A Dynamic Decision Making on Outsourcing from Suppliers with Different Innovation Capabilities

Subjects: supply-chain operations (sourcing and contracting, location and capacity);
global coordination (siting and sourcing)

Methodology: optimal control theory, distributed parameter systems approach

November 1997

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In this paper, we develop economic insights into a dynamic outsourcing decision when a firm can use multiple suppliers with different innovation capabilities. Many firms focus primarily on current cost structures when they choose their suppliers or make a plant location decision. We show that focusing on the present cost structures might lead to a short-term solution with possible long-term sub-optimality. In order to achieve long-term effectiveness, the firm must take into account the dynamic evolution of cost structures, which is often carried out by the suppliers' innovating activities. The setting can be handily extended to a market-level analysis, where an individual supplier can be analogized with a pool of suppliers in the particular market. To support our propositions, we employ an optimal control theory modeling and its numerical examples.

1. Introduction

A firm's production process requires inputs in forms of raw materials, intermediate products, and semi-finished inventories, from suppliers (Starr 1993). Suppliers can often provide the firm with innovative inputs that can influence the firm's cost structures (Ittner and Larcker 1997). Because of the fundamental nature of organizational capability, however, different suppliers can have unlike levels of innovation capabilities (Helfat 1994, Peteraf 1993).

In this paper, we explore manufacturing firms' dynamic decision making on outsourcing from suppliers with varying innovation capabilities, assuming that the innovation capabilities affect the dynamic cost structures faced by the firms. In the literature, researchers worked on similar issues in the context of various uncertainties and/or global supply chain management. Hodder and Jucker (1985) investigated a location decision related with outsourcing alternatives, emphasizing price uncertainty (Hodder and Dincer 1986). Cohen and Lee (1988) dealt with the outsourcing issues from a broad supply chain management perspective. Kogut (1985) viewed having multiple sourcing options as retaining operational flexibility in a global market context. With varying emphases on uncertainties, relevant research has been carried out by many researchers (Huchzermeier and Cohen 1996, Pomper 1976, Kogut and Kulatilaka 1994).

In essence, we try to explore an issue of outsourcing capacity allocation when there are alternative suppliers to provide the firm with (from raw, work-in-process, to semi-finished or even finished) materials/products. For analytical simplicity, we primarily deal with a situation involving two suppliers that can offer a full line of intermediate products. In addition, we assume that each supplier's innovative capability is different, and the capability affects the dynamics of cost structure faced by the firm.
One supplier is assumed to offer low prices currently, but does not have much possibility to lower them through innovation over time. In contrast, the other supplies at higher prices, but can offer lower prices over time through innovative activities. This kind of setting readily extends to a global market environment, where a supplier implies a group of suppliers in the industry and the innovation is of autonomous nature at the society and/or industry level.

In the next section, we develop an analytical model to describe the research setting as outlined above. The model is based on an optimal control theory model, a distributed parameter systems approach, which employs two dimensions, time and processing level. Section 3 solves the optimal control problem and derives important economic interpretations. In section 4, we show numerical examples and make more practical inferences from them. Finally, we close the paper with suggesting managerial implications and some issues of discussion for future research.

2. Model Development

In order to explore the research question, we employ an optimal control model, utilizing a distributed parameter systems approach (Butkovskiy 1969, Derzko, et al. 1980). Before developing key variables and parameters in the model, we need to specify the model setting in detail. As Cohen and Lee (1988) suggested, we view a manufacturing system as a network of processing centers/stages and stocking points, e.g., multi-stage processing lines.

2.1. Two-supplier Outsourcing Case

A manufacturing firm on a supply chain requires materials from suppliers external to the manufacturing system (Davis 1993). An entire process to produce a finished good can be decomposed into several stages. Figure 1 shows such an example in which the process is divided into $N$ stages, e.g., $N$ different sequential operations required to finish a product. Albeit a little simplified, it seems to describe an actual production process, in particular a discrete production process, reasonably well. Between two consecutive operations, the manufacturing system has a work-in-process to which its suppliers supply materials. The supplied materials can be anything from raw materials, intermediate inputs, to semi-finished or finished goods, depending on how much processed the materials are at the time they are supplied. We say that certain materials are $ith$-stage processed when they are supplied to a WIP following the $ith$ operation.

Viewing each operation stage as a processing center, we can regard the structure in Figure 1 as either a set of operation stages in a plant or a chain of plants. For the purpose of our research, it does not matter whether the plants are owned by one firm or not as long as a decision making entity is assumed possessing a channel perspective to optimize the entire chain’s performance rather than individual plants’ (Hammond 1992). Vendors supply a wide range of intermediate products to the plants, which in turn can be grouped into intermediate product plants in various processing levels and final product plant (Cohen and Lee 1988). The outsourcing options in Figure 1 can be thought as either an individual supplier that supplies a full range of intermediate
products or a group of individual suppliers in an industry each of which can supply a few intermediate products in different processing stages to the firm. If the second situation is considered, a global operations management setting can be more appropriate since we can conceptually pool different suppliers in a foreign market as an outsourcing option sharing similar innovation capability due to the same infrastructure for technological innovation: that is, we can assume that each outsourcing option is in a different factor market, possibly in a different country. As alluded already, also in a global supply chain management, a key issue is to determine which suppliers should supply each plant for each class of parts, i.e., in our context, which supplier in the chain should supply ‘how much processed’ materials/products to the manufacturing system (Arntzen, et al. 1995).

Figure 1. A Schematic of Outsourcing Decision (Discrete Case)

An extension of the discrete production process is its continuous version (Figure 2). In the continuous-version production process, we denote the level of ‘being processed’ by ‘\( h \)-processed,’ \( 0 \leq h \leq H \). ‘\( h=0 \)’ implies that the supplies are completely raw materials, while ‘\( h=H \)’ implies that they are finished goods. In addition to analytical tractability, this continuous production process model captures most of the key features of the discrete case. In this paper, we develop a model based on this continuous case.

Since we are interested in a dynamic decision making, the decision variables will also involve a time dimension, \( \tau \). The firm’s decision time horizon is \( 0 \leq \tau \leq T \). In the ensuing analysis, we assume that the unit of \( h \) is scaled so as to be equivalent to the time unit. That is, it is assumed that a \( h \)-processed product would have taken \( h \) time units if it had been processed inside the manufacturing system. Consistent with this assumption, \( H \) can be regarded as a throughput/cycle time of the product.

As shown in Figure 2, we consider two suppliers for the modeling purpose. With the possibility to extend to situations involving more than two suppliers, assuming two suppliers does not impose a serious distortion on the analytical procedure. In fact,
based on their empirical study on Japanese supply chain practices, Dyer and Ouchi (1993) suggested that utilizing two suppliers is usually enough to give the firm competitive benefits which could be derived from having more than two suppliers.

Figure 2. A Schematic of Outsourcing Decision (Continuous Case)

2.2. State Variable, Control Variables, and Cost Structures

As described in the last section, the relevant decision variables are of two dimensions, \((\tau, h)\). Assuming there are two full-line suppliers, the model needs two control variables, \(u_i(\tau, h)\)'s, each of which represents the amount of \(h\)-processed materials at time \(\tau\) from supplier \(i\), \(i = 1\) or 2. Accordingly, the state variable, \(x(\tau, h)\), stands for the stock of products \(h\)-processed at \(\tau\) in the firm's manufacturing system. For clarity of analysis, we assume \(x(0, h) = x(\tau, 0) = 0\) as initial conditions without loss of generality. There are two types of cost, purchase cost and internal processing cost. The purchase costs occur when the firm buys materials from the suppliers, i.e., \(P_i(\tau, h), i = 1, 2\), is the 'unit purchase cost' associated with \(u_i(\tau, h)\). The unit cost of internal processing is denoted by \(P_e(\tau, h)\) associated with \(x(\tau, h)\).

**Innovation Capability captured by Learning Capability.** Suppliers' innovation capabilities are captured in \(P_i(\tau, h)\)'s. We assume that each supplier's innovation capability can be well represented by its learning rate (Yelle 1979). Another critical assumption necessary to be consistent with the traditional learning curve theory is that the relationship between time and cumulative production unit is linear, i.e., we can accurately approximate the supplier's cumulative production unit with the time variable. By converting the unit of production into an equivalent time unit, we can uphold this assumption without loss of generality. In a global market context, we might want to consider this learning capability as related with an autonomous innovation at the society or industry level, primarily correlated with a time-related variable (Arrow 1962, Adler and Clark 1991).
In addition, it is reasonable to impose that the more processed the materials are more expensive. Taking into account the two conditions, we can establish the cost structures so that \( \frac{\partial P_i(\tau, h)}{\partial \tau} \leq 0 \) and \( \frac{\partial P_i(\tau, h)}{\partial h} \geq 0 \).

Following an widely-applied economic modeling, we specify the cost structures as follows: \( P_i(\tau, h) = K_i h^\alpha \tau^\alpha \) for supplier 1 and \( P_i(\tau, h) = K_2 h^\beta \tau^\beta \) for supplier 2, where \( K_i \) and \( K_2 \) are constant initial purchase costs not affected by learning capabilities. In addition, we impose \( 0 \leq \alpha \leq 1, \ 0 \leq \beta \leq 1, \ n = \frac{\ln \phi_n}{\ln 2}, \) and \( m = \frac{\ln \phi_m}{\ln 2}, \) where \( \phi_n \) and \( \phi_m \) are learning rates for supplier 1 and 2, respectively. Consistent with the convention, the smaller \( \phi_n (\phi_m) \), the faster the learning. The formulation conforms with a well-accepted learning function (Yelle 1979) and a production function, e.g., Cobb-Douglas production function (de Neufville 1990).

In this paper, we assume that the supplier 2 has higher innovation capability than the supplier 1 does, while its initial unit cost/price is more expensive than its counterpart’s. Accordingly, we further impose the following.

Cost parameter constraints:
1) \( K_1 \leq K_2 \), initial cost is cheaper for the first supplier
2) \( \alpha \leq \beta \), the first supplier’s rate of cost increase as \( h \) increases is not higher than the second supplier’s
3) for \( n = \frac{\ln \phi_n}{\ln 2} \) and \( m = \frac{\ln \phi_m}{\ln 2}, \ 1 \geq \phi_n \geq \phi_m \geq 0, \) i.e., the second supplier’s potential to reduce the future cost through innovation is higher than the first supplier’s.

Finally, we assume that each supplier has its own ‘economic scale’ for the supplies, denoted as \( \bar{u}_i \) and \( \bar{u}_2 \) for supplier 1 and 2, respectively. This economic scale can be regarded as an economic order quantity, and any deviation from the economic scale causes a penalty cost to be paid by the firm. We assume the cost incurs in proportion to the quadratic deviation of the actual purchase amount from the economic scale: let’s denote \( r_i \)’s as such deviation costs for supplier \( i \). Then, the manufacturing firm pays total deviation costs that amount to \( r_1 (u_1 - \bar{u}_1)^2 + r_2 (u_2 - \bar{u}_2)^2 \). The manufacturing firm’s own processing capacity will be denoted as \( \bar{u} \).

**Firm’s Revenues and Salvage Values.** Fully processed products, i.e., \( H \)-processed products, can sell in the market at a unit rate of \( V(\tau) \) at \( \tau \). For simplicity without excessive loss of generality, we assume \( V(\tau) = V_0, \) a constant, and thus the firm’s sales revenue at \( \tau \) amounts to \( V_0 x(\tau, H) \). At the end of the current decision horizon, unfinished products can generate salvage values according to the schedule of \( S(h) \), which is further assumed to be \( S_0 h, \) where \( S_0 \) is constant. Thus, the total salvage value of \( h \)-processed products at \( T \) would be \( S_0 h x(T, h) \).
Table 1. Summary of Decision Variables and Model Parameters

- \( u_i(\tau, h) \): purchase amount of materials \( h \)-processed at \( \tau \) from supplier \( i \)
- \( x(\tau, h) \): stock of intermediate products \( h \)-processed at \( \tau \) in the firm
- \( \bar{u}_i \): economic order quantity for the materials from supplier \( i \)
- \( r_i \): penalty cost of purchase deviation with regard to \( \bar{u}_i \)
- \( \bar{u} \): the firm’s processing capacity
- \( 0 \leq \tau \leq T \): current decision time horizon
- \( 0 \leq h \leq H \): product throughput/cycle time
- \( x(0, h) = x(\tau, 0) = 0 \): initial condition for \( x(\tau, h) \)
- \( P_i(\tau, h) = K_i h^\alpha \tau^\beta \): purchase price from supplier 1
- \( P_2(\tau, h) = K_2 h^\alpha \tau^\beta \): purchase price from supplier 2
- \( P_c(\tau, h) = K_c h^\alpha \tau^\beta \): the firm’s internal processing cost
- \( K_1 \leq K_c \): initial purchase cost; cheaper for supplier 1
- \( \phi_n, \phi_m \): learning rates for supplier 1 and 2, respectively, \( 1 \geq \phi_n \geq \phi_m \geq 0 \)
- \( n = \frac{ln \phi_n}{ln 2} \) and \( m = \frac{ln \phi_m}{ln 2} \): learning coefficients
- \( V(\tau) = V_0 \): sale price/revenue of a finished product
- \( S(h) = S_0 h \): salvage value at \( T \) of an \( h \)-processed intermediate product

2.3. A Complete Model Formulation

In accordance with the formulation in the previous section, this control problem can be set up as follows.

Maximize

\[
Z = \int_0^T \int_0^H \left\{ r_i (u_i - \bar{u}_i)^2 + r_2 (u_2 - \bar{u}_2)^2 + P_i(\tau, h)u_i + P_2(\tau, h)u_2 + P_c(\tau, h)x(\tau, h) \right\} dh \, d\tau
+ V_0 \int_0^T x(\tau, H) d\tau + S_0 \int_0^T hx(T, h) dh
\]

\[
= \int_0^T \int_0^H \left\{ r_i (u_i - \bar{u}_i)^2 + r_2 (u_2 - \bar{u}_2)^2 + K_i h^\alpha \tau^\beta u_i + K_2 h^\alpha \tau^\beta u_2 + K_c h^\alpha \tau^\beta x(\tau, h) \right\} dh \, d\tau
+ V_0 \int_0^T x(\tau, H) d\tau + S_0 \int_0^T hx(T, h) dh
\]......(P.1)

Subject to

\[
\frac{\partial x(\tau, h)}{\partial \tau} = -\frac{\partial x(\tau, h)}{\partial h} + u_i(\tau, h) + u_2(\tau, h) \]......(P.2)

\[
u_i + u_2 \leq \bar{u} \]......(P.3)

\[
0 \leq u_i \]......(P.4)
0 \leq u_2 \quad \ldots \quad (P.5)

The constraint (P.2) can be derived in the following way. Since \( h \) is scaled so as to be comparable with the time unit and \( x(\tau, h) \) represents the stock of products \( h \)-processed at \( \tau \), \( x(\tau + \Delta \tau, h) \) is comprised of three elements, \( x(\tau, h - \Delta \tau) \), i.e., one element due to the internal processing of products \( (h - \Delta \tau) \)-processed at \( \tau \), and \( u_i(\tau, h)\Delta \tau \) and \( u_i(\tau, h)\Delta \tau \), i.e., the \( h \)-processed materials from the two suppliers at the instantaneous moment. Therefore,

\[
x(\tau + \Delta \tau, h) = x(\tau, h - \Delta \tau) + u_1(\tau, h)\Delta \tau + u_i(\tau, h)\Delta \tau. \quad \ldots \quad (P.6)
\]

By dividing both sides of (P.6) with \( \Delta \tau \) and let \( \Delta \tau \to 0 \), we obtain (P.2).

The associated Hamiltonian is, after related Lagrange multipliers are assigned,

\[
H_\varepsilon = -\left\{ r_1(u_1 - \bar{u}_1)^2 + r_2(u_2 - \bar{u}_2)^2 + K_i h^\nu \tau^\nu u_1 + K_i h^\nu \tau^\nu u_2 + K_i h^\nu x_\nu \right\} + \\
\lambda (-\frac{\partial x}{\partial h} + u_1 + u_2) + \mu (\bar{u} - u_1 - u_2) + \mu_1 u_1 + \mu_2 u_2, \quad \ldots \quad (P.7)
\]

where \( \lambda \) is the costate variable for (P.2), and \( \mu \), \( \mu_1 \), and \( \mu_2 \) are Lagrangian multipliers for (P.3), (P.4), and (P.5), respectively.

Assuming that the firm can sell all of its products in the market, i.e., the market has a sufficient demand for the finished products, it is reasonable to assume that \( u_1 \) and \( u_2 \) are always positive. Therefore, for the remaining analysis, we can safely ignore the two nonnegativity constraints, (P.4) and (P.5). One way to make sure that our assumption is valid is to obtain optimal solutions with the proposed relaxation, and then confirm that the \( u_1^*(t) \)'s are indeed positive for \( 0 \leq t \leq T \). If the assumption cannot be confirmed as suggested, then we should reinstate the relaxed constraints.

In addition to relaxing the nonnegative constraints, we consider two different cases with respect to (P.3). For the first case, we assume that the firm does not have any limit on its capacity to process the intermediate products (or, materials/supplies), i.e., the constraint (P.3) can be dropped. For the other case, (P.3) remains as a constraint. In the second case, we again consider two separate situations. The first situation is when the firm can sell its processing (i.e., production) capacity in the market, and the second when it does not do so. Each of these cases will be revisited in the following sections.

3. Economic Analysis and Implications

Before solving the formulated control problem, we need to consider one more complication stemming from dealing with a 2-dimensional space, i.e., \((\tau, h)\)-space, rather than 1-dimensional like in most other simple control models. One example of such complication is that since each product needs to be processed for \( H \) time period to generate a full market value and the current time horizon is \( T \), the materials (e.g.,
intermediate products) processed less than \( \tau + (H - T) \), i.e., \( h < \tau + (H - T) \) or \( T - H < \tau - h \), and supplied after time \( T - H \) can not be expected to be finished by \( T \). In other words, the values generated by those materials bought after \( T - H \) must differ from others' that contribute full market values.

For analytical accuracy, we need to divide the \((\tau, h)\)-space into three regions, taking into account \( H \) and \( T \). The division results in regions \( O_1 \), \( O_2 \), and \( O_3 \) with two internal boundaries \( R_1 \) and \( R_2 \), as in Figure 3. Greater detail must be referred to references (Butkovskiy, 1969, Derzko, et al. 1980).

Figure 3. Division of \((\tau, h)\)-space

3.1. Capacity-unconstrained Process

For this capacity-unconstrained process case, we use a relaxed Hamiltonian, omitting the capacity limit constraint (P.3) from (P.7),

\[
H_2 = -r_1(u_1 - \bar{u}_1)^2 + r_2(u_2 - \bar{u}_2)^2 + K_i h^\alpha \tau^\alpha u_1 + K_i h^\beta \tau^\beta u_2 + K_i h^\gamma \tau^\gamma \]

\[
\lambda \left( - \frac{\partial \lambda}{\partial h} + u_1 + u_2 \right).
\]

......(P.8)

For \( u_i \), we set \( \frac{\partial H_2}{\partial u_i} = - \left\{ 2r_i(u_i - \bar{u}_i) + K_i h^\alpha \tau^\alpha \right\} + \lambda = 0 \).

Thus, \( u_i^* = u_i + \frac{1}{2r_i}(\lambda - K_i h^\alpha \tau^\alpha) \). ..................................(P.9)

Likewise, we can obtain for \( u_2 \), \( u_2^* = u_2 + \frac{1}{2r_2}(\lambda - K_i h^\beta \tau^\beta) \). ..................................(P.10)

To ensure we have a maximized solution, we take \( \frac{\partial^2 H_2}{\partial u_i^2} = -2r_i \), which is always non-positive as long as \( r_i \geq 0 \), as assumed throughout the analysis. Once \( u_i^* \)'s are obtained, we can calculate optimal state and costate variables. As mentioned before,
the detailed structures of $x^*$ and $\lambda^*$ depend on the $(\tau, h)$-space division. Detailed analytical procedures to calculate (P.11) and (P.12) can be referred to Butkovskiy (1969). The optimal $x^*$'s can be determined as follows.

$$x^*(\tau, h) = \begin{cases} \int_0^r [u_1(q, h - \tau + q) + u_2(q, h - \tau + q)] dq & (\tau, h) \in O_1 \\ \int_0^h [u_1(\tau - h + q, q) + u_2(\tau - h + q, q)] dq & (\tau, h) \in O_2 \cup O_3 \end{cases} \quad \text{......(P.11)}$$

$$\lambda^*(\tau, h) = \begin{cases} V_0 - \int_0^h K_c \rho^d d\rho & (\tau, h) \in O_1 \cup O_2 \\ (T - \tau + h) S_0 - \int_0^{\tau - r + h} K_c \rho^d d\rho & (\tau, h) \in O_3 \end{cases} \quad \text{......(P.12)}$$

After plugging (P.12) into (P.9) and (P.10), the $u_i^*$'s can be used to attain $x^*$'s from (P.11). Using the optimal values, $u_i^*$ and $x^*$, in (P.1), we can finally determine the optimal objective value of the optimal control problem, $Z^*$. Although computing $x^*$ and $Z^*$ is also interesting, we will focus more on the behavior of $u_i^*$ since our primary research question is about the dynamics of control variables, i.e., optimal outsourcing decisions from the suppliers over time.

An interesting question is when the firm starts outsourcing more from the innovative supplier, i.e., supplier 2 in this paper, than from the other. Suppose that such timing is $\bar{\tau}$. Then, $\bar{\tau}$ is the smallest $\tau$ that can satisfy (P.13).

$$\bar{u}_1 + \frac{1}{2r_1}(\lambda - K_i h^d \tau^n) \leq \bar{u}_2 + \frac{1}{2r_2}(\lambda - K_i h^d \tau^n) \quad \text{......(P.13)}$$

For simplicity, assume that $\bar{u}_i = \bar{u}_2$ and $r_1 = r_2$, i.e., the two suppliers have the same 'efficient production scale' and same penalty cost for the deviation from the efficient scale. With this additional assumption, (P.13) can be simplified as follows.

$$\tau \geq \left[ \frac{K_2}{K_1} \right]^{1-n-m} h^{\alpha-a} h^{n-m} \quad \text{and therefore} \quad \bar{\tau} = \left[ \frac{K_2}{K_1} \right]^{1-n-m} h^{\alpha-a} h^{n-m}, \text{given } h. \quad \text{......(P.14)}$$

We just proved the following theorem.

**Theorem 1. Timing to Change the Primary Supplier.** Suppose the two suppliers have the same efficient economic scale and the associated deviation cost. With the same formulation as in (P.1-12), the firm's outsourcing amount from the supplier
with more innovation capability starts surpassing that from the other at
\[ \tau = \left( \frac{K_2}{K_1} \right)^{\frac{1}{m}} \frac{h_a}{h^a}, \text{ given } h. \]

The implications of \( \tau \) include:

i) As the second supplier's innovative capability improves, i.e., its learning rate, thus -m, increases, \( \tau \) decreases. It takes less time for the second supplier to become a major supplier, the more capable it is in innovation.

ii) As the purchased materials are more advanced, i.e., more highly processed and thus with larger \( h \), \( \tau \) increases. In other words, it takes more time for the innovative supplier to become a major supplier with regard to more processed (intermediate) materials/products.

3.2. Capacity-constrained Process

For the capacity-constrained process, we reinstate the constraint (P.3), \( u_1 + u_2 \leq \bar{u} \). We further assume that the firm's unused processing capacity can be sold in the market at the rate of \( \kappa \). With this information, we can set up an optimal control problem.

Maximize
\[ Z = \int_{0}^{\tau} \left[ \left( r_1(u_1 - \bar{u}) \right)^2 + r_2(u_2 - \bar{u})^2 + K_1h^a \tau^x u_1 + K_2h^b \tau^w u_2 + K_3h^c x - \kappa(u - u_i - u_2) \right] dh \right] d\tau + V_0 \int_{0}^{\tau} x(\tau, H) d\tau + S_0 \int_{0}^{\tau} x(T, h) dh \]

...(P.15)

with (P.2) and (P.3) as constraints.

The related Hamiltonian is,
\[ H_2 = -\left\{ r_1(u_1 - \bar{u})^2 + r_2(u_2 - \bar{u})^2 + K_1h^a \tau^x u_1 + K_2h^b \tau^w u_2 + K_3h^c x - \kappa(u - u_i - u_2) \right\} \]
\[ + \lambda \left( -\frac{\partial x}{\partial t} + u_1 + u_2 \right) + \mu (u - u_i - u_2). \]

...(P.16)

As in the previous sections, for an optimal solution we take
\[ \frac{\partial H_2}{\partial u_i} = -\left\{ 2r_1(u_i - \bar{u}_i) + K_1h^a \tau^x \right\} + \lambda - \kappa - \mu = 0. \]

Thus, \( u_i^* = \bar{u}_i + \frac{1}{2r_1} (\lambda - \kappa - \mu - K_1h^a \tau^x) \)

...(P.17)
and following the same procedure,

\[ u_2^* = \bar{u} + \frac{1}{2r_2} (\lambda - \kappa - \mu - K_2 h^a \tau^m) \]  

......(P.18)

Because of the additional constraint, we have the following set of Kuhn-Tucker conditions as well:

\[ \mu \geq 0, u_1 + u_2 \leq \bar{u}, \text{ and } \mu (\bar{u} - u_1 - u_2) = 0. \]  

......(P.19)

We will consider (P.19) separately in either \( u_1 + u_2 < \bar{u} \) or \( u_1 + u_2 = \bar{u} \).

3.2.1. Case when \( u_1 + u_2 < \bar{u} \)

If \( u_1 + u_2 < \bar{u} \), then \( \mu = 0 \) from (P.19).

Thus, \( u_1^* = \bar{u} + \frac{1}{2r_1} (\lambda - \kappa - K_1 h^a \tau^m) \) and \( u_2^* = \bar{u} + \frac{1}{2r_2} (\lambda - \kappa - K_2 h^a \tau^m) \)

......(P.20)

Given the present problem formulation, the firm might want to know what is the minimum value of \( \kappa \) that makes it more profitable for the firm to sell its unit processing capacity in the market rather than producing a product within the manufacturing process. For example, such \( \kappa \) might be the price the firm charges other firms for processing a unit product for them.

**Theorem 2. Minimum Value of \( \kappa \) for Selling Production Capacity.** Suppose \( r_1 = r_2 \) and \( \bar{u}_1 + \bar{u}_2 = \bar{u} \), i.e., the two suppliers have the same deviation penalty and the firm has a balanced processing capacity equivalent to the sum of the two suppliers’ efficient economic scale’ capacities. Let \( \bar{\kappa} \) be the minimum value of \( \kappa \) at \((\tau, h)\) that makes it more attractive to sell the production capacity. Then, \( \bar{\kappa} \) is determined by the average net contribution made by the intermediate materials and products provided by the two suppliers. More specifically, \( \bar{\kappa} = \frac{1}{2} \left( (\lambda - K_1 h^a \tau^m) + (\lambda - K_2 h^a \tau^m) \right) = \lambda - \frac{1}{2} (K_1 h^a \tau^m + K_2 h^a \tau^m) \).

**Proof.** We can obtain \( \bar{\kappa} \) by evaluating \( \kappa \) that holds up \( u_1 + u_2 < \bar{u} \). That is, \( \kappa \) that satisfies (P.21).

\[ u_1 + u_2 = \bar{u} + \frac{1}{2r_1} (\lambda - \kappa - K_1 h^a \tau^m) + \bar{u} + \frac{1}{2r_2} (\lambda - \kappa - K_2 h^a \tau^m) < \bar{u} \]  

......(P.21)

By rearranging (P.21) with regard to \( \kappa \), we have

\[ \frac{2r_1 r_2 (\bar{u} + \bar{u} - \bar{u})}{r_1 + r_2} + \frac{r_1 r_2}{r_1 + r_2} \left[ \frac{\lambda - K_1 h^a \tau^m}{r_1} + \frac{\lambda - K_2 h^a \tau^m}{r_2} \right] < \kappa \]  

......(P.22)
By assumption, \( r_1 = r_2 \) and \( \bar{u}_1 + \bar{u}_2 = \bar{u} \). Accordingly, (P.22) becomes

\[
\frac{1}{2} \left[ (\lambda - K_1 h^a \tau^n) + (\lambda - K_2 h^b \tau^n) \right] < \kappa. \tag{P.23}
\]

That is, if the market value of a unit capacity is greater than the average contribution made by producing a final product with resources from the two suppliers at \((\tau, h)\), it can be more profitable for the firm to sell its unit processing capacity rather than producing a product within the manufacturing process.

If \( \kappa = 0 \), \( u^* \) would be exactly the same as in the case of capacity-unconstrained process.

3.2.2. Case when \( u_1 + u_2 = \bar{u} \)

If \( u_1 + u_2 = \bar{u} \) and therefore \( \mu \geq 0 \), we can simplify the Hamiltonian by substituting \( u_1 \) with \( u_2 = \bar{u} - u_1 \). Now the simplified Hamiltonian becomes

\[
H_2 = -\left\{ r_1 (u_1 - \bar{u}_1)^2 + r_2 (\bar{u} - \bar{u}_2 - u_1)^2 + K_1 h^a \tau^n u_1 + K_2 h^b \tau^n (\bar{u} - u_1) + K_2 h^2 \tau^n \right\} + \\
\lambda \left( \frac{\partial x}{\partial h} + \bar{u} \right) \quad \tag{P.24}
\]

with only one control variable \( u_1 \). To derive an optimal solution, we take

\[
\frac{\partial H_2}{\partial u_1} = -\left\{ 2r_1 (u_1 - \bar{u}_1) - 2r_2 (\bar{u} - \bar{u}_2 - u_1) + K_1 h^a \tau^n - K_2 h^b \tau^n \right\} = 0,
\]

\[
\therefore \quad u_1^* = \frac{r_1 \bar{u}_1 + r_2 (\bar{u} - \bar{u}_2)}{r_1 + r_2} + \frac{K_1 h^a \tau^n - K_2 h^b \tau^n}{2(r_1 + r_2)} \quad \text{and} \quad u_2^* = \bar{u} - u_1^*. \tag{P.25}
\]

4. Numerical Examples and Inferences

In the previous sections, we developed and analyzed the optimal control model describing our research setting. The analysis was useful to generate economic insights into an optimal dynamic outsourcing decision. In order to enrich the insights, we present a few numerical examples with the parameter values in Table 2. The example results are graphed in Figure 4 to 7. We found that given the particular parameter values, the four figures represent most situations one can encounter for the dynamic decision making.

Figure 4 and 5 contrast the optimal dynamic decisions of the two cases, capacity-unconstrained and capacity constrained, over time given \( h=2 \). On the other hand, Figure 6 and 7 compare the optimal decisions with respect to \( h \) at a given time, \( \tau = 15 \). Other situations with varying \( h \) and \( \tau \) showed qualitatively similar patterns.
Table 2. Parameters for Numerical Examples

<table>
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<th>$T$</th>
<th>$H$</th>
<th>$V_0$</th>
<th>$S_0$</th>
<th>$K_C$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\phi_m$</th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$\bar{u}$</th>
<th>$\bar{u}_1$</th>
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<td>1</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 4. Dynamic Outsourcing Decision (over $r$, capacity-unconstrained, $h=2$)

Figure 4 depicts a dynamic decision to purchase intermediate materials/products ($h=2$ processed) from supplier 1, $u_1^*$, with no learning effect, and three different scenarios for $u_2^*$ with $\phi_m = 0.80, 0.85, \text{and} 0.90$, denoted as $u_3(0.80)$, $u_4(0.85)$, and $u_5(0.90)$, respectively. Although $u_1^*$ was much larger than $u_2^*$ in the early period, $u_2^*$ becomes gradually surpassing $u_1^*$, the exact timing being determined by the learning rates according to Theorem 1. This change of primary supplier occurs since the more innovative supplier can offer a gradually reduced price schedule as it innovates more or learns faster over time. We can see that the faster the learning (i.e., the smaller $\phi_m$), the earlier the timing for $u_1^*$ to surpass $u_2^*$. Since there is no limit on the firm’s processing capacity, there is no explicit tradeoff between $u_1^*$ and $u_2^*$. The tradeoff becomes unambiguous for the capacity-constrained process case in Figure 5. In Figure 4, $u_1^*$ stays constant until $r = 27$ at which it shows a sharp decrease due to the fact that the finished goods require $H=3$ processing time. Unlike in Figure 4, $u_1^*$ displays a sharply at first and then gradually decreasing pattern as $u_1^*$ increases in Figure 5. That is, increases in $u_1^*$ must be compensated with equivalent decreases in $u_1^*$. Figure 5
shows two dynamic paths of each $u^*$ with $\phi_m = 0.80$ and 0.90, and $\phi_n = 1$: since $\phi_n$ is constant, the figure denotes $u_*(0.8)$ when $\phi_m = 0.80$ and $u_*(0.9)$ when $\phi_m = 0.90$.

Figure 5. Dynamic Outsourcing Decision (over $\tau$, capacity-constrained, $\bar{u} = 10$, $h = 2$)

Figure 6 and 7 depict the optimal dynamics with regard to $h$ at a given time $\tau = 15$. As observed previously in Figure 5, Figure 7 shows a clear tradeoff between $u_1^*$ and $u_2^*$ whereas Figure 6 displays a rather intriguing convex shape. We can see that with the parameter values in Table 2, the capacity-unconstrained case indicates the firm needs to purchase 'more processed' materials from the suppliers: it partly implies that processing externally by the suppliers costs less than processing internally by the firm itself. $u_1^*$ and $u_2^*$ are almost the same when $\phi_m = 0.90$, i.e., there is little difference in innovative capability between the two suppliers.

From the numerical examples presented in Figure 4 to 7, we can make some observations.

- An optimal dynamic decision pattern can be significantly affected by the difference in the suppliers' innovation capabilities.

- When there is a limit on the firm's processing capacity, optimal decisions can show a dramatic pattern when the terminal time approaches mainly because the product throughput time is finite. We can observe such examples in Figure 4. In many real situations, however, $H$ can be very small in comparison with $T$, and thus one can ignore a terminal behavior of an optimal outsourcing decision.
- We can construct other optimal decision patterns under different scenarios with varying parameter values. The main thrust of these numerical examples is to demonstrate the resilience of the optimal control model not only as an analytical tool, but also as a more intuitive mechanism to develop realistic scenarios. Much richer economic insights can be derived from similar exercises.
5. Managerial Implications and Discussion

We have two primary objectives in this paper. The first is to suggest a novel way to use an existing analytical tool to solve a managerial problem. We developed an optimal control model, based on a distributed parameter systems approach, to analyze an outsourcing decision problem. Unlike most other simple control models, this model was able to examine dynamics involving two dimensions, time and processing level. Investigating the 'processing dimension' in addition to the time dimension enabled us to answer more complex dynamic questions than other one dimensional control models would. The second objective is to generate economic insights into a critical role played by suppliers' innovation capability in determining a long-term strategic optimality. We employed numerical examples to support our propositions established with the optimal control model and its analysis.

From the analysis results and numerical examples, we derived important managerial implications. An optimal dynamic outsourcing decision can be significantly influenced by the difference in the suppliers' innovation capabilities. This implies that if a firm makes an outsourcing decision based only on short-term competitive factors (such as currently low material/labor costs) without taking into account more dynamic factors (like supplier's innovation capability), the resulting decision can be far off from an optimal one from a long-term strategic perspective.

As the numerical examples further indicate, the analysis also enables the firm to predict the shifting point at which it changes its primary supplier from less innovative to more innovative one. This timing can be an important decision making element when the firm makes long-term contracts with the suppliers possessing different innovation capabilities. If the firm knows when it will shift the supplier's status, it can design a contract that takes into account such future changes in advance so as to minimize the future penalty costs. But, this shifting differs from the 'production shifting' for an arbitrage purpose as suggested by Kogut (1985) since it is motivated not by short-term competitive responses, but by a long-term strategic learning/innovation capability. A global setting pertains well to the research context since in a global market an industry in one country can show a distinct innovation capability that is clearly different from another in a different country.

We propose the analytical model can be used as a communication/planning tool and/or mechanism for a strategic formulation. With appropriate adjustments, the model developed in this paper can be applied to other managerial situations as well. For instance, one can explore a human resource planning problem with this model; \( H \) can now be regarded as a required education period, and the suppliers are two sources of new and/or skilled employees. There is still room for further improvement. Our assumption that \( h \) has a unit equivalent to the time unit can be sometimes very restrictive. In addition to relaxing the assumption, we might need to explore possible ways to generalize the implications by freeing another limited supposition related with the nature of \( u \)'s: \( u \)'s were assumed to be anything from raw, intermediate, to semi-finished materials. Although theoretically possible, in some industries it can be hard to find a supplier that is willing to provide such a wide range of materials to a firm. Future research also needs to take a look at analytical modeling allowing more than two suppliers simultaneously. In order to simulate a specific real-world case, varying
values of decision parameters can be tried in numerical examples. With a few limits on its generalization as mentioned so far, the analytical model we developed in this paper seems to shed constructive light on an optimal outsourcing decision making.

References


