d/a\right) \left(\frac{1}{\sqrt[1.7]{d}} + 0.07\right) \]

for \( d \geq 250 \text{ mm} \) (9.84 in.)

The accuracy of the simplified equation is nearly identical compared with the original equation and is considered to be better than those of the other equations compared in this study.

5. The effect of concrete strength on the shear strength of beams with \( ald < 3.0 \) is estimated well by introducing failure mode index \( \alpha \).

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NOTATION

- \( a_{eff} \) = effective length of shear span
- \( A_s \) = area of tension reinforcement
- \( ald \) = shear span-to-depth ratio
- \( b \) = width of beam
- \( b_{ct} \) = net width of beam section
- \( c_1, c_2, ..., c_17 \) = constants
- \( D \) = characteristic size of structure
- \( d \) = effective depth of beam
- \( d_a \) = maximum aggregate size
- \( d_b \) = diameter of reinforcing bar
- \( E_c \) = elastic modulus of concrete
- \( E_s \) = elastic modulus of steel
- \( f'_c \) = compressive strength of concrete
- \( f'_t \) = tensile strength of concrete
- \( G_f \) = fracture energy
- \( jd \) = internal lever arm
- \( k_1, k_2, ..., k_7 \) = constants
- \( kd \) = neutral axis depth
- \( l, m, p, s, t \) = constants
- \( M_u \) = ultimate moment
- \( n \) = ratio of elastic moduli of steel and concrete
- \( s_{cr} \) = spacing of primary cracks
- \( V_a \) = shear force carried by aggregate interlock
- \( V' \) = shear resistance of compression zone
- \( V_d \) = dowel force
- \( v_u \) = shear capacity of beams
- \( v' \) = shear strength of beams
- \( \alpha \) = failure mode index
- \( \beta \) = brittleness number
- \( \Delta n \) = crack width
- \( \Delta n_{u} \) = ultimate crack width
- \( e_t \) = strain of tension reinforcement
- \( \lambda(\Delta) \) = size factor
- \( \rho, \rho_w \) = longitudinal steel ratio
- \( \sigma_N \) = nominal strength of specimen at failure
- \( \sigma_f \) = nominal strength of specimen with reference size
- \( \tau_{ju} \) = maximum shear stress along crack surface

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![Fig. 11—Comparison between experimental data and calculated values by using design equation](image-url)