FAILURE BEHAVIOR OF REINFORCED CONCRETE FRAMES BY THE COMBINED LAYERED AND NONLAYERED METHOD

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Abstract—The entire nonlinear behavior of reinforced concrete frames up to collapse is analyzed by the displacement control method and the combined layered and nonlayered method. The method to calculate the rigidities at any incremental or iterative stage is automatically selected in this analysis according to the extreme strain of actual strain distribution of the section. The spurious sensitivity to the chosen element size in the result of analysis by the finite element method for the materials with strain-softening can be overcome by modifying the rigidities and the curvature for each element based on the concept of concentrated inelastic rotations at a plastic hinge considering the applied axial load.

INTRODUCTION

For the nonlinear analysis of reinforced concrete structures in the finite element method, both the nonlayered (modified stiffness) and the layered methods have been developed and have been widely used in recent years. The layered method, made up of a series of layers idealized as being in a state of plane stress, leads to success, but the computational time in finite element analysis is too great [1] because of using the stress-strain relationships of materials for each layer of all elements. Therefore, it is not efficient in the economical aspects to use nonlinear analysis for the wide range of structures with the layered method. In the nonlayered method, the result of analysis is speedily obtained by using the models or the integral forms for the sectional behaviors (for example, moment-curvature relation, etc.) of all elements. The nonlayered method, however, cannot be considered the effect of the loading history, thus only limited success or sometimes an incorrect result is achieved. Consequently, the result of analysis on the inelastic region (i.e., a different result depends on the loading history) is not reliable.

When loaded to destruction, reinforced concrete frames may exhibit a softening response in which the load, after reaching its peak value, does not follow a constant-load yield plateau but gradually declines with increasing displacement [2] because of the limited ductility and strain-softening behavior of typical reinforced concrete sections. For this reason, it is attempted here to analyze the problem by the displacement control method, which makes it possible to follow the response up to the snapback point, if such a point exists.

This paper deals with nonlinear analysis of softening frames by using the combined layered and nonlayered method. Failure due to shear effect, bond slip between steel and concrete, long-term loading, and strain rate effect will not be included. The purpose of this paper is to obtain the entire load-deflection behavior for reinforced concrete frames considering the strain-softening in order to determine the maximum load and the load at which collapse occurs, and thereby to provide an efficient and reliable finite element method for the assessment of collapse load due to flexural failure.

SOLUTION ALGORITHM

Finite element formulation

The element and the positive directions and numbering of the end forces \( \{f\} \) and the corresponding displacements \( \{u\} \) expressed in terms of the member local \( x \) - and \( y \) -axes, are shown in Fig. 1. Assuming the member deflection of \( y \)-axis to be a cubic polynomial, the element local stiffness matrix [2] is given by

\[
[k] = \begin{bmatrix}
\frac{EA}{l} & 0 & -\frac{ER}{l} & \frac{EA}{l} & 0 & \frac{ER}{l} \\
-\frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\
\frac{4EI}{l} & \frac{ER}{l} & -\frac{6EI}{l^2} & -\frac{2EI}{l} \\
\frac{EA}{l} & 0 & -\frac{ER}{l} \\
-\frac{12EI}{l^3} & \frac{6EI}{l^2} \\
\end{bmatrix}
\]

(1)
in which $EA$ and $EI$ are the axial and the flexural rigidity, respectively; $ER$ is the combined axial and flexural rigidity; $l$ is the length of element.

**Conversion into the displacement control**

To be able to calculate the postpeak behavior, the structure is analyzed by the displacement control method, i.e., the increment of the $c$th displacement at which the $c$th applied load exists, is described and then the applied load vector and displacements which are not described are solved. Since each load which acts on a structure can be expressed in terms of a certain load $F_p$, the resultant simultaneous equation [3] is written by

$$
\begin{bmatrix}
    K_{11} & K_{12} & \cdots & -a_1 & \cdots & K_{1n} \\
    K_{21} & K_{22} & \cdots & -a_2 & \cdots & K_{2n} \\
    \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
    K_{c1} & K_{c2} & \cdots & -a_c & \cdots & K_{cn} \\
    \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
    K_{n1} & K_{n2} & \cdots & -a_n & \cdots & K_{nn}
\end{bmatrix}
\begin{bmatrix}
    U_1 \\
    U_2 \\
    \vdots \\
    F_p \\
    U_n
\end{bmatrix}
= \begin{bmatrix}
    -K_{1c} \\
    -K_{2c} \\
    \vdots \\
    -K_{cc} \\
    \vdots \\
    -K_{nc}
\end{bmatrix}
U_c
$$

in which, $a_1$, $a_2$, ..., $a_n$ are proportional constants for $F_p$. Figure 2 shows the algorithm for the proposed analytical procedure, and the direct iteration method (iterative secant stiffness algorithm) for the analysis is adopted.

**CONSIDERATION OF MATERIAL NONLINEARITY**

**Constitutive laws for materials**

The uniaxial compressive stress–strain relationship in concrete including confinement is assumed to be as the following expression [4], in which the variables in Hognestad's model [5] for ascending, and in Fafitis and Shah's model [6] for descending, respectively, were recalculated properly by regression analysis with test data.

**Ascending part:**

$$
\sigma_c = f_0 \left[ A \left( \frac{\varepsilon_c}{\varepsilon_0} \right) - (A - 1) \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^{A(A - 1)} \right]
$$

**Descending part:**

$$
\sigma_c = f_0 \exp[-B(\varepsilon_c - \varepsilon_0)^C],
$$

where

$$
A = \frac{E_c \varepsilon_0}{f_0} \\
B = \left( 260 + \frac{100}{f_c'} \right) \exp \left( -30 \frac{f_c}{f_c'} \right) \\
C = 1.2 - 0.006f_c' \\
f_0 = f_{c'} + 4.2f_{cl} \\
\varepsilon_0 = 7 \times 10^{-4} \varepsilon_{c0} + 0.06 \frac{f_{cl}}{f_c'} \\
f_{cl} = \frac{\rho_s f_{ct}}{2} \left( 1 - \frac{s}{d_c} \right)
$$

in which, $\sigma_c$ and $\varepsilon_c$ are the stress (MPa) and the corresponding strain of concrete; $f_0$ and $\varepsilon_0$ are the ultimate compressive strength of confined concrete (MPa) and the corresponding strain; $E_c$ is the elastic modulus of concrete (MPa); $f_c'$ is the ultimate compressive strength of plain concrete (MPa); $f_{cl}$ is the yield strength of confining reinforcement (MPa); $f_{ct}$ is the confinement stress (MPa); $d_c$ and $s$ are the depth of the core (mm) and the spacing (mm), respectively.

Even for monotonically increasing displacement, unloading may take place at points adjacent to a strain-softening zone. The unloading-reloading curve of concrete is used in Otter and Naaman's model [8] for compression, and is assumed to be a straight line passing through the origin for tension. Figure 3 shows the typical stress–strain curve of plain concrete.
For a reinforcing bar, meanwhile, an elasto-perfectly plastic stress-strain relationship in both tension and compression is adopted in this analysis, and the unloading slope is modeled as same as the elastic slope.

Calculation of rigidities

The basic assumptions for this analysis are as follows:
1. A linear strain distribution across the depth of a section is adopted.
2. The axial force is applied at the uncracked sectional centroid when calculating the applied moment.
3. Shear deformations are neglected.

The secant rigidities at the mid-point of a element are defined [9] as

\[
\begin{bmatrix}
P \\
M
\end{bmatrix} =
\begin{bmatrix}
EA & -ER \\
-ER & EI
\end{bmatrix}
\begin{bmatrix}
\varepsilon_a \\
\phi_a
\end{bmatrix}
\]

in which, \( P \) and \( M \) are the axial force and the bending moment applied to the sectional centroid, respectively; \( \varepsilon_a \) is the axial strain of the section measured at the sectional centroid; \( \phi_a \) is the curvature of the section.

The method to calculate the rigidities at any incremental or iterative stage is automatically selected in this analysis according to the extreme strain of actual strain distribution of the section. For example, the
nonlayered method is selected as shown in Fig. 4 when the extreme strain of the section is less than \( \varepsilon_N \) for concrete (assuming that the sign of the compression is plus), because it is possible to use the integral regardless of increase and/or decrease of strain. And if the extreme strain of the section is greater than \( \varepsilon_N \), the nonlayered method is used for the increase of the strain, and the layered method is adopted for the decrease of the strain.

In the layered method, each cross-section is divided into \( N \) layers, some of which may represent the reinforcing bars. Accordingly, the values of \( EA \), \( ER \), and \( EI \) are calculated approximately by a sum over all the layers

\[
EA = \sum_{i=1}^{N} E_i A_i,
\]

\[
ER = \sum_{i=1}^{N} E_i y_i A_i,
\]

\[
EI = \sum_{i=1}^{N} E_i y_i^2 A_i
\]

In which, \( E_i \) is the secant modulus; \( A_i \) and \( y_i \) are the area and the centroidal coordinate for layer number of the section, respectively. The values of \( EA \), \( ER \), and \( EI \), however, are calculated exactly by the integral formulation in the nonlayered method instead of a sum over all the layers

\[
EA = \int A E_{ct} \, dA + E_{ct} A_s,
\]

\[
ER = \int A E_{ct} y \, dA + E_{ct} y A_s,
\]

\[
EI = \int A E_{ct} y^2 \, dA + E_{ct} y^2 A_s
\]

in which, \( E_{ct} \) and \( E_{ct} \) are the secant modulus of concrete and reinforcing bar, respectively; \( A_s \) is the cross-sectional area of reinforcing bar.

Modification due to element size

As expected on the basis of other previous experience [3, 10, 11] with strain-softening problems, the result of the finite element analysis on the inelastic range is found to be highly sensitive to the chosen element size. The reason is that failure occurs within the element size adopted not within the plastic length \( l_p \) by the finite element method, thus the energy dissipation differs with the chosen element size. Accordingly, the rigidities and the curvature need to be modified in order to obtain consistent results. To illustrate it, consider two different element lengths \( l_p \) and \( l \) (arbitrarily selected) with a uniform distributed bending moment as shown in Fig. 5. The fracture energy of two different elements must be the same, i.e.

\[
(\phi_a - \phi_y) \times l_p = (\phi_m - \phi_y) \times l
\]

in which \( \phi_a \), \( \phi_m \) and \( \phi_y \) are the actual, the modified, and the yield curvatures, respectively. Replacing the total changes by the incremental changes, eqn (11) has the form

\[
(\phi_a - \phi_{a-1}) \times l_p = (\phi_m - \phi_{m-1}) \times l
\]

in which, \( \phi_{a-1} \) and \( \phi_{m-1} \) are the actual and the modified curvatures at the previously converged stage, respectively.

Fig. 4. Selection of a method applicable.

Fig. 5. Influence of element size adopted.
Solving for $\phi_m$, we have

$$\phi_m = \phi_{m-1} + (\phi_a - \phi_{a-1}) \frac{l}{l'}$$  \hspace{1cm} (13)

Substituting $\phi_m$ instead of $\phi_a$ in eqn (8), we see that for a change of the element size from $l_p$ to $l$, the modified rigidities are obtained by

$$ER_m = ER \frac{\phi_a}{\phi_m}$$  \hspace{1cm} (14)

$$EI_m = \frac{M - ERm\varepsilon_a}{\phi_m}$$  \hspace{1cm} (15)

in which $ER_m$ is the modified combined axial and flexural rigidity; $EI_m$ is the modified flexural rigidity.

Several equations are available to calculate the plastic length. In this study, the mean value of following two equations is taken as the plastic length for beam

Corley [12]: $l_p(\text{beam}) = 0.5d + 0.032 \frac{z}{\sqrt{d}}$  \hspace{1cm} (16)

Mattock [13]: $l_p(\text{beam}) = 0.5d + 0.05z$  \hspace{1cm} (17)

in which $d$ is the effective depth (mm); $z$ is the distance from the critical section to the point of contraflexure (mm). For a column, it is obvious that the plastic length tends to grow with increasing axial compressive load level. Therefore, Zahn’s model [14] is used as the plastic length considering the effect of the axial load

$$l_p(\text{column}) = l_p(\text{beam}) \left(1.5 + 1.67 \frac{P}{f_c A_g}\right)$$

$$\geq l_p(\text{beam})$$  \hspace{1cm} (18)

in which, $P$ is the axial compressive force (N); $A_g$ is the gross area of the section (mm$^2$).

**COMPARISON WITH TEST RESULTS**

The analytical results from the proposed method are compared with the experimental results for the following reinforced concrete frames, and with the analytical results and the computational times by the present method. The computer program was written in double-precision in FORTRAN 77 and compiled in FORTRAN compiler version 5.0 for the PC 386-DX system which exists as a coprocessor, and then uses 277 kilobytes of core storage. The computational time means the time required during the analysis in that system.

**Ernst et al.’s test**

Ernst et al. [15] conducted a series of tests on reinforced concrete portal frames which were loaded at two or three points. In this study, two of the frames, A40 and B40, are analyzed. Due to symmetry, we analyzed only one half of the frame.

Firstly, the frame B40 which was exhibiting softening, and its cross-section are shown in Fig. 6(a). The reinforcement in the beam consists of No. 4 steel bars placed at the bottom and No. 6 bars at the top. In order to check the difference of analysis due to the element size adopted, three cases of different length, i.e., cases 1–3 are analyzed with element sizes 150, 200, and 250 mm. As can be seen in Fig. 6(b), the results of the analysis are not consistent after yielding when the rigidities and the curvature are used with the actual values, but the results using the modification technique are quite consistent. The analytical results are compared with the results obtained experimentally and analytically by El-Metwally and Chen [9]. El-Metwally and Chen analyzed B40 up to a maximum load using only the nonlayered method which does not consider the unloading condition on the load control approach. The results from the proposed method show good agreement with the experimental results as shown in Fig. 6(b).

Secondly, frame A40 and its material properties are shown in Fig. 7(a). It can be seen from Fig. 7(b) that the analytical result agrees well with the experimental result. The ultimate load of the frame, however, is

![Fig. 6. Frame B40 tested by Ernst.](image-url)
somewhat overestimated and the corresponding displacement is underestimated by both the proposed and the layered method. The reason is that the stiffness of the beam and the column is the same and the compressive steel ratio in the beam is large. Accordingly, it shows linear behavior without decreasing stiffness until the yielding point in the analysis.

The proposed method reduces computational time by about 25% and saves about 34% main memory space except in the register of the PC 386-DX system as shown in Figs 6(c) and 7(c), and Table 1 in comparison with the present layered method, and is able to obtain the softening state which cannot be analyzed by the nonlayered method alone.

**Cranston's test**

Cranston [16] tested ten pinned portal frames, and one of them, P2, is chosen for analysis. The cross-section and its material properties are shown in Fig. 8(a). The reinforcement in beam segment $AB$ consists of four steel bars placed at the bottom and two at the top. This arrangement is reversed for the beam segment $BC$, i.e., two bars are placed at the bottom and four at the top. In analysis due to symmetry, only one half of the frame is analyzed. Figure 8(b) shows the analytical load-deflection curve compared with the experimental and the analytical by Bazant et al. [2]. The analysis by Bazant et al. uses only the layered method on the displacement control approach, and parameters which were determined so as to optimize the fit of the test data. It is apparent that the load-deflection curve of the frame under investigation was predicted with reasonable accuracy by the proposed method at all displacement stages. It is seen that failure occurs quickly in the proposed method and slowly in the method by Bazant et al. at this result because of the use of the different modification technique.
As explained in the previous section, this proposed method reduces computational time by about 20% and saves about 26% of memory space as shown in Fig. 8(c) and Table 1.

SUMMARY AND CONCLUSIONS

A nonlinear finite element analysis method for reinforced concrete frames including strain-softening has been presented. Using this method, load–deformation curves up to the point of collapse are obtained with reasonable agreement to experimental results. The proposed method reduces the computational time and needs less memory in comparison with the layered method alone, and it is also able to show the softening behavior that is not apparent in the use of the nonlayered method alone. Therefore, this method provides a more efficient, economical, and accurate method for a complete analysis of reinforced concrete structures over other available methods. Accordingly, it is possible to apply the proposed method for the analysis of larger scale structures without the use of an additional auxiliary system.

The analytical result leads to spurious sensitivity to the chosen element size. This problem can be overcome by modifying the rigidities and the curvature for each element based on the concept of concentrated inelastic rotations at a plastic hinge, considering the applied axial load.

REFERENCES