Size Effect on Compressive Strength of Plain and Spirally Reinforced Concrete Cylinders

by Jin-Keun Kim, Seong-Tae Yi, Chan-Kyu Park, and Seok-Hong Eo

Many experimental and theoretical investigations have been carried out to examine the reduction phenomenon of compressive strength of cylindrical concrete specimens with size, but up until now, an adequate analysis technique has not been developed. In this paper the fracture mechanics type size effect on the compressive strength of cylindrical concrete specimens was studied, with the diameter, the height/diameter ratio, and the volumetric spiral ratio of cylinder considered as the main parameters. For this purpose, theoretical and statistical analyses were conducted. First, a size effect equation was proposed to predict the compressive strength of cylindrical concrete specimens with various diameters and height/diameter ratios. Second, the model equation derived from the plain concrete specimens with various diameters and height/diameter ratios was extended for predicting the compressive strength of spirally reinforced concrete cylinders. The proposed equation showed good agreement with the existing test results for concrete cylinders with and without spiral reinforcement.

Keywords: compressive strength; cylinders; diameter; height/diameter ratio; spiral reinforcement.

INTRODUCTION

Many experimental and theoretical investigations have been carried out to examine the size effect in concrete structures. The early works on the size effect up until the 1970s relied primarily on extensive test results, and there was no generally accepted theory for predicting the size effects.\(^1\)\(^-\)\(^3\) Apart from the foregoing, more sound theoretical bases on the size effect have been established by many investigators. The fictitious crack model by Hillerborg et al.\(^4\) and Petersson,\(^5\) the crack band model by Bazant and Oh,\(^6\) and the notch sensitivity analysis by Zaitsev and Kovler\(^7\) give a representative theoretical basis on the size effect in concrete structures. On the basis of the previous theories, extensive research has been carried out to verify the fracture mechanics type size effect for various types of failure of concrete structures—for example, diagonal shear failure of beams,\(^8\) punching shear failure of slabs,\(^9\) pullout failure of bars,\(^10\) and failure of other structures.\(^11\)\(^-\)\(^13\) Although the size effect on the compressive strength of plain concrete is not so remarkable as that on the tensile, flexural, and shear strength, previous tests\(^14\)\(^-\)\(^18\) show that the compressive strength tends to decrease with an increasing size of the specimen.

In the previous studies,\(^19\)\(^-\)\(^20\) a model equation for prediction of the compressive strength of plain concrete cylinders was proposed. But the equation is theoretically valid for geometrically similar specimens such as the cylindrical specimens with height-to-diameter ratio of 2. In this study a generalized equation was derived for predicting the compressive strength of plain cylindrical concrete specimens with various height-to-diameter ratios. Based on this equation derived from plain concrete and previous test results for specimens with spiral reinforcement, the size effect for specimens with spiral reinforcement subjected to concentric axial compressive load was investigated, and an equation for the prediction of the compressive strength of specimens with spiral reinforcement was also proposed.

RESEARCH SIGNIFICANCE

An adequate analysis technique for reduction trend of compressive strength of cylindrical concrete specimens with size has not yet been presented. The research described is intended to propose model equations that predict the compressive strength of cylindrical specimens with and without spiral reinforcement in case of various height/diameter ratios based on nonlinear fracture mechanics. The proposed equations could be applicable to the strength correction of core samples from concrete structures and the prediction of compressive strength of circular columns.

SIZE EFFECT IN COMPRESSION STRENGTH OF PLAIN CONCRETE

Theoretical review of size effect law

Considering the energy balance at crack propagation in concrete, Bazant\(^21\) derived the size effect law from the dimensional analysis for geometrically similar members, as follows

$$\sigma_n = \frac{P}{bD} = \frac{Bf_t^p}{\sqrt{1 + \frac{D}{\lambda_n d_a}}}$$

where \(\sigma_n\) is nominal strength, \(P\) is maximum load, \(b\) is thickness, \(D\) is characteristic dimension, \(f_t^p\) is direct tensile strength of concrete, \(d_a\) is maximum aggregate size, and \(B\) and \(\lambda_n\) are empirical constants. Thereafter, introducing the size independent strength \(\sigma_n (=\alpha f_t^p)\), Kim and Eo\(^19\) proposed a modified size effect law, which was also proposed by Bazant\(^22\)\(^-\)\(^24\) in a different approach, given by

$$\sigma_n = \alpha f_t^p + \frac{Bf_t^p}{\sqrt{1 + \frac{D}{\lambda_a d_a}}}$$

In derivation of Eq. (1), the hypotheses include that total energy release is proportional to the area of the fracture process zone \(nd_a\), where \(n\) is a constant and \(a\) is the length of the crack...
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ACI member Seek-Hong Eo is an assistant professor in the Department of Civil Engineering at Changwon National University, Changwon-City, Korea. He obtained his MS and PhD degrees from KAIST. His research interests include nonlinear behavior band. However, it seems to be reasonable to assume that the fracture process zone width does not vary linearly with the maximum aggregate size $d_{ag}$, since cracks occur at a narrowly strained concentrated region. In other words, $n$ is a function of the maximum aggregate size rather than a constant. Therefore, the width of microcrack zone $n d_{ag}$ can be simply expressed as $n_1 d_{ag}^m$ ($m = constant$, $0 < m < 1$) even though it needs to be analyzed more precisely by experiments or in a theoretical manner. The $n_1$ may be, of course, a function of the strength of concrete since the microcrack zone for high-strength concrete is smaller than that of normal strength concrete.

For the uniaxial compression strength, substituting $f'_c$ in Eq. (2) for the compressive strength of standard cylinder $f'_c$, Kim and Eo 19 and Kim et al. 20 proposed a model equation for the prediction of compressive strength of cylindrical concrete specimens with a height/diameter of 2. Since the main crack in uniaxial compression usually occurs at a stress of 0.7 to 0.85 $f'_c$, the size independent strength $\alpha' f'_c$ can be taken as 0.7 to 0.85 $f'_c$. However, tests by Smadi and Slate 25 have shown that the value of $\alpha$ is increased with increasing concrete strength. So the size independent strength $\alpha' f'_c$ can be expressed as $\alpha(f'_c) f'_c$. From the above discussion and the previous model equation, 19, 20 the nominal compressive strength of cylindrical specimens with a height/diameter of 2 can be expressed as follows

$$f'_o = \alpha(f'_c) f'_c + \frac{B f'_c}{1 + \frac{d}{\lambda_n(f'_c)} d^m}$$  \hspace{1cm} (3)

where $f'_o$ is the compressive strength of cylindrical concrete specimen with diameter $d$.

Derivation of modified size effect law for nonstandard cylinder specimen

Eq. (3) can be applied to predict uniaxial compressive strength of cylindrical concrete specimens with height/diameter of 2. In order to apply for cylindrical specimens with different height/diameter, the equation should be modified to reflect the width of the microcrack zone and the characteristic dimension that provides the main crack zone.

When a cylindrical concrete specimen is subjected to uniaxial compression loads, it tends to expand in the lateral direction. However, there exists a frictional force between the machine platens and the specimen. This frictional force creates a lateral compressive force that is responsible for the formation of a cone at failure. When the lateral constraint is eliminated, the lateral compressive force disappears, and a splitting type rupture is obtained. However, it seems to be valid to assume that the lateral constraint is produced to some extent since it is very difficult to eliminate the frictional force in practice.

In Fig. 1, when the frictional force is produced at failure, the characteristic dimension is represented by $(h_1 - \beta d_1)$. It can be replaced by $h_i$ or $d_i$, especially when the specimens are geometrically similar since the ratios of the characteristic dimension $(h_1 - \beta d_1)/(h_2 - \beta d_2)$, $h_1/h_2$ and $d_1/d_2$ have the same value. But $(h_1 - \beta d_1)/(h_2 - \beta d_2)$ is not equal to $h_1/h_2$ if the specimens have the same diameter ($d_1 = d_2$) as shown in Fig. 1(b). In other words, the specimen that exhibits the size effect when the size is twice the size of the specimen denoted $ABCD$ is not the specimen denoted $A'B'C'D'$ which satisfies $h_2 = 2h_1$, but the specimen denoted $A'B'C'D'$ or the specimen denoted $A''B''C''D''$ which satisfies $h_2 - \beta d_2 = 2(h_1 - \beta d_1)$ or $(h_2' - \beta d_2') = 2(h_1 - \beta d_1')$ respectively. This conclusion results from the condition that only the effects of the microcrack zone width and the characteristic dimension are considered as the factors on the size effect.

On the other hand, the size effect in uniaxial compressive strength is affected by end restraints and the energy release zone (denoted by the dotted area given in Fig. (b)) as well as the microcrack zone width and the characteristic dimension. Unless
of reinforced concrete structures, particularly based on fracture mechanics of concrete.
the confinement effect and the energy release zone are considered, the specimens $A'B'C'D'$ and $A''B''C''D''$ show the same size effect because the size effect is only a function of the microcrack zone width and the characteristic dimension. The areas denoted $A'E'D'$ and $A''E'D''$ represent the confinement effects for specimens $A'B'C'D'$ and $A''B''C''D''$, respectively. Thus, the specimen $A'B'C'D'$ has the greater load-resistant capacity than the specimen $A''B''C''D''$, as the confinement is related to the volume, i.e., $(d_2/d_1)^3$ while the stress is related to the area, i.e., $(d_2/d_1)^2$. But if the energy release zones are regarded for the specimens, the specimen $A'B'C'D'$ has more energy per unit volume—that is, the lower load-resistant capacity per unit area (i.e., stress)—than the specimen $A''B''C''D''$ has since the same energy is required for the unit crack to be created. As a result, the effects of confinement and the energy release zone are considered to act contradictory to each other on the size effect of uniaxial compressive strength. Furthermore, it is difficult to consider them for derivation of a size effect model as well, since they have minor importance within practical size range compared with the effects of microcrack zone width and the characteristic dimension. Consequently, Eq. (3) can be written as follows

$$f_o = \alpha(f'_c)' + \frac{Bf'_c}{\sqrt{1 + \frac{d}{\lambda_o(f'_c)^m}} (h/d - \beta)}$$  \hspace{1cm} (4)$$

It should be noted that the application of Eq. (4) is limited for cases $h \geq \beta d$, as shown in Fig. 2(b) and (c). If $h < \beta d$, as shown in Fig. 2(a), the confinement zone extends through the specimen to lead failure by crushing—not by cracking. In this study, the value of $\beta$ shown in Fig. 1 was selected as approximately 45 deg.

**Fig. 3**—Comparison of analytical and experimental strength values of plain concrete cylinders (1 MPa = 145 psi).

**Fig. 4**—Relationship between $1 + (h - d)/50$ and relative concrete strength ($f_o/f'_c$, 1 mm = 0.0394 in.).

**Considering effects of maximum aggregate size and concrete strength**

From the statistical analysis of existing experimental data of Gornnerman (172 specimens), Blanks and McNamara (26 specimens), U.S. Department of the Interior (20 specimens), Kesler (337 specimens), and Murdock and Kesler (123 specimens), the empirical constants in Eq. (4) were determined. In this case data numbers of specimens with $h/d = 2$ and $h/d \neq 2$ are 222 and 456, respectively, and the range of the maximum aggregate size is between 12.7 and 76.2 mm. From the regression analysis based on Eq. (4), it can be observed that the power of $d_u$ is $m = 0.00055$. This means that since the value of $d_u^{0.00055}$ approaches 1.0, the effect of maximum aggregate size can be negligible within the practical range of size. It was shown that the effect of the concrete strength in Eq. (4) is also negligible.

**Not considering effects of maximum aggregate size and concrete strength**

From statistical analysis, the following equation was derived for the same test results in the previous section

$$f_o = 0.8f'_c + \frac{0.4f'_c}{\sqrt{1 + (h - d)/50}}$$  \hspace{1cm} (5)$$

where $f_o$ and $f'_c$ are in MPa, and $h$ and $d$ are in mm. Fig. 3 shows the comparison of the analytical and experimental values of plain concrete. The comparison indicates that the proposed equation gives a good prediction. Fig. 4 shows the relationship between $1 + (h - d)/50$ and $f_o/f'_c$. From the same figure, it can be seen that most of the data are concentrated in a certain particular range since the diameters of most cylinders used in tests were 76, 100, and 150 mm. When the value of $h/d$ approaches 1.0, it is shown that the scatter of data is increased due to the effects of
confinement and energy release zone. Fig. 4 also shows the compressive strength of concrete would be 80 percent of the laboratory test results, since the confinement effects by frictional force would be negligible if the ratio, $h/d$, becomes very large.

Most of the experiments were performed about 50 years ago, and at that time experimental techniques, testing machines, and quality assurance were very poor, so there can be little reliability. Considering the papers used in this study, conditions of tests were not described in detail, and every paper had its own conditions. However, in this study a new theory is introduced to obtain a prediction model, and satisfactory results are obtained through the regression analyses with the experimental data.

**Strength correction factor**

When nondestructive testing (NDT) of concrete structures is performed, generally, strength correction factors based on ASTM C 42-94 can be used to predict the strength of sampled cores with various height/diameter ratios. The experimental data of Gonnerman, Kesler, and Murdock and Kesler are used to compare the prediction results with the suggested model and ASTM standard. Fig. 5 shows that the prediction values of the proposed equation are less than those of the ASTM standard, but the difference is not great. If, however, the height/diameter ratio exceeds the ASTM limit (1.94), Eq. (5) can be used with sufficient correctness. It should be noted that Eq. (5) has a theoretical basis in fracture mechanics of concrete, while the ASTM standard comes from the pure empirical bases.

**SIZE EFFECT IN SPIRALLY REINFORCED CONCRETE CYLINDERS**

It is expected that when the confining stress is applied to concrete, the size effect on the compressive strength could be less than that of plain concrete. The trend of the size effect could also be influenced by the magnitude of confining stress or volumetric spiral ratio. In the case of concrete cylinders with heavy spiral reinforcement, the size effect will probably be eliminated because concrete will behave in a plastic manner under very high confining stress. In this case, the compressive strength can be expressed approximately as follows

$$f_{oc} = kf_c' + k_1 f_s'$$

where $k$ and $k_1$ are empirical constants or functions of the confining stress and/or the concrete strength, and $f_s'$ is confining stress by spiral reinforcement. Many equations by the previous researchers have been proposed to estimate the confining stress by spiral. In this study, a model equation proposed by Iyengar et al. was used, which is given by the following equation

$$f_s' = \frac{2A_{sp} f_y}{d_s s} \left( 1 - \frac{s}{d_c} \right)$$

where $A_{sp}$ is the cross section area of the spiral reinforcement, $f_y$ is the yield strength of the spiral reinforcement, $d_c$ is the outer diameter of the spiral reinforcement, and $s$ is the spacing of the
spiral reinforcement. As shown in Fig. 6, the phenomenon discussed earlier can also be observed from the previous test results listed in Table 1. Fig. 6 shows the relationship between the normalized strength of specimens with spiral reinforcement and the size and the normalized confining stress. As can be seen in Fig. 6, when the confining stress is large, the strength of specimens with spiral reinforcement is not affected by the size of specimens. As the confining stress is decreased, however, the strength tends to decrease with increasing specimen size. This means that the parameter \( k \) in Eq. (6) is affected by compressive strength of concrete, volumetric spiral ratio, yield strength of spiral reinforcement, and size of specimen, but the parameter \( k_1 \) is only a function of the normalized confining stress. Therefore, using Eq. (5) and Eq. (6), the compressive strength of specimens with spiral reinforcement can be expressed by the following equation

\[
f_{oc} = 0.8f'_{c} + \frac{0.4f'_{c}}{1 + \frac{d_{c}}{h_{o}}(h/d_{c} - 1)}
\]  

(8)

The value of the parameter \( 1/\lambda_{oc} \) in Eq. (8) is between zero and 0.02, which corresponds to the case of the confining stress being equal to zero as given in Eq. (5). The value of \( 1/\lambda_{oc} \) is decreased with increasing volumetric spiral ratio. When the confining stress or the volumetric spiral ratio is large, the value of \( 1/\lambda_{oc} \) is zero, i.e., the characteristic dimension is zero. In this case, the strength of specimens with spiral reinforcement is calculated as \( 1.2f'_{c} + k_1f'_{s} \), as shown in Fig. 6.

Saatcioglu and Razvi\textsuperscript{37} pointed out that for specimens subjected to high strain rates, the unconfined concrete strength should be determined under the same strain rate for which the confined concrete strength is sought. Thus, prior to analysis, a model equation proposed by Dilger et al.\textsuperscript{38} was used to calculate the compressive strength of concrete for the test results by Mander et al.\textsuperscript{32} and Ahmad and Shah\textsuperscript{35} listed in Table 1. First, based on the previous available test results summarized in Table 1 and using the relationship between the normalized confining stress and the normalized strength as shown in Fig. 6, \( k_1 \) in Eq. (8) was determined. From the statistical analysis in this study, the value of \( k_1 \) in Eq. (8) was approximately obtained as a constant of 2.7 regardless of the magnitude of confining stress when the normalized confining stress is large. A comparison of Eq. (8) with the formula of Richart et al.,\textsuperscript{39} which can be used in the range of concrete compressive strength (13.8 to 20.0 MPa), was shown in Fig. 6. When the confining stress is large, the formula overestimates the trend of test data, whereas Eq. (6) predicts it relatively well in that region.

To obtain an expression for the value of the parameter \( 1/\lambda_{sp} \), a total of 130 data were analyzed with Eq. (8). But the 26 data that gave the value \( 1/\lambda_{oc} \geq 0.03 \) were neglected in determining the final expression. As a result of a trial and error approach, it was shown that the parameter \( 1/\lambda_{oc} \) can be expressed as a function of the volumetric spiral ratio and the compressive strength of concrete, as shown in Fig. 7. Finally, the compressive strength of specimens with spiral reinforcement can be written as follows

\[
f_{oc} = 0.8f'_{c} + \frac{0.4f'_{c}}{1 + \frac{8000A_{sp}(1 - s/d_{c})}{d_{c} sf_{c}'}} + \frac{5.4A_{sp}f'_{s}}{d_{c}s}(1 - s/d_{c})
\]  

(9)

where \( f'_{c} \) and \( f_{s} \) are in MPa and \( d_{c} \), \( h \), and \( s \) are in mm. Hereby, the expression within brackets under the square root should not be negative, i.e., it should be equal to zero if \( 8000A_{sp}(1 - s/d_{c}) > d_{c} sf_{c}' \). Eq. (9) indicates that as the volumetric spiral ratio is increased, the size effect is mitigated, and the minimum volumetric spiral ratio needed to eliminate the size effect is increased with increasing compressive strength of concrete. Fig. 8 shows the comparison between the predicted values and the experimental values of strength of spiral specimens. The comparison shows that the compressive strengths of confined concrete predicted by the proposed equation agree well with the experimental results.

**Table 1—Summary of available tests**

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Number of data</th>
<th>( f_{c}' ) MPa</th>
<th>( f_{y} ) MPa</th>
<th>( d_{c} ) mm</th>
<th>( h/d_{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iyengar et al.\textsuperscript{27}</td>
<td>7</td>
<td>25 to 34</td>
<td>319</td>
<td>150</td>
<td>2</td>
</tr>
<tr>
<td>Kim et al.\textsuperscript{28}</td>
<td>11</td>
<td>26 to 78</td>
<td>451</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>Martinez et al.\textsuperscript{29}</td>
<td>18</td>
<td>25 to 68</td>
<td>380 to 414</td>
<td>100 to 132</td>
<td>4 to 4.8</td>
</tr>
<tr>
<td>Sakino and Sun\textsuperscript{30}</td>
<td>9</td>
<td>32 to 86</td>
<td>1110</td>
<td>240</td>
<td>3.1</td>
</tr>
<tr>
<td>Sakino\textsuperscript{31}</td>
<td>14</td>
<td>41 to 74</td>
<td>1043</td>
<td>260</td>
<td>2.1 to 3.2</td>
</tr>
<tr>
<td>Mander et al.\textsuperscript{32}</td>
<td>15</td>
<td>27 to 33</td>
<td>307 to 340</td>
<td>450</td>
<td>3.3</td>
</tr>
<tr>
<td>Sudo et al.\textsuperscript{33}</td>
<td>27</td>
<td>39 to 113</td>
<td>571</td>
<td>150</td>
<td>2</td>
</tr>
<tr>
<td>Ahmad and Shah\textsuperscript{34}</td>
<td>9</td>
<td>25 to 62</td>
<td>414</td>
<td>76</td>
<td>2</td>
</tr>
<tr>
<td>Ahmad and Shah\textsuperscript{35}</td>
<td>11</td>
<td>30 to 36</td>
<td>414</td>
<td>76</td>
<td>2</td>
</tr>
<tr>
<td>Desayi et al.\textsuperscript{36}</td>
<td>9</td>
<td>14 to 29</td>
<td>299 to 677</td>
<td>150</td>
<td>2</td>
</tr>
</tbody>
</table>

\( f_{c}' \): compressive strength for standard cylinder of \( 150 \times 300 \) mm.

\( \dagger \) Tested for high strain rate.

\( \dagger \) MPa = 145 psi.

\( \dagger \) mm = 0.0394 in.
CONCLUSIONS

On the basis of the theoretical and statistical analyses for the size effect of compressive strength of plain and spirally reinforced concrete cylinders, the following conclusions are drawn.

1. Model equations for predicting the compressive strength of concrete cylinders with and without spiral reinforcement are suggested based on nonlinear fracture mechanics.

2. The effect of maximum aggregate size on the size effect of the compressive strength is negligible within the practical size range. This means that the effect of maximum aggregate size on the width of the microcrack zone can be ignored compared with the effect of the characteristic dimension defined as \( h_j - \beta d_j \).

3. The size effect is mitigated with increasing volumetric spiral ratio, and the minimum volumetric spiral ratio needed to eliminate the size effect is increased with increasing compressive strength of concrete.

ACKNOWLEDGMENT

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CONVERSION FACTORS

\[
\begin{align*}
1 \text{ in.} &= 25.4 \text{ mm} \\
1 \text{ kip} &= 4.448 \text{ kN} \\
1 \text{ ksi} &= 6.895 \text{ MPa} \\
1 \text{ kip-ft} &= 1.356 \text{ kN-m}
\end{align*}
\]

NOTATION

- \( A_{sp} \) = area of spiral
- \( a \) = length of crack band
- \( B \) = empirical constant in size effect equation
- \( b \) = thickness of specimen
- \( D \) = characteristic dimension
- \( d_a \) = maximum aggregate size
- \( d_c \) = outer diameter of spiral reinforcement
- \( d_r, d_r' \) = diameters of cylinder
- \( f' \) = compressive strength of standard cylinder
- \( f_s \) = confining stress by spiral reinforcement
- \( f_{oc} \) = compressive strength of general cylinder
- \( f'_{oc} \) = direct tensile strength
- \( f_{oc}' \) = compressive strength of concrete confined by spiral reinforcement
- \( f_y \) = yield strength of spiral reinforcement
- \( h, h_1, h_2 \) = heights of cylinder

REFERENCES


