Values of the balanced decision making between supply chain partners

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Abstract

Coordination between supply chain partners is critical to effective supply chain management. In this paper, we consider two cases of decision-making structure for a simple supply chain consisting of two players: the first case in which a supply chain partner dominates the decision-making process, and the other in which two players share the decision-making process equally. According to the literature, we know \textit{a priori} that the balanced decision making forges a better (financial) outcome than the dominating case does. Therefore, our primary research objective is to analyze detailed dynamics of resource allocation and sources of the benefit of the balanced decision making. Our analysis indicates that the value of the balanced decision making arises from more effective resource utilization than the dominating case does: each of the cooperative partners knows how to optimize its resource utilization better than the other does.

\textit{Keywords:} balanced decision making; supply chain cooperation; optimal control theory model

1. Introduction

For effective supply chain management, coordination between supply chain partners is critical (Kim, 2000). But, there is a subtle issue: depending on the bargaining power balance between the supply chain participants, it is determined who exerts more influence in making decisions related to such coordination. For example, Munson et al. (1999) postulated that often one company in a chain might attempt to influence other members in order to achieve its own goals and promote its own interests.

In this paper, we consider two cases of decision-making structure in the context of a simple supply chain consisting of two players: the first case in which a supply chain partner dominates the decision-making process and the other partner passively follows the dominant player’s decision,
and the other case in which the two players share the decision-making process equally in a balanced way. For instance, in the automobile industry, a supplier that does not have a strong bargaining power cannot help but act passively vis-à-vis its stronger partner, e.g. the carmaker.

In the game theory literature, it is well established that a collusive or a cooperative case generates a better result than a competitive case from the entire system’s perspective: this result is mainly due to the existence of free riding (Fershtman and Nitzan, 1991). Applying the reasoning to the current context, we know a priori that the balanced decision making forges a better outcome than the dominating case does for the supply chain system as a whole. In this paper, therefore, rather than asking about which decision-making structure is better for the supply chain, our primary questions are “How do the resource allocation dynamics between the supply chain partners behave?” and “From where comes the benefit of the balanced case?” To answer the questions, we set up optimal control theory models and derive analytical solutions. In the literature, many of the studies on a similar issue have used static game-theoretic approaches. Although a game-theoretic approach is useful in examining competitive interactions between players participating in a static context (e.g. a single- or two-period time horizon), it is not usually effective in analyzing dynamic changes over time. That is, the game-theoretic methodology analyzes static, i.e. snapshot, portrayals, whereas the optimal control theory model can capture dynamic changes occurring over an entire decision-time horizon. Our optimal control analysis outcome indicates that the value of the balanced decision making arises from more effective resource utilization than in the dominating case.

Our paper is structured as follows: in the next section, we review the relevant literature. In Section 3, we develop optimal control models to answer our main research questions. Analysis of these models is carried out in detail in Section 4, focusing on where the benefit of the balanced decision making arises from. In Section 5, we present numerical examples to show the dynamics of resource allocation and profit increase throughout the decision horizon: the numerical analysis is based on a real-world case in the automobile industry. Finally, the concluding section summarizes our findings and discusses key managerial implications. Detailed analytical procedures to solve the optimal control models are summarized in Appendix A.

2. Literature review

In the literature, there is an abundance of relevant references, which can be grouped under several headings.

2.1. Supply chain coordination

Kim (2000) emphasized the importance of a long-term strategic relationship between a manufacturer and its suppliers in order to create more value in the supply chain (Spekman, 1988; Doyle, 1989; Choi and Hartley, 1996). It is fundamentally based on the belief that such a long-term relationship would eventually benefit all of the supply chain participants (Iyer and Bergen, 1997; Parlar and Weng, 1997). Without such a mutual benefit, supply chain collaboration may not be sustainable (Crewe and Davenport, 1992; Sparks, 1994; Ellram and Edis, 1996; Boddy

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et al., 1998; Stank et al., 1999; Ireland and Bruce, 2000; Lee and Whang, 2000; McIvor and McHugh, 2000; Barratt, 2004).

Jorgensen and Zaccour (2003) modeled a two-member channel of distribution, consisting of a manufacturer and a retailer (Chintagunta and Jain, 1992; Jorgensen and Zaccour, 1999). They postulated that in the absence of coordination, channel members determine their decision variables independently and non-cooperatively, but such uncoordinated decision making creates “channel inefficiency,” that is, channel members’ strategies are not at their joint profit-maximization levels and their profits are inferior to what could be achieved with coordinated behavior. They implied that there exist incentives to coordinate among supply chain partners so as to overcome such channel inefficiencies. In the literature, some argued that successful SCM improves financial performance for all chain members (Wisner and Tan, 2000; Tan, 2002; Crook and Combs, 2007). Others doubted it: Ketchen and Giunipero (2004) suggested “a chain member may exploit its partners for its own gain.”

At the essence of any strategic supply chain relationship, there must be effective coordination between supply chain partners: Weng (1999) put forth that effective coordination can lead to either increasing expected joint profit, increasing the probability of achieving the desired profit level, or a combination of both (Reyniers, 1992; Whang, 1995; Sox et al., 1997).

2.2. Coordination mechanisms

Coordination has been an important research issue not only in operations but also in marketing (Monahan, 1984; Chandra and Fisher, 1994). However, despite the weighty emphasis on the advantages of close supply chain coordination, Goffin et al. (2006) pointed out that there has been little research carried out on identifying the specific attributes of such relationships.

As Jeuland and Shugan (1983) suggested, coordination or cooperation is not a natural behavioral pattern for the channel members (Coughlan, 1985). That is, in order to accomplish effective coordination, the supply chain partners must design appropriate contractual arrangements (Cachon and Lariviere, 2005). These contracts need to specify such essential aspects as how to share the revenues as well as the costs (Taylor, 2000; Dana and Spier, 2001; Pasternack, 2002; Gerchak and Wang, 2004; Cachon and Lariviere, 2005), how to allocate decision rights among the participating supply chain members (Whang, 1995; Tsay et al., 1999), and so forth.

Now rather than just highlighting the importance of coordination itself, finding out effective mechanisms, i.e. coordination structures, to achieve high-performing coordination has become the focus in the literature (Anand and Mendelson, 1997; Moses and Seshadri, 2000). Narayanan and Raman (2004) suggested that companies have to create monetary incentive systems to induce the supply chain partners to behave in ways that are best for everyone in the chain. Sahin and Robinson (2005) highlighted information sharing and physical flow coordination for tighter supply chain integration in make-to-order supply chains, which is supposed to improve the supply chain’s economic performance.

Through longitudinal case studies, Spina and Zotteri (2000) showed that negotiations on how the benefits of joint cost-saving efforts between supply chain partners will be shared should be conducted up-front, suggesting “Once a player knows how the cake will be split, all he can do to get a larger slice is to contribute to making a larger cake; otherwise players will spend a lot of time fighting for a relatively larger slice rather than working to produce a larger cake.”
2.3. Decision-making structures

Sahin and Robinson (2002) suggested that there are two decision-making structures for accomplishing coordination, i.e. centralized and decentralized decision making. Munson et al. (1999) investigated the issue from a bargaining power perspective, postulating that although firms in a supply chain depend on each other and work together for mutual benefit, the relationships among them are rarely symmetrical. That is, in reality, there can be an imbalance of bargaining power among the supply chain partners. This unequal distribution of bargaining power might cause a supply chain partner with a larger bargaining power to impose its own decisions on the other weaker partners in the supply chain. Similarly, Crook and Combs (2007) noted that the benefits of supply chain coordination vary according to how bargaining power is used: a stronger member might calculate other weaker members’ dependency and exert its power during negotiation to appropriate a larger percentage, if not all, of the gains from supply chain coordination. At the extreme, the stronger member might even negotiate all supply chain gains plus some of the weaker members’ pre-SCM profits. In a similar vein, Dyer (1996b) argued that due to the bargaining power bias or imbalance, US automobile manufacturers and their suppliers had higher procurement costs than their Japanese counterparts.

Under what circumstances, then, is a supply chain member powerful? Pfeffer (1992) advocated five different sources of a stronger power: a supply chain member has larger bargaining power if (1) other members of the chain depend on it for their essential needs, (2) it has control over financial resources, (3) it plays a central role in the chain, (4) it is not substitutable, and (5) it has the ability to reduce critical uncertainty.

We already pointed out that it is ineffective or suboptimal for a single player in the supply chain to dominate the decision-making process. When they studied a supply chain system where the manufacturer dominated the supply chain decision making, Xu et al. (2001) implied that the manufacturer was the main beneficiary of coordination in terms of safety stock and resource waste reduction, while there was little effect on the retailer in these two areas. We can infer that when one member in the supply chain wields an unconstrained decision-making power, it can be difficult to avoid resource wastes in the supply chain system as a whole. Consistent with this inference, Munson et al. (1999) postulated that in order to maximize the effectiveness of coordination, strong firms can often maintain solid channel relationships by including other channel members in the decision-making process and/or providing concessions to eliminate potential losses. That is, allowing other supply chain partners to be involved in the decision-making process, it will be possible for the supply chain system to achieve effective coordination by eliminating possible wastes of resources.

2.4. Methodology

Various research methods for studying issues related to supply chain coordination have been used in the literature. There are researchers who have used traditional inventory models, sometimes with numerical analysis: for references, see Iyer and Bergen (1997), Parlar and Weng (1997), Chen (1998), Cachon and Fisher (2000), Donohue (2000), Chen et al. (2001), Fisher et al. (2001). Others adopted more complicated analytical models based on game theory: for a few notable examples,

There have been more empirical attempts as well: see Frohlich and Westbrook (2001), Griffith et al. (2006). There are researchers who favored using simulation techniques: Waller et al. (1999), Towill (1991), Wikner et al. (1991), Towill et al. (1992).

In the literature, it is easy to find papers utilizing usual game-theoretic models. The game-theoretic approach has advantages, one of which is to enable the researcher to analyze competitive interactions between players (e.g. supply chain partners) rigorously. However, with the optimal control theory model, we are more capable of analyzing the optimal dynamics and understanding intricate changes of the fundamental forces that drive such dynamics, i.e. analyzing the competitive situation dynamically over a longer period of time in a continuous time line.

3. Optimal control theory models

There are many areas of supply chain coordination (Kim, 2005). Supply chain partners can coordinate for sharing information about market demand, improving the quality of an existing product, conducting a joint marketing and/or promotion, making joint decisions on plant capacity or the inventory level, and the like. In this paper, we focus on a specific area of coordination: new product development (NPD).

We consider a simple supply chain consisting of two players, e.g. a supplier and a manufacturer (Jorgensen and Zaccour, 2003). To highlight the difference in decision-making structures, two cases are studied. The first case portrays a situation where the manufacturer dominates the decision-making process (Xu et al., 2001). The manufacturer’s domination is embodied in its ability to make use of the entire resources in the supply chain including the supplier’s as well as its own. In the second case, on the other hand, the supplier is also actively involved in the decision-making process, i.e. participating in the decision making for resource allocation.

3.1. Research motivation – the automotive industry

Our research was motivated by the automobile industry, where it is easy to find the situations described above. In the automobile industry, it is well known that there usually exists a huge imbalance in bargaining power between the supply chain partners, in particular, a carmaker (manufacturer) and its supplier(s): in general, it is the manufacturer that has the larger bargaining power. Sometimes the player with a dominating bargaining power wields its excessive influence on the weaker player to gain its own profit at the cost of the other. In Liker and Choi (2004), there is a quote from a director of a major supplier to the big three US carmakers, saying “The Big Three (US automakers) set annual cost-reduction targets (for the parts they purchase). To realize those targets, they’ll do anything. (They’ve unleashed) a reign of terror and it gets worse every year.” This is a situation comparable to the manufacturer-dominating decision-making structure mentioned above, where the manufacturer exerts its excessive bargaining power to coerce the weaker supplier into passively accepting the terms and conditions without any meaningful
bilateral negotiations. Therefore, the manufacturer obtains a \textit{de facto} right to utilize the supplier’s resources at its own “arbitrary” discretion. The question is, “Is this type of manufacturer-dominating decision structure in a supply chain good even for the manufacturer itself?”

But this kind of coercive relationship is not always the rule in the industry. In the same reference, there is another quote from a supplier to Toyota, saying “Toyota helped us dramatically improve our production system. We started by making one component, and as we improved, [Toyota] rewarded us with orders for more components. Toyota is our best customer.” Dyer (2000) described a situation that exactly matches the “balanced decision-making” structure in a supply chain mentioned above: “Instead of relying solely on its own engineers to create the concept for a new car and then to design all the car’s components, Chrysler now involves suppliers. And instead of Chrysler dictating prices to suppliers, regardless of whether the prices are realistic or fair, the two sides now strive together to find ways to lower the costs of making cars and to share the savings. . . . The results have been astounding. The time Chrysler needs to develop a new vehicle is approaching 160 weeks, down from an average of 234 weeks during the 1980s. The cost of developing a new vehicle has plunged an estimated 20–40 percent during the last decade. And, at the same time, Chrysler has managed to produce one consumer hit after another.”

3.2. The single player-dominating decision-making structure (coercive case)

For our first case, we develop an optimal control theory model that describes the manufacturer-dominating supply chain. In general, the manufacturer has larger bargaining power vis-a-vis its supplier, implying that the manufacturer can exert a much larger decision-making power than its supplier (Munson et al., 1999). We embody the manufacturer’s domination in the objective function of the optimal control model in P1-M. Table 1 explains the variables and parameters in the model

\[
P1-M: \text{Maximize} \int_0^T \{(x_1 + x_2)(a_1 - (x_1 + x_2)) - \rho_1 u_1^2\} \, dt; \tag{1}
\]

Subject to

\[ \dot{x}_1 = u_1, \tag{2} \]

\[ u_1 \leq \bar{u}_1, \tag{3} \]

\[ \dot{x}_2 = u_2, \tag{4} \]

Table 1
Notation

| \(x_i\) | Firm \(i\)’s knowledge/technology stock for new product development (state variable), \(i = 1, 2\); the manufacturer, \(i = 1\) |
| \(u_i\) | Firm \(i\)’s investment rate (control variable) at \(t\), \(i = 1, 2\) |
| \(\bar{u}_i\) | Limit on firm \(i\)’s resource investment rate at \(t\) |
| \(a_i\) | Parameter that determines firm \(i\)’s profitability |
| \(\rho_i\) | Parameter that determines the cost to use firm \(i\)’s resources |

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\[ u_2 \leq \bar{u}_2, \quad x_1(0) = x_1, \ x_2(0) = x_2. \]

P1-M is the manufacturer’s decision problem: this optimal control model consists of an objective function and seven constraints. We explain each of the components in detail.

- \( \int_0^T \left\{ (x_1 + x_2)(a_1 - (x_1 + x^2)) - \rho_1 u_1^2 \right\} dt \): In the objective function (1), the profit maximization is carried out from the manufacturer’s perspective and only the cost of using the manufacturer’s resource is included in the model. That is, when the manufacturer can wield its dominating bargaining power, it can make a decision on how the supplier’s resource should be utilized, but does not take into account the cost of using it. Although this kind of decision-making structure or arrangement is unfair from the supplier’s viewpoint, in reality, it can happen when the supply chain partners have extremely skewed bargaining powers: refer to an example in the automobile industry mentioned in the previous section.

- \((x_1 + x_2)(a_1 - (x_1 + x_2))\): The first part in the integrand of the objective function is the manufacturer’s profit, jointly determined by the manufacturer’s knowledge stock and the supplier’s at the same time, and concave in the combined knowledge stock, \(x_1 + x_2\): this format of profit function is widely used, in particular when competing or coordinating players have to devote effort jointly in order to create a common ground (e.g. a public good) on which each of them can earn its own profit (Fershtman and Nitzan, 1991). See Fig. 1. That is, the manufacturer needs the supplier’s knowledge stock, i.e. investment in NPD, in order to generate its own profit, and vice versa. In this sense, it is important for the supply chain partners to coordinate with each other. As we can see in Fig. 1, there might be a technical issue: there is an effective range of the combined knowledge stock, during which an SC partner’s profit increases as the knowledge stock increases. Once the combined knowledge stock exceeds the range, the profit decreases even when the knowledge stock increases. Although this kind of phenomenon can happen if we allow “diseconomies of scale,” to avoid any unnecessary (and unrealistic)
complications in the ensuing discussion, we would like to consider only the cases where the combined knowledge stock is within the effective range.

- Using the example of the automobile industry (as in this paper), for example, we propose that a carmaker needs knowledge or technology to develop new models continuously. The carmaker invests in improving such knowledge stock, \( x_1 \), and the supplier also invests in enhancing its own knowledge stock, \( x_2 \). Finally, the carmaker’s NPD capability is determined by the combined knowledge stock of both the carmaker and the supplier, i.e. \( X = x_1 + x_2 \).

- Now we assume that the carmaker’s profit or payoff is dependent on its NPD capability, i.e. the higher the NPD capability, the larger the potential payoff. Based on empirical observations in the literature, we further put forth that the relationship between the payoff and the NPD capability is concave, implying an existence of diminishing returns to scale as the firm’s knowledge stock accumulates. More specifically, we express this relationship as follows: “the carmaker’s payoff at \( t = \kappa X(a_1 - X) \)” and without loss of generality, we normalize so as to have \( \kappa = 1 \).

- The relationship between the firm’s NPD capability (i.e. knowledge/technology stock) and its payoff (related with NPD) is based on the literature as well as empirical findings. As the firm invests in enhancing NPD knowledge, its NPD knowledge stock increases and so does the firm’s NPD capability. As the firm’s NPD capability grows, it will be able to speed up its NPD process, i.e. to reduce the NPD lead-time. Reduced NPD lead-time implies faster introduction of new models into the market, and therefore increased market share, which helps the firm achieve higher profit (Datar et al., 1997; Hendricks and Singhal, 1997).

- Developing the knowledge stock is costly, i.e. each SC partner needs to pay its share of cost. However, as alluded to in the automobile industry, it does happen that the manufacturer makes decisions on how to utilize the supplier’s resources when it has dominating bargaining power vis-à-vis the supplier. When this kind of situation indeed happens, from the manufacturer’s perspective, the supplier’s resources are free and the cost to utilize them is not included in the manufacturer’s decision problem. That is why there is only one cost element, \( \rho_1 u_1^2 \), in P1-M.

- In P1-M, (2) and (4) show the dynamics of the knowledge/technology stock. Without compromising generalizability, we assume that the investment is scaled so that investing one unit of resource increases one unit of knowledge.

- (3) and (5) specify the resource availability that can be utilized at \( t \) for each of the players. For instance, the manufacturer cannot spend more than \( \tilde{u}_1 \) at \( t \) on developing its knowledge stock for NPD.

- (6) presents the initial values of the state variables.

With P1-M, we formulate the Hamiltonian as in (7) and solve the optimal control model as in Appendix Section A.1. The problem’s Hamiltonian or Lagrangian is structured as follows:

\[
L = H = (x_1 + x_2)(a_1 - (x_1 + x_2)) - \rho u_1^2 + \lambda_1 u_1 + \lambda_2 u_2 + w_1(\tilde{u}_1 - u_1) + w_2(\tilde{u}_2 - u_2).
\]  
(7)

Now the profit of the weaker player in the supply chain, i.e. the supplier, is completely determined by the solutions to P1-M: “”” indicates the value is part of the optimal solution determined in the manufacturer’s problem.
P1-S: Supplier’s profit
\[
\int_0^T \left\{ (x_1^r + x_2^r)(a_2 - (x_1^r + x_2^r)) - \rho_2(u_2^r)^2 \right\} dt.
\] (1 – 1)

In the single player-dominating case, where the manufacturer wields an excessively large bargaining power, the supplier can only react passively, allowing the manufacturer to decide how to utilize the supplier’s resources. Therefore, the supplier’s decision problem is simply to calculate its profit by plugging in its profit function the solved decision variables, both state and control variables.

3.3. Balanced decision-making structure (balanced case)

In this section, we develop an optimal control theory model for the second case, the balanced decision-making structure. For this case, a shared decision making or a balanced allocation of bargaining power between SC partners is assumed: what we mean by “balanced” in this paper is that the supplier’s profit and cost are taken into account “equally or equitably” when the resource allocation decision is made. That is, the decision maker adopts a balanced (i.e. not biased towards one side’s profit maximization only) perspective when making the resource allocation decision.

We already discussed cases about Toyota’s relationship with its suppliers and Chrysler’s coordination in developing new models with its suppliers. Even if the manufacturer can wield a dominating bargaining power, it might not want to make a decision unilaterally because it knows that such a biased decision making will not improve the profitability not only for the supplier but also for the manufacturer itself.

The most significant difference from the single player-dominating case is the objective function. That is, in the objective function of the balanced decision-making structure, the profit to be maximized is the sum of those for the supplier and the manufacturer. In addition, the cost to utilize the supplier’s resources is also included in the objective function, implying that the supplier’s resources are not free any longer even to the manufacturer. This arrangement in the objective function implies that the manufacturer and the supplier are now carrying out joint decision making, i.e. it is a balanced decision-making structure. That is, we embody the balanced decision-making scheme by taking into account the costs and benefits of the supplier fully in the objective function. P2 is the optimal control theory model for this case.

P2: Maximize
\[
\int_0^T \left\{ (x_1 + x_2)(a_1 + a_2 - 2(x_1 + x_2)) - \rho_1 u_1^2 - \rho_2 u_2^2 \right\} dt,
\] (8)

Subject to
\[
\dot{x}_1 = u_1,
\] (9)
\[
u_1 \leq \bar{u}_1,
\] (10)
\[
\dot{x}_2 = u_2,
\] (11)
\[
u_2 \leq \bar{u}_2,
\] (12)
\[
x_1(0) = x_1, \; x_2(0) = x_2.
\] (13)
The relevant Hamiltonian or Lagrangian is as follows:

\[ L = H \]
\[ = (x_1 + x_2)(a_1 + a_2 - 2(x_1 + x_2)) - \rho_1 u_1^2 - \rho_2 u_2^2 + \lambda_1 u_1 + \lambda_2 u_2 + w_1(\bar{u}_1 - u_1) \]
\[ + w_2(\bar{u}_2 - u_2). \]  

(14)

In P2, (9)–(13) are exactly the same as (2)–(6). Using the Hamiltonian (14), we derive the analytical solution to P2, the procedures of which are well documented in Appendix Section A.2.

4. Analysis of the model

In Section 3, we have developed optimal control theory models for two cases: the coercive (single player-dominating) and the balanced decision-making situation. Because the detailed mathematical derivations are completely listed in Appendix Sections A.1 and A.2, we do not have to repeat the problem-solving procedures here. Rather, in this section, we would like to discuss the dynamics of the solutions by depicting them in figures: we focus on how to analyze the patterns of resource allocation dynamics and where the benefits of the balanced decision making arrive from. For the single player-dominating case, we show why the manufacturer (with the superior bargaining power) always uses up the supplier’s resource first before it starts using its own resource: we prove the SC participants’ behavioral dynamics by analyzing their marginal valuations reflected in co-state variables and Lagrangian multipliers. We conduct a similar analysis for the balanced case. More importantly, with the solutions obtained from this section, we will be able to perform the numerical analysis in the next section.

4.1. Dynamics of the single player-dominating case (the coercive case)

In Appendix Section A.1, we have detailed the solution procedures for the single player-dominating case: we assume the dominant player is the manufacturer, reflecting the reality in general (Munson et al., 1999). We have paid special attention to the behaviors of \( \lambda_i \) and \( w_i \) in order to understand under what circumstances the sole decision maker, e.g. the manufacturer in our case, uses the resources to their limits, i.e. the dynamics of \( u_i^* \leq \bar{u}_i \). Our analysis indicates that because the supplier’s resources are essentially free to the manufacturer, assuming the resources are valuable, the manufacturer always consumes the supplier’s resources to the limit, i.e. \( u_i^* = \bar{u}_i \). In addition, the result shows that the relative magnitude between \( \lambda_i(0) \) and \( 2\rho \bar{u}_i \) is critical to determining whether the manufacturer uses up its own resources completely or not: \( \lambda_i(0) \) is the shadow price or the marginal value of the unit resource at the beginning of the decision time horizon; because \( \rho \) is a cost coefficient and \( \bar{u}_i \) is the limit of the manufacturer’s resources, \( 2\rho \bar{u}_i \) somehow represents the total cost to utilize its own entire resources.

There are two cases:

- \( \lambda_i(0) < 2\rho \bar{u}_i \), i.e. the marginal value of unit resource is less than a measure related to the total cost to use up the manufacturer’s entire resources.
Figure 2 describes this situation. As in Appendix Section A.1 and Fig. 2, we see that under this condition, the manufacturer never consumes its own resources completely, while it always uses up the supplier’s: \( u_1^* < \bar{u}_1 \) and \( u_2^* = \bar{u}_2 \) throughout \( 0 \leq t \leq T \). It is, therefore, reasonable to infer that under this circumstance, it might be unavoidable for the resources to be utilized inefficiently and ineffectively. Such inefficient resource utilization easily causes sub-optimality of performance in the supply chain as a whole.

- \( \lambda_1(0) > 2 \rho \bar{u}_1 \), i.e. the marginal value of unit resource is larger than a measure related to the total cost to use up the manufacturer’s entire resources.

In the second case, there can exist \( \hat{t} \) such that \( u_1^* = \bar{u}_1 \) for \( 0 \leq t < \hat{t} \) and \( u_1^* < \bar{u}_1 \) for \( \hat{t} \leq t \leq T \): see Fig. 3. Appendix Section A.1 proves that

\[
\hat{t} = \sqrt{\frac{-\lambda_1(0)}{2 \rho \bar{u}_1} + \frac{\bar{u}_1}{\bar{u}_2}}.
\]  

(A22) or (15) indicates the following:

\[
\begin{align*}
\text{(a)} & \quad \frac{d}{d \bar{u}_1} \left( \frac{-\lambda_1(0) - 2 \rho \bar{u}_1}{\bar{u}_1 + \bar{u}_2} \right) = -\frac{\lambda_1(0) + 2 \rho \bar{u}_2}{(\bar{u}_1 + \bar{u}_2)^2} < 0, \\
\text{(b)} & \quad \frac{d}{d \bar{u}_2} \left( \frac{-\lambda_1(0) - 2 \rho \bar{u}_1}{\bar{u}_1 + \bar{u}_2} \right) = \frac{\lambda_1(0) + 2 \rho \bar{u}_1}{(\bar{u}_1 + \bar{u}_2)^2} < 0,
\end{align*}
\]

implying that the larger the manufacturer’s resource limit, the shorter \( \hat{t} \), i.e. the shorter the period during which \( u_1^* = \bar{u}_1 \), other things being equal. That is, when the manufacturer’s resources are abundant, it becomes less likely that the manufacturer consumes its resources fully.
implying that the larger the supplier’s resource limit, the shorter \( \hat{t} \), i.e. the shorter the period during which \( u_1^* = \bar{u}_1 \), other things being equal. That is, when the supplier’s resources are abundant, it becomes less likely that the manufacturer consumes its resources fully because it would be sufficient to use up the supplier’s resources first.

Which situation is more likely to occur in reality: \( \lambda_1(0) < 2\rho \bar{u}_1 \) or \( \lambda_1(0) > 2\rho \bar{u}_1 \)? It might depend on the specific context in point. But \( \lambda_1(0) \) is the shadow price or the marginal value of the unit resource, while \( 2\rho \bar{u}_1 \) somehow represents the total cost to utilize the full resources, and therefore, in general, \( \lambda_1(0) < 2\rho \bar{u}_1 \) seems more likely to occur. This observation implies that it is more likely to have \( u_1^* < \bar{u}_1 \) and \( u_2^* = \bar{u}_2 \) throughout \( 0 \leq t \leq T \), i.e. the manufacturer always consumes the supplier’s resources completely, whereas it never uses up its own resources, regardless of the relative efficiency or effectiveness between the two different types of resources. Hence, it is more likely to waste valuable resources.

4.2. Dynamics of the balanced decision-making case (the balanced case)

Now, we would like to analyze the dynamics of resource allocation for the balanced decision-making case. Again, the details of mathematical derivations are well documented in Appendix Section A.2. Similar to our analysis for the single player-dominating case, the precise dynamics of resource allocation for the balanced case are dependent on the relative magnitudes among \( \rho_1 \bar{u}_1 \), \( \rho_2 \bar{u}_2 \), and \( \lambda(0) \): see Fig. A1 and Table A1.

Figure 4 depicts the case where \( 2\rho_2 \bar{u}_2 < 2\rho_1 \bar{u}_1 < \lambda(0) \). Under this condition, \( \hat{t}_1 \) and \( \hat{t}_2 \) can be determined as in (A43) and (A48)

\[
\hat{t}_1 = \sqrt{\frac{\lambda_1(0) - 2\rho_1 \bar{u}_1}{2(\bar{u}_1 + \bar{u}_2)}},
\]

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\[
\hat{t}_2 = T - \frac{\rho_2 \bar{u}_2}{\rho_1 \bar{u}_1} (T - \hat{t}_1).
\] (17)

We can make a similar interpretation for \(\hat{t}_1\) to that for \(\hat{t}\) in (15). It is noteworthy that \(\hat{t}_2\) is determined mainly by two elements: the difference between \(\hat{t}_1\) and \(T\) as expressed in \((T - \hat{t}_1)\) and the relative efficiency between the two different resource utilizations as represented in \(\rho_2 \bar{u}_2 / \rho_1 \bar{u}_1\).

The biggest difference between the single player-dominating and the balanced case is that the decision makers in the balanced case use resources by taking into account their relative efficiency, whereas in the coercive case the resources owned by the player with less bargaining power are used up completely regardless of their efficiency vis-à-vis that of the resources possessed by the dominant decision maker.

5. Numerical examples

To cast our analysis in the real-world context, we conduct a numerical analysis. We already mentioned two real-world cases in the automobile industry, i.e. GM and Chrysler: we compare the GM case to the single player-dominating structure and Chrysler to the balanced decision-making structure. The setting can be described as follows. The carmaker gains benefit as it becomes more capable of developing a new model faster, i.e. reducing the NPD lead-time. In order to improve its NPD speed, the carmaker needs support from its supplier. Now we have the following one-to-one relationship.

- \(x_1\): the carmaker’s knowledge stock for speeding up its NPD process,
- \(x_2\): the supplier’s knowledge stock for speeding up the NPD process,
• $x_1 + x_2$: the combined knowledge stock developed by both the carmaker and its supplier, which reflects (e.g. proxies) the supply chain’s overall NPD capability,

• $(x_1 + x_2)(a_i - (x_1 + x_2))$: the payoff the carmaker ($i = 1$) or the supplier ($i = 2$) earns when the NPD capability (i.e. the combined knowledge stock) is $x_1 + x_2$ at $t$.

Table 2 presents the parameter values used in the numerical analysis: these parameter values are based on the information from the Chrysler case (Dyer, 1996a, 2000).

- The primary resources are engineering hours. The carmaker can invest as much as 3000 engineering hours in increasing its NPD knowledge at a time: i.e. $\bar{u}_1 = 0.03$.
- Through an appropriate adjustment (e.g. normalization), one unit of NPD knowledge stock is equal to 1 engineering hour, i.e. $\bar{x}_1 = u_1$.
- The carmaker’s instantaneous payoff is determined by its NPD knowledge stock developed by both the manufacturer and the supplier. For instance, if the combined knowledge stock for NPD is 0.3 at $t$, the firm’s instantaneous payoff (before subtracting the investment cost) at $t$ would be $210$ million, i.e. $0.3 \times (1 - 0.3) = 0.21$ and the unit is $1$ billion. As the firm’s NPD capability improves, its payoff increases as well.
- The relationship between the firm’s NPD capability (i.e. knowledge/technology stock) and its payoff (related with NPD) is based on the following reasoning. As the firm invests in NPD knowledge/technology development, its NPD knowledge/technology stock increases and so does the firm’s NPD capability. As the firm’s NPD capability grows, it will be able to speed up its NPD process, i.e. to reduce the NPD lead-time. Reduced NPD lead-time implies faster introduction of new models into the market, and therefore increased market share, which helps the firm achieve higher profit (Datar et al., 1997; Hendricks and Singhal, 1997).
- The total cost due to investing $u_1$ engineering hours is $\rho_1 u_1^2$. Using $\rho_1 = 120$, the relationship between invested engineering hours and total cost resembles that in Fig. 5.
- The supplier has a similar condition in terms of available engineering hours, although the cost structure is better than the carmaker’s ($\rho_1 > \rho_2$). That is, it is less expensive to utilize a supplier’s engineering hour, possibly because it pays its engineers less, runs less expensive plant/equipment, has less costly legacy systems, and so forth.
- The functional relationships are derived from the estimates based on our empirical investigation of the carmaker, using data from secondary sources as well.
- Because we would like to see the long-term effect of NPD collaboration, we use a 10-year time horizon, i.e. $T = 10$.

Using the parameter values in Table 2, we conducted numerical analyses for the two different scenarios. The numerical analysis results are reported in Table 3, which we recapitulate as follows:
Total payoff (combining the manufacturer’s and the supplier’s together) of the balanced case is much larger than that of the coercive case, i.e. the single player-dominating case. It is intriguing to note that the balanced case total payoff is larger, despite the fact that the total cumulative investment of the balanced case is smaller than that of the coercive case. It implies that the SC partners utilize resources much more effectively under the balanced decision-making structure than under the single player-dominating structure. We infer that when it comes to utilizing its own resources, each partner in the SC knows better than the other. Under the balanced case, the manufacturer must increase its investment, while the supplier can reduce its spending significantly.

When we look at the individual profits, we notice that the supplier’s increases significantly on shifting from the coercive to the balanced case, whereas the manufacturer’s actually decreases slightly. Here, we can see a potential dilemma faced by the manufacturer. When the manufacturer was exerting its huge bargaining power without any restraints, it was able to enjoy higher profit, although the supply chain as a whole was operating at an inefficient level. In order to make the entire SC more efficient, the manufacturer needs to change its decision-making structure so that a more balanced decision making becomes possible. By doing so, the entire supply chain performs better, but it appears to be doing that at the cost of the

### Table 3
Numerical analysis results at $T$

<table>
<thead>
<tr>
<th></th>
<th>Single player-dominating (coercive)</th>
<th>Balanced decision making</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total payoff</td>
<td>20.90</td>
<td>24.44</td>
</tr>
<tr>
<td>Manufacturer’s payoff</td>
<td>14.19</td>
<td>13.33</td>
</tr>
<tr>
<td>Supplier’s payoff</td>
<td>6.71</td>
<td>11.11</td>
</tr>
<tr>
<td>Cumulative engineering hours invested: total</td>
<td>0.39</td>
<td>0.32</td>
</tr>
<tr>
<td>Cumulative engineering hours invested: manufacturer</td>
<td>0.09</td>
<td>0.15</td>
</tr>
<tr>
<td>Cumulative engineering hours invested: supplier</td>
<td>0.30</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Payoffs are in $\text{\$ billion}$ and engineering hours are in 100,000.

- Total payoff (combining the manufacturer’s and the supplier’s together) of the balanced case is much larger than that of the coercive case, i.e. the single player-dominating case.
- It is intriguing to note that the balanced case total payoff is larger, despite the fact that the total cumulative investment of the balanced case is smaller than that of the coercive case. It implies that the SC partners utilize resources much more effectively under the balanced decision-making structure than under the single player-dominating structure. We infer that when it comes to utilizing its own resources, each partner in the SC knows better than the other. Under the balanced case, the manufacturer must increase its investment, while the supplier can reduce its spending significantly.
- When we look at the individual profits, we notice that the supplier’s increases significantly on shifting from the coercive to the balanced case, whereas the manufacturer’s actually decreases slightly. Here, we can see a potential dilemma faced by the manufacturer. When the manufacturer was exerting its huge bargaining power without any restraints, it was able to enjoy higher profit, although the supply chain as a whole was operating at an inefficient level. In order to make the entire SC more efficient, the manufacturer needs to change its decision-making structure so that a more balanced decision making becomes possible. By doing so, the entire supply chain performs better, but it appears to be doing that at the cost of the
Table 4  
Comparison between the two cases

<table>
<thead>
<tr>
<th></th>
<th>Balanced case value – Coercive case value</th>
<th>Coercive case value</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total payoff</td>
<td>17</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Manufacturer’s payoff</td>
<td>-6</td>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>Supplier’s payoff</td>
<td>66</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Cumulative investment: total</td>
<td>-18</td>
<td>1</td>
<td>-18</td>
</tr>
<tr>
<td>Cumulative investment: manufacturer</td>
<td>67</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Cumulative investment: supplier</td>
<td>-43</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Total payoff.

manufacturer’s profit. Is there any solution that can eliminate or at least mitigate such a dilemma faced by the manufacturer? One possible way is for the SC partners, the manufacturer and its supplier, to agree upon a scheme to share the total profit in a more equitable manner, e.g. each player’s payoff under the balanced case should be strictly larger than that under the coercive case. If they can find a mechanism that measures how much contribution was made by each player to enlarging the total profit, the profit-sharing issue under the balanced case can be much simpler. Table 4 summarizes the incremental changes as the SC shifts from a single player-dominating to a balanced decision-making structure.

Figure 6 depicts the dynamics of total payoffs whereas Fig. 7 presents the dynamics of the manufacturer’s payoff and Fig. 8 the supplier’s. For the manufacturer, it is always better to have a coercive decision-making structure (of course, where the manufacturer is the dominant player). On the contrary, for the supplier, it is always better to have a balanced decision-making structure, where both the manufacturer and the supplier share the decision-making power equitably. One interesting observation is about the dynamics of total payoffs in Fig. 6. Although the total payoff under the balanced decision-making structure becomes much better than that under the single player-dominating structure eventually, it actually remains lower than the coercive payoff almost
until the half of the decision-time horizon: that is, the total payoff under the coercive case is larger than that under the balanced case in the early period of the decision-time horizon. It is due to this that the balanced decision-making structure requires relatively more initial investment from the SC partners, which can be utilized fully in the later period: this kind of early commitment is possible because the SC partners have trust in each other. As such, we can infer that if the SC partners have a relatively short decision-time horizon (i.e. a myopic perspective), the balanced decision-making structure might not be the ideal choice, because it may generate less total profit than the coercive case does.

6. Managerial implications

Our primary research objective in this paper is to understand the detailed dynamics of resource allocation and profit generation under the two important supply chain decision-making
mechanisms, i.e. the single player-dominating (coercive) and the balanced decision-making structure. Essentially, we would like to answer the research question, “Where does the benefit of the balanced decision making between supply chain partners come from and how can we analyze its dynamics?” by developing optimal control theory models to describe two contrasting cases. By solving the optimal control theory models, we have been able to identify the general sources and dynamics of inefficiency or efficiency for each of the decision-making structures.

Under the arrangement of a single player-dominating decision making, the dominant partner always consumes the less-powerful partner’s resources completely before it starts using its own resources. This is due to the fact that the less powerful partner’s resources are essentially free to the dominant player in the supply chain: when something is free, a human being tends to use it fully without paying due attention to whether such usage is efficient or economical from the entire system’s perspective. That is, under the single player-dominating case, it is unavoidable to waste resources to a great extent, and such resource waste causes the entire supply chain to perform sub-optimally.

Under the balanced decision-making structure, it costs to use not only the dominant partner’s resources but also the less powerful player’s resources: in the objective function, both the manufacturer’s and the supplier’s benefits and costs are equally represented. Our analysis indicates that under this balanced decision-making structure, resource utilization is disciplined in that the decision makers determine how much of each player’s resources should be utilized by taking into account their relative efficiency. Such disciplined resource utilization enables the entire supply chain to perform more optimally.

To cast our analysis in a more realistic setting, we presented numerical examples based on the automobile industry. Our numerical examples clearly showed the detailed dynamics of the resource allocation and profit generation for the two cases. Some of the key results are as follows: the total payoff of the balanced case is much larger than that of the single player-dominating case, despite the fact that the total cumulative investment of the balanced case is smaller, indicating that the SC partners utilize resources more effectively under the balanced decision-making structure, but the supplier’s profit increases significantly on shifting from the coercive to the balanced case, whereas the manufacturer’s actually decreases slightly; to fix this dilemma, the manufacturer and its supplier need to agree upon a scheme to share the total profit in a more equitable manner. If they can find a mechanism that measures how much contribution was made by each player to enlarging the total profit, the profit-sharing issue under the balanced case can be much simpler. Finding out such a mechanism can be a promising area for future research.

Coordination between supply chain partners becomes an important issue not only from a theoretical standpoint but also from an empirical perspective. Although it seems simple enough to guess that the performance from close collaboration, i.e. coordinated or balanced decision making, can be higher than without such collaboration, research has not been performed extensively to analyze the sources and dynamics of such improved performance. The major contribution made by our paper is that it pinpointed the sources and dynamics of the benefit from such a collaboration by developing and solving optimal control theory models. In order to highlight the fundamental nature of such sources and dynamics of benefit, we adopted relatively simple models, which can have both merits and demerits, which are potential shortcomings of this research.

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Appendix A

A.1. A single player-dominating (coercive) case

We have the following Hamiltonian or Lagrangian:

\[ L = H = (x_1 + x_2)(a_1 - (x_1 + x_2)) - \rho u_1^2 + \lambda_1 u_1 + \lambda_2 u_2 + w_1(\bar{u}_1 - u_1) + w_2(\bar{u}_2 - u_2). \]  \( \text{(A1)} \)

The set of necessary conditions is

\[ L_{u_1} = -2\rho u_1 + \lambda_1 - w_1 = 0, \quad \text{(A2)} \]

\[ L_{u_2} = \lambda_2 - w_2 = 0, \quad \text{(A3)} \]

\[ \dot{\lambda}_1 = -L_{x_1} = -(a_1 - (x_1 + x_2) - (x_1 + x_2)) = -(a_1 - 2(x_1 + x_2)) = 2(x_1 + x_2) - a_1, \quad \text{(A4)} \]
\[ \dot{x}_2 = -L_{x_2} = -((a_1 - (x_1 + x_2) - (x_1 + x_2)) = 2(x_1 + x_2) - a_1, \]  
\[ w_1 \geq 0, \quad w_1(\bar{u}_1 - u_1) = 0, \]  
\[ w_2 \geq 0, \quad w_2(\bar{u}_2 - u_2) = 0. \]  

- Behavior of \( \dot{\lambda}_i \)

If \((x_1 + x_2) < \frac{\alpha_1}{2}, \dot{\lambda}_1 < 0 \) and \( \dot{\lambda}_2 < 0 \). Likewise, if \((x_1 + x_2) > \frac{\alpha_1}{2}, \dot{\lambda}_1 > 0 \) and \( \dot{\lambda}_2 > 0 \).

Because \( \dot{x}_1 = u_1 \geq 0 \) and \( \dot{x}_2 = u_2 \geq 0, \dot{\lambda}_1 < 0 \to \dot{\lambda}_1 > 0 \) and \( \dot{\lambda}_2 < 0 \to \dot{\lambda}_2 > 0 \) as time increases. Moreover, \( \dot{\lambda}_1(T) = \dot{\lambda}_2(T) = 0 \).

Therefore, we reach the following conclusions.

(a) \( \dot{\lambda}_1 < 0 \) and \( \dot{\lambda}_2 < 0 \) for \( 0 \leq t \leq T \). \hspace{1cm} (A8)

(b) Because \( \dot{\lambda}_1 = \dot{\lambda}_2 \) and \( \lambda_1(T) = \lambda_2(T) = 0 \), \( \dot{\lambda}_1 = \dot{\lambda}_2 \) throughout \( 0 \leq t \leq T \). \hspace{1cm} (A9)

(c) Therefore, from (A3), \( \dot{\lambda}_1 = \dot{\lambda}_2 = w_2 \). \hspace{1cm} (A10)

(d) Because \( w_2 > 0 \) for \( 0 \leq t < T \), \( u^*_2 = \bar{u}_2 \) throughout \( 0 \leq t < T \). \hspace{1cm} (A11)

- Behavior of \( w_1 \)

Now, from (A2), \( \dot{\lambda}_1 = 2\rho u_1 + w_1 \geq 0 \).

(a) If \( \dot{\lambda}_1(0) < 2\rho \bar{u}_1 \), then \( w_1 = 0 \) and \( u^*_1 < \bar{u}_1 \) for \( 0 \leq t \leq T \). \hspace{1cm} (A12)

(b) But if there exists \( \hat{t} \) such that \( \dot{\lambda}_1(t) = 2\rho \bar{u}_1 + w_1(t) \) for \( 0 \leq t \leq \hat{t} < T \) and \( w_1(\hat{t}) = 0 \), i.e. \( \dot{\lambda}_1(0) > 2\rho \bar{u}_1 \), then \( u^*_1 = \bar{u}_1 \) for \( 0 \leq t \leq \hat{t} \) and \( u^*_1 < \bar{u}_1 \) for \( \hat{t} \leq t \leq T \). \hspace{1cm} (A13)

- How to determine \( \hat{t} \)?

First, suppose \( w_1 > 0 \) and thus \( u^*_1 = \bar{u}_1 \). From (A4), \( \dot{\lambda}_1 = 2(x_1 + x_2) - a_1 \).

\[ \dot{\lambda}_1 = 2(\dot{x}_1 + \dot{x}_2) = 2(\bar{u}_1 + \bar{u}_2), \]  
\[ \dot{\lambda}_1 = 2(\bar{u}_1 + \bar{u}_2)t + k_1, \]  
\[ \dot{\lambda}_1 = (\bar{u}_1 + \bar{u}_2)t^2 + k_1t + k_2, \]  

where \( k_1 \) and \( k_2 \) are constants to be determined.
Now, from (A2) and \( w_1 > 0 \), we have

\[
w_1 = \lambda_1 - 2\rho \bar{u}_1 = (\bar{u}_1 + \bar{u}_2)\bar{t}^2 + k_1\bar{t} + k_2 - 2\rho \bar{u}_1. \tag{A17}
\]

(A17) indicates that \( w_1 \geq 0 \) is a quadratic function and we can find the minimum of \( w_1 \) by arranging (A17) as follows:

\[
w_1 = (\bar{u}_1 + \bar{u}_2) \left\{ t + \frac{k_1}{2(\bar{u}_1 + \bar{u}_2)} \right\}^2 - \frac{k_1^2}{4(\bar{u}_1 + \bar{u}_2)} + k_2 - 2\rho \bar{u}_1. \tag{A18}
\]

Therefore, in order for \( \hat{\bar{t}} > 0 \) in (A13) to exist, it must hold that

\[
- \frac{k_1^2}{4(\bar{u}_1 + \bar{u}_2)} + k_2 - 2\rho \bar{u}_1 = 0, \tag{A19}
\]

and

\[
\hat{\bar{t}} = \frac{-k_1}{2(\bar{u}_1 + \bar{u}_2)} > 0. \tag{A20}
\]

(A19) implies \( k_1^2 - 4(\bar{u}_1 + \bar{u}_2)(k_2 - 2\rho \bar{u}_1) = 0 \) and thus

\[
k_1 = \pm 2\sqrt{(\bar{u}_1 + \bar{u}_2)(k_2 - 2\rho \bar{u}_1)}. \tag{A21}
\]

From (A16), we know that \( k_2 = \lambda_1(0) \).

Substituting (A21) into (A20) appropriately, we finally have

\[
\hat{\bar{t}} = \sqrt{\frac{\lambda_1(0) - 2\rho \bar{u}_1}{\bar{u}_1 + \bar{u}_2}}. \tag{A22}
\]

### A.2. The balanced decision-making case

We have the following Hamiltonian or Lagrangian:

\[
L = H = (x_1 + x_2)(a_1 + a_2 - 2(x_1 + x_2)) - \rho_1 u_1^2 - \rho_2 u_2^2 + \lambda_1 u_1 + \lambda_2 u_2
\]

\[
+ w_1(\bar{u}_1 - u_1) + w_2(\bar{u}_2 - u_2). \tag{A23}
\]

The set of necessary conditions is

\[
L_{u_1} = -2\rho_1 u_1 + \lambda_1 - w_1 = 0, \tag{A24}
\]

\[
L_{u_2} = -2\rho_2 u_2 + \lambda_2 - w_2 = 0, \tag{A25}
\]

\[
\dot{\lambda}_1 = -(a_1 + a_2 - 2(x_1 + x_2) - 2(x_1 + x_2)) = 4(x_1 + x_2) - a_1 - a_2, \tag{A26}
\]

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\[ \dot{\lambda}_2 = -(a_1 + a_2 - 2(x_1 + x_2) - 2(x_1 + x_2)) = 4(x_1 + x_2) - a_1 - a_2, \]  
(A27)

\[ w_1 \geq 0, \ w_1(\bar{u}_1 - u_1) = 0, \]  
(A28)

\[ w_2 \geq 0, \ w_2(\bar{u}_2 - u_2) = 0. \]  
(A29)

- Behavior of \( \lambda_i \)

From (A26) and (A27), \( \dot{\lambda}_1 = \dot{\lambda}_2 < 0 \). Using the same reasoning as in Appendix Section A.1 along with \( \lambda_1(T) = \lambda_2(T) = 0 \), we conclude:

\[ (a) \ \lambda_1 = \lambda_2 \geq 0 \ and \ \lambda_i \ is \ decreasing \ throughout \ 0 \leq t \leq T. \]  
(A30)

- Behavior of \( w_i \)

If \( w_i > 0 \), then \( u_i = \bar{u}_i \) according to (A28) and (A29). From (A24) and (A25), we know that while

\[ w_i > 0, \ w_i = \lambda_i - 2\rho_i\bar{u}_i. \]  
(A31)

From (A31), we know that \( w_i \) and \( \lambda_i \) have the same slope while \( w_i > 0 \). That is,

\[ \dot{w}_i = \dot{\lambda}_i \ while \ w_i > 0. \]  
(A32)

Suppose \( w_i \) is for the player/company \( i \); then, we define \( w_{-i} \) for the other player/company. Therefore, we have: using \( \lambda_1 = \lambda_2 = \dot{\lambda}_i \),

\[ w_i = \lambda_i - 2\rho_i\bar{u}_i = \dot{\lambda}_i - 2\rho_i\bar{u}_i \ while \ w_i > 0, \]  
(A33)

\[ w_{-i} = \lambda_{-i} - 2\rho_{-i}\bar{u}_{-i} = \dot{\lambda}_{-i} - 2\rho_{-i}\bar{u}_{-i} \ while \ w_{-i} > 0. \]  
(A34)

From (A33) and (A34), we know that if

\[ \rho_i\bar{u}_i > \rho_{-i}\bar{u}_{-i}, \ w_i < w_{-i}. \]  
(A35)

Figure A1 depicts the situation specified in (A35).

Now suppose that \( \hat{i}_i \) and \( \hat{i}_{-i} \) exist such as in Fig. A1. Then, we can suggest the dynamics of \( u_i^* \) and \( u_{-i}^* \) at least in a figurative manner: see Table A1.

- How to determine \( \hat{i}_i \)?

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In order to determine the specific \( \hat{t}_i \)s, we assume \( \rho_1 \bar{u}_1 > \rho_2 \bar{u}_2 \) and thus \( w_2 > w_1 \). While

\[
\begin{align*}
w_2 > w_1 > 0, \quad u_1 &= \bar{u}_1 \text{ and } u_2 = \bar{u}_2. \tag{A36}
\end{align*}
\]

From (A26), we derive the following:

\[
\begin{align*}
\dot{\lambda}_1 &= 4(u_1 + u_2) = 4(\bar{u}_1 + \bar{u}_2), \\
\ddot{\lambda}_1 &= 4(\bar{u}_1 + \bar{u}_2)t + k_1, \tag{A37}
\end{align*}
\]

\[
\begin{align*}
\lambda_1 &= 2(\bar{u}_1 + \bar{u}_2)t^2 + k_1 t + k_2.
\end{align*}
\]
Now, from (A33),
\[ w_1 = \dot{\lambda}_1 - 2\rho_1 \dot{u}_1 = 2(\dot{u}_1 + \ddot{u}_2)\dot{t} + k_1 \dot{t} + k_2 - 2\rho_1 \ddot{u}_1. \] (A38)

By rearranging (A38), we obtain:
\[ w_1 = 2(\dot{u}_1 + \ddot{u}_2) \left[ \dot{t} + \frac{k_1}{4(\dot{u}_1 + \ddot{u}_2)} \right]^2 - \frac{k_1^2}{8(\dot{u}_1 + \ddot{u}_2)} + k_2 - 2\rho_1 \ddot{u}_1. \] (A39)

We carry out similar steps with (A19)–(A22).

From (A39), if \( \hat{t}_1 < T \) indeed exists such that \( w_1(\hat{t}_1) = 0 \), then
\[ \hat{t}_1 = \frac{-k_1}{4(\dot{u}_1 + \ddot{u}_2)} \text{ and } -\frac{k_1^2}{8(\dot{u}_1 + \ddot{u}_2)} + k_2 - 2\rho_1 \ddot{u}_1 = 0. \] (A40)

From (A40),
\[ k_1 = \pm 2\sqrt{2(\dot{u}_1 + \ddot{u}_2)(k_2 - 2\rho_1 \ddot{u}_1)}. \] (A41)

From (A37),
\[ k_2 = \dot{\lambda}_1(0). \] (A42)

Using (A40)–(A42) appropriately, we obtain:
\[ \hat{t}_1 = \sqrt{\frac{\dot{\lambda}_1(0) - 2\rho_1 \ddot{u}_1}{2(\dot{u}_1 + \ddot{u}_2)}}. \] (A43)

Now we would like to determine \( \hat{t}_2 \).

From (A36), because
\[ \rho_1 \ddot{u}_1 > \rho_2 \ddot{u}_2, \dot{\lambda}(0) > 2\rho_1 \ddot{u}_1 > 2\rho_2 \ddot{u}_2. \] (A44)

If \( \rho_1 \ddot{u}_1 > \rho_2 \ddot{u}_2 \) and there exists \( \hat{t}_1 \) such that \( \hat{t}_1 < T \), there must exist \( \hat{t}_2 \) such that
\[ \hat{t}_1 < \hat{t}_2 < T. \] (A45)

As in (A32), \( \dot{w}_i = \dot{\lambda}_i \) while \( w_i > 0 \), implying that \( w_1(t) \) and \( w_1(t) \) have the same slope as that of \( \dot{\lambda}(t) \) as long as \( w_i > 0 \). Using this property, along with the relationship in Fig. A2, we derive the following:
\[ \frac{\dot{\lambda}(0) - w_2(0)}{T - t_2} = \frac{\dot{\lambda}(0) - w_1(0)}{T - t_1}. \] (A46)

From (A31), we know
\[ w_i(0) = \dot{\lambda}_i(0) - 2\rho_1 \ddot{u}_i. \] (A47)
Therefore, by substituting (A47) into (A46), we obtain:

\[ \hat{\tau}_2 = T - \frac{\rho_2 \bar{u}_2}{\rho_1 \bar{u}_1} (T - \hat{\tau}_1). \]  \hspace{2cm} (A48)