Phasor Transformation and its Application to the DC/AC Analyses of Frequency Phase-Controlled Series Resonant Converters (SRC)

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Abstract—A new modeling technique based on phasor transformation that provides the unified model of series resonant converters (SRC) is proposed. The proposed approach gives explicit and simple equations with fruitful physical insight. In the case that the switching frequency deviates from the resonant frequency, the SRC is modeled as the first order, and in the case of resonance the SRC is modeled as the second order. It is shown that the frequency band in which the SRC is modeled as the second order is very narrow in practice. The time constant, small signal gains, and system order are highly depend on the switching frequency, load resistor, and output capacitor.

I. INTRODUCTION

Many researchers [1]–[7] have shown that the well-known resonant converters are nonlinear switching systems which are described as the very complex and implicit equations. The complete dc and ac analyses of switching frequency controlled SRC are found in [1]–[2]. Several useful curves based on dc analysis, which can be utilized in designing the SRC, are proposed in [3]. The small signal circuit models for ac analysis are suggested in [4]–[6].

It is worthy to note that the experimental or the simulational output voltage waveforms are very similar to either the first or the second order system responses in a certain operating range and the dc voltage gain curves are similar with resistance-inductance-capacitance (RLC) filter frequency characteristic curves. Therefore, in view of practical applications, it is an important work to find a simplified equivalent circuit of an SRC which can show the simple behaviors though not so accurate for the efficient analysis and design.

Some solutions to the above problem have been given in [8]–[11]. The diode rectifier and output filter is substituted by an equivalent resistor [8]–[9]. So the analysis is drastically simplified by this substitution and the resemblance of resonant converters with RLC filters is well explained in this way. This technique is, however, valid only for the dc analysis and is no longer valid for the ac analysis. On the other hand an approximated small signal model is suggested in a recent paper with good intuition [10].

It is thought that the previous models have at least one of the following disadvantages in view of the practical application of them.

1) The equations are not of explicit form.
2) The model is too complex to get physical insights from it.
3) Any rigorous explanation for that the system order changes according to the switching frequency is not provided.
4) DC/ac analyses of frequency [2]/phase [11] control methods are not performed by a unified principle.

In this paper, a new systematic approach based on phasor transformation which gives an equivalent time-invariant circuit that is described as the explicit equations, is suggested. Also the first and second order models of SRC are given. It is further shown that the simulation results based on the proposed model agree with the experimental results.

II. PHASOR TRANSFORMATION

It is assumed that the SRC shown in Fig. 1 is composed of ideal switches and ideal circuit components. Also it is assumed that the converter operates in the continuous conduction mode (CCM) and that the quality factor is sufficiently high enough for the sufficient reduction of harmonic current in the LC resonant tank [1]–[2].

And the switching frequency which is lower than a half of the resonant frequency is not considered due to the difficulties in dealing with the harmonic resonances such as 3rd, 5th switching harmonics. Though these assumptions arise some error in the analysis result and reduce the generality, these are still useful for the practical application of the SRC models.

A. Equivalent Transformer Circuit

Since it is proved that a switch set is generally a time-varying transformer [14], an equivalent circuit of Fig. 1 can be drawn as shown in Fig. 2 where the switch sets are substituted by their corresponding transformers. The turn-ratios are just the same as the switching functions as shown in Fig. 3(c). Here the influence of harmonics is
Fig. 1. Series resonant converter (SRC).

Fig. 2. An equivalent transformer circuit.

Fig. 3. Switching functions. (a) Original $s_1^*(t)$. (b) Original $s_2^*(t)$. (c) Fundamentals of (a) and (b).

ignored since the magnitudes of harmonics are small and these small harmonics are filtered by the LC resonant circuit. The analysis is greatly simplified by this approximation. It should be, however, reminded that this may not be true when the quality factor is small or the switching frequency is much lower than the resonant frequency.

The fundamental component, $s_1(t)$, $s_2(t)$ of the original switching functions are represented as

$$s_1(t) = \frac{4}{\pi} \cos \phi \cdot \cos \omega_1 t$$

$$s_2(t) = \frac{4}{\pi} \cos (\omega_1 t - \theta)$$

where $\phi$ is the controlled phase, $\theta$ is the diode switch phase delay, and $\omega_1$ is the switching frequency.

Since the $s_1(t)$ is a function of inductor current, the SRC is described by nonlinear equations and so we cannot directly use the Laplace transformation technique for the analysis of the SRC.

It is noted that no switching element is appeared in Fig. 2. The system equation is now analytical, which is a very important basic condition for the following linear transformation.

B. Phasor Transformation

It is well known that the D-Q transformation is powerful in the analysis of poly-phase ac systems. The basic principle of the D-Q transformation is to find a stationary circuit for a given rotational circuit, which makes it greatly easy to find the envelopes of rotational variables of a rotational circuit [14]. Unfortunately the powerful D-Q transformation can not be applied to the single phase ac circuits as well as SRC. Up to now, there has been no appropriate transformation which is directly applicable to the analysis of the single phase ac systems.

In this section the way to obtain a stationary circuit for a given rotational one is suggested. Since the conventional phasor represents the magnitude and the phase of a sinusoidal signal in the steady state it can not represent the signal in the transient state. So a modified phasor which can represent any sinusoidal signal is considered:

$$x(t) = \text{Re} \left[ \sqrt{2} x(t) e^{j \omega t} \right].$$

where the $x(t)$ indicates the complex time-varying variable. None of the $x(t)$ and $\omega$ need to be sinusoidal or constant, however, the $x(t)$ can be non-sinusoidal function only when the $x(t)$ is sinusoidal with the frequency of $\omega$. In the steady state, $x(t)$ becomes just the conventional phasor. By applying (2) to the time-varying circuit variables appeared in the rotational circuit of Fig. 2, a stationary circuit is obtained.

The five basic circuit elements of the single phase ac system, which are inductor, capacitor, transformer, resistor and source, are phasor-transformed, respectively, as follows.

Inductor Phasor Transformation

The procedure of the phasor transformation for the inductor is shown in Fig. 4. The rotational circuit equation is

$$L \frac{di_L}{dt} = v_L.$$  

The phasor transformations of $i_L$ and $v_L$ are

$$i_L = \text{Re} \left[ \sqrt{2} i_L e^{j \omega t} \right]$$

and

$$v_L = \text{Re} \left[ \sqrt{2} v_L e^{j \omega t} \right].$$


respectively. Applying (4) to (3) results in

\[ L \frac{d}{dt} \left[ \text{Re} \left( \sqrt{2} i e^{j\omega t} \right) \right] = \text{Re} \left[ \sqrt{2} v e^{j\omega t} \right], \]

or

\[ L \text{Re} \left( \frac{d}{dt} i e^{j\omega t} + j\omega i e^{j\omega t} \right) = \text{Re} \left( L \frac{d}{dt} i e^{j\omega t} + j\omega L i e^{j\omega t} \right) = \text{Re} \left( v e^{j\omega t} \right). \]

Equation (5) is just equivalent to

\[ L \frac{d}{dt} i + j\omega i = v. \]  

The equivalence of (5) with (6) can be proved as follows.

**Theorem**

For any \( x, y \) and \( \omega_i \neq 0 \)

\[ \text{Re} \left[ xe^{j\omega t} \right] = \text{Re} \left[ ye^{j\omega t} \right] \]

if and only if \( x = y \).

**Proof**

\[ \text{Re} \left[ xe^{j\omega t} \right] = \text{Re} \left[ |x| \left( e^{i\text{arg}(x)} e^{j\omega t} \right) \right] \]

\[ = |x| \text{Re} \left[ e^{i\text{arg}(x)} + j\omega t \right] \]

\[ = |y| \text{Re} \left[ e^{i\text{arg}(y) + j\omega t} \right] \]

\[ \leftrightarrow |x| = |y| \text{ and } \text{arg} (x) = \text{arg} (y) + 2\pi n, \quad n = 0, 1, 2, \cdots \]

\[ * x = y : Q.E.D. \]

It is necessary to make (6) be of circuit form for the benefit of well established circuit analyses techniques as shown in Fig. 4(d). There exists an imaginary resistor, whose meaning is just conventional inductor impedance in the steady state. However it should not be misunderstood that Fig. 4(d) is no longer valid in the transient state. The imaginary resistor does not vanish even though it is in the transient state, therefore it's worthy to distinguish this from a conventional reactance.

As can be seen from Fig. 4 (b)-(c) and (e)-(f) the stationary circuit can be used to find the envelopes of the sinusoidal waveforms of the rotational circuit. Since the imaginary resistor does not serve the damping, the response needs not to be exponential; in this example there is no time delay.

**Capacitor Phase Transformation**

By a similar procedure with the inductor case, the rotational circuit equation of Fig. 5(a),

\[ C \frac{dv_c}{dt} = i_c \]  

is changed by the phasor transformation to

\[ C \frac{dv_c}{dt} + j\omega C v_c = i_c. \]

The circuit reconstruction of (8) is shown in Fig. 5(b).

**Transformer Phasor Transformation**

The voltage source single phase inverter as shown in Fig. 6(a) is composed of stationary and rotational parts. Hence the phasor transformation is taken for the rotational part only as

\[ v_o = \text{Re} \left[ \sqrt{2} v e^{j\omega t} \right] \]

\[ s(t) = \text{Re} \left[ \sqrt{2} s e^{j\omega t} \right] \]

\[ i_v = \text{Re} \left[ \sqrt{2} i e^{j\omega t} \right]. \]

On the other hand the following relations exist in the transformer of Fig. 6(a):

\[ v_o = s(t) v_i \]

\[ i_v = s(t) i_v. \]

Applying (9) to (10) results in

\[ v_o = \text{Re} \left[ \sqrt{2} v e^{j\omega t} \right] = \text{Re} \left[ \sqrt{2} s v e^{j\omega t} \right] \]

\[ i_v = \text{Re} \left[ \sqrt{2} s e^{j\omega t} \right] \cdot \text{Re} \left[ \sqrt{2} i e^{j\omega t} \right] \]

\[ = \text{Re} \left[ s^2 i_v \right] + \text{Re} \left[ si_v e^{2j\omega t} \right]. \]

The symbol, asterisk (*) represents the complex conjugate.

As previously proved, (11a) becomes

\[ v_o = sv_v. \]
And (11b) can be rewritten as follows considering that the ac current of it does not contribute to any voltage change in the source or any energy transfer. Furthermore the frequency of the ac current is so high that this is thoroughly filtered by the adjacent circuits. Therefore,

\[ i_2 = s^*i_1, \]  

where a dummy variable \( i_2 \) whose imaginary part may be arbitrary is introduced as

\[ i_2 = \text{Re}[i_1]. \]  

(12c) may not be true for the parallel resonant converter case where the imaginary current can not be set to be arbitrary.

The circuit reconstruction based on (12) is shown in Fig. 6(b), where an unusual complex turn-ratio is found.

**Resistor Phasor Transformation**

The phasor transformation of a resistor is straightforward since

\[ v_R = \text{Re}[\sqrt{2}v_Re^{j\omega t}] = i_R\bar{R} = \text{Re}[\sqrt{2}i_Re^{j\omega t}R] \]  

or

\[ v_R = i_R\bar{R}. \]  

(13b)

**C. Phasor Transformed SRC**

Since the stationary circuits for individual rotationary circuits are now available, the stationary SRC for the rotationary SRC of Fig. 2 can be drawn as shown in Fig. 8 using the results of Figs. 4–7. Auto-transformers are used now since the turn-ratios never exceed unities. They are determined from (1) and (9b) as

\[ s_1 = \frac{2\sqrt{2}}{\pi} \cos \phi \]  

\[ s_2 = \frac{2\sqrt{2}}{\pi} e^{-i\beta}. \]  

Now the circuit shown in Fig. 8 is no longer time-varying and all the variables in it are just equivalent to those in Fig. 2 with respect to the phasor transformation, (2).

It took many steps and discussions in finding this time-invariant circuit, however this procedure can be drastically simplified only if one is get used to this transformation; Fig. 8 can be directly drawn from Fig. 2 or Fig. 1 without any manipulation of equations in case that one knows Figs. 4–7.

**III. THE ANALYSIS OF SRC**

**A. DC Analysis**

From the phasor transformed circuit of Fig. 8, the dc circuit which is utilized in the dc analysis can be obtained by removing the inductor and the capacitors. This stationary circuit should not be confused with the rotationary circuit; the inductors of Fig. 8 are not substituted by the ac reactances since Fig. 8 is not the rotationary circuit.

Since the input, output voltages of the two circuits of Figs. 2 and 9 are identical (note that phasor transformation is not applied to these dc side variables), the dc gain of Fig. 9 is the same as that of Fig. 2. Then the dc gain is calculated from Fig. 9 as

\[ G_v = \frac{V_o}{V_i} = \frac{\cos \phi}{\sqrt{1 + Q^2 \left( \frac{\omega_f}{\omega_i} - \frac{\omega_i}{\omega_r} \right)^2}}, \]  

where

\[ Q = \frac{\pi \omega_L}{8 R_L}, \quad \omega_r = \frac{1}{\sqrt{LC}}. \]  

This confirms the previous simplified dc analyses results where intuitive models are used [8]–[9]. There is a little discrepancy between (15) and the result of [1], which stems not from the inaccuracy of the phasor transformation but from the neglection of the high order harmonics. Recall that the only approximation used in the phasor transformation is the neglection of the dc side ac current which does not affect the fundamental components of dc or ac side signals as shown in (11b) and (12b).

**B. Variation of System Order**

Since Fig. 8 is still too complex to analyze, let’s simplify it considering the fact that the impedances of imaginary resistors are much larger than those of reactor impedances. This can be justified since the output capacitor is so large that it may govern the overall system response time and, as a result, this time is much larger than...
a switching period. Under this assumption that \( s \ll \omega_r \), it can be seen that

\[
\frac{1}{sC + j\omega_r C} = \frac{1}{j\omega_r C \left( 1 + \frac{s}{j\omega_r} \right)}
\]

\[
\approx \frac{1}{j\omega_r C \left( 1 - \frac{s}{j\omega_r} \right)} = \frac{1}{j\omega_r C} + sL \left( \frac{\omega_r}{\omega_i} \right)^2.
\]

(17)

Surprisingly the capacitor is changed to an equivalent inductor, which reduces the system order as shown in Fig. 10(a)-(b). Furthermore it is found that this can be once more simplified as shown in Fig. 10(c), where the equivalent inductor and imaginary resistors are

\[
L_{eq} = L \left( 1 + \left( \frac{\omega_r}{\omega_i} \right)^2 \right);
\]

\[
X_{eq} = \omega_r L \left( 1 - \left( \frac{\omega_r}{\omega_i} \right)^2 \right).
\]

(18)

The simplified SRC for ac analysis then can be drawn as shown in Fig. 11. The reason why the system order changes according to the switching frequency can be explained by Fig. 11 and (18).

As \( \omega_r \) deviates from \( \omega_i \), the absolute of the impedance of the inductor \( sL_{eq} \) becomes smaller than that of the imaginary resistor \( X_{eq} \) as seen from (18) since the magnitude of the switching frequency is much larger than the inverse of the system response time. This can be justified by the fact that the time constant of an output filter—the products of \( C_r \) and \( R_L \)—is several hundreds times larger than the switching period in practice. Under this condition the system becomes the first order as shown in Fig. 12.

On the other hand as \( \omega_r \) is close to \( \omega_i \), now \( X_{eq} \) becomes smaller than \( sL_{eq} \) as seen from (18). Under this condition the system becomes second order as shown in Fig. 13. Especially when \( \omega_r \) is identical to \( \omega_i \), \( X_{eq} \) becomes zero, which is just the case of quantum converter [12]-[13].

It is important to determine the boundary switching frequencies \( \omega_b \) at which the system order changes since this is essential to select a correct model between the two models of Fig. 12 and Fig. 13. Obviously the exact determination of \( \omega_b \) is impossible since the system order does not change abruptly in practice, however the rough estimation may not be impossible assuming that the equivalent LC resonant circuit impedances of Figs. 12 and 13 become identical at \( \omega_b \). Under this condition the both circuits are physically equivalent.

Since it is observed by the simulation and the experiment that \( \omega_b \) is very close to \( \omega_i \), (18) can be approximated as

\[
|X_{eq}| = \left| (\omega_r \pm \omega_d) \left( 1 - \left( \frac{\omega_r}{\omega_i \pm \omega_d} \right)^2 \right) \right| \approx 2\omega_d L;
\]

(19a)

\[
|sL_{eq}| = |s| \left| L \left( 1 - \left( \frac{\omega_r}{\omega_i \pm \omega_d} \right)^2 \right) \right| = \frac{2}{\tau_s} L;
\]

(19b)

where it is defined as

\[
\omega_b = \begin{cases} 
\omega_r + \omega_d & \text{for } \omega_r > \omega_i, \\
\omega_r - \omega_d & \text{for } \omega_i < \omega_r.
\end{cases}
\]

(19c)

\( \tau_s \) is the system time constant when it is modeled as the first order system and \( \omega_d \) is the frequency deviation from \( \omega_i \).

From the above discussion it can be concluded that

\[
|X_{eq}| = |sL_{eq}| \quad \text{or} \quad \omega_d = \frac{1}{\tau_s}.
\]

(20)

This very simple relationship shows that the boundary switching frequency is directly related with the system time constant. And it is found that the SRC is first order...
at almost every where except the very narrow frequency band around \( \omega \), since \( \tau \) is very much larger than a switching period. This is true in practice since the output filter time constant is so largely designed compared with the switching period that the output voltage ripple may be very small (about 1 percent of regulated output voltage). Equation (20) will be completed in later by the determination of the value, \( \tau \).

C. AC Analysis When \( \omega \) Deviates from \( \omega_c \)

The first order model as shown in Fig. 12 is now analyzed. Since this model is nonlinear due to the non-linear turn-ratio, \( s_2 \), the small signal perturbations in source voltage, phase, and frequency are to be applied to the first order model to linearize it. Before applying small signal perturbations let's find an output side equivalent circuit. The current \( i \) is determined from Fig. 12 and (14) as

\[
i = i_x s^+_2 = \frac{V_x s_1 - V_x s_2}{jX_{eq}} s^+_2
\]

or

\[
i = \frac{8 V_x \cos \phi}{\pi^2} \cos \frac{\theta}{\pi^2} (21a)
\]

where

\[
v_x = \frac{V_x \cos \phi}{\pi^2} (21b)
\]

Since the diode rectifier switching function, \( s_2 \), is always synchronized with the resonant tank current, \( i_t \), the rectified current \( i \) should be positive real. So (21) becomes

\[
\text{Im}(i) = 0, \quad \text{Re}(i) > 0
\]

or

\[
\frac{\theta}{\pi^2} = \begin{cases} 
\cos^{-1} \frac{v_x}{v_x \cos \phi} & \omega > \omega_c \\
-\cos^{-1} \frac{v_x}{v_x \cos \phi} & \omega < \omega_c 
\end{cases} (22b)
\]

(22) completely determines \( s_2 \) of (14). Applying (22) to (21) yields

\[
i = \frac{8 V_x \cos \phi \sin \theta}{\pi^2} (23a)
\]

or

\[
i = \frac{\sqrt{(v_x \cos \phi)^2 - v_x^2}}{X_c} (23b)
\]

where

\[
X_c = \frac{\pi^2}{8} X_{eq}. (23c)
\]

As discussed above, (23) is nonlinear functions of input/output voltages, hence perturbations are now applied:

\[
i = \frac{\partial i}{\partial v_x} + \frac{\partial i}{\partial v_0} + \frac{\partial i}{\partial \phi} + \frac{\partial i}{\partial X_c} \omega_c
\]

(24a)

where the following relations are used:

\[
I_0 = \frac{V_x}{V_0} , \quad V_0 = \frac{1}{\sqrt{1 + \left( \frac{X_c}{R_L} \right)^2}} (24b)
\]

The output voltage perturbation is determined by applying (24) to Fig. 14, as

\[
\text{Im}(i) = \frac{\omega_c}{\omega_c + \omega_2} [G_1 \hat{i}_x - G_2 \hat{\phi} - G_3 \omega_2] (25a)
\]

where

\[
G_1 = G_v = \frac{\cos \phi}{\sqrt{1 + \left( \frac{X_c}{R_L} \right)^2}}, \quad G_2 = \frac{V_x \sin \phi}{\sqrt{1 + \left( \frac{X_c}{R_L} \right)^2}}
\]

or

\[
G_3 = \frac{8 V_x L_{eq} R_L}{X_c^2} \left[ 1 + \left( \frac{R_L}{X_c} \right)^2 \right]^{1/2}.
\]

(25b)

The expressions of (25) are of explicit and simple forms, which are what we have searched for.

D. AC Analysis When \( \omega \) is Close to \( \omega_c \)

Now the second order model as shown in Fig. 13 is used for the analysis. The circuit without transformer is drawn as shown in Fig. 15 considering the fact that the turn-ratios of (14) are all real; this is justified since the \( \theta \) is zero as can be identified from (15) and (22b). The ac analysis of this circuit is then straightforward.

\[
\hat{e}_x = G_o(s) \left[ \hat{i}_x \cdot \cos \phi - V_x \sin \phi \cdot \hat{\phi} \right] (26a)
\]
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Fig. 14. Perturbed first order model.

Fig. 15. Transformerless second order model.

where

\[ G_n(s) = \frac{\omega_n^2}{s^2 + as + \omega_n^2}, \quad a = \frac{1}{C_wRL}, \quad \omega_n = \frac{2}{\pi \sqrt{LC_w}}. \]  

(26b)

The similar result has been also obtained by our previous work [13], as a quantum converter analysis. All SRC control methods such as frequency, phase and time domain ones have been analyzed by a unified principle, the phasor transformation.

IV. SIMULATIONS

Since the proposed model includes several assumptions and approximations, the verification of the model is quite essential. By comparing the exact solution with the response obtained by the proposed model this can be accomplished.

To obtain the exact solution, the following state equation for the original circuit of Fig. 1 which valids not only for CCM but also for discontinuous conduction mode (DCM) is solved numerically:

\[
\begin{bmatrix}
i_L \\
i_C \\
i_o
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{L} s_2^2(t) & -\frac{1}{L} \\
\frac{1}{C} & 0 & 0 \\
-\frac{1}{C_wRL} & 0 & \frac{1}{C} \\
\end{bmatrix}
\begin{bmatrix}
i_L \\
i_C \\
i_o
\end{bmatrix} +
\begin{bmatrix}
s_1^1(t) \\
0 \\
0
\end{bmatrix} v_i.
\]

(27a)

where

\[ s_1^1(t) = s(t), \quad s_2^2(t) = \text{sgn}(i_L) = \begin{cases}
1 & i_L > 0 \\
-1 & i_L < 0.
\end{cases} \]

(27b)

The switching function, \( s(t) \) is externally controllable, however, \( s_2^2(t) \) which represents the diode rectification is internally controlled by the inductor current. These are the same as those shown in Fig. 3.

The following parameters used in the simulations are selected:

\[ L = 78 \ \mu\text{H}, \quad C = 0.20 \ \mu\text{F}, \quad V_i = 5.0 \ \text{V}. \]

Here the frequency and the phasor perturbation simulations are done for various \( \omega, \phi, C_w, \) and \( R_L \). Large signal operation waveforms are shown in Fig. 16. The vertical scale is relative scale, whereas the horizontal scale is absolute scale. From a lot of simulations it is identified that the large signal system behaviors are nearly either first or second order as shown here. Step change in switching frequency or phase is applied to the SRC in a steady state to identify the small signal behaviors. Comparisons of the simulated waveforms (with harmonic noise) with the approximated model waveforms (without harmonic noise) are shown in Figs. 17-21 for the first order model and in Fig. 22 for the second order model.

By a lot of simulations as well as the above mentioned it is identified that the proposed models contain errors of about 15 percent when the quality factor is too small as shown in Fig. 20, or when the switching frequency is neither close to nor far from the resonant frequency as shown in Fig. 21. The proposed models, however, have no problem in practice since the quality factor is usually not smaller than unity and the switching frequency is either close to (for quantum converter) or far from (for frequency control method) the resonant frequency.

V. EXPERIMENTS

The proposed models are to now be experimentally verified for the feasibility test of the practical application. The experimental circuit is selected as a half bridge SRC instead of a full bridge one, however the analytical result of the full bridge SRC valids for the half bridge one either with slight changes in variables as shown in Fig. 23.

The parameters are

\[ L = 78 \ \mu\text{H}, \quad C = 0.24 \ \mu\text{F}, \quad V_i = 5.0 \ \text{V}, \]

\[ R_i = 0.40 \ \Omega, \quad R_Q = 0.46 \ \Omega, \quad R_D = 0.10 \ \Omega, \]

\[ L_m = 130 \ \mu\text{H}, \quad V_D = 0.60 \ \text{V}, \quad f_s = 36.8 \ \text{kHz}. \]

The circuit is constructed on a breadboard and is open loop controlled by the frequency and the phase. The typical experimental output voltage waveforms correspond to the first and the second order models are shown in Figs. 24-25, respectively. Since the second order model is well verified by the literature [12]-[13], only the first order model is extensively studied here.
Fig. 16. Large signal simulation waveform examples with zero initial conditions. (a) Inductor current. (b) Output voltage.

Fig. 17. Frequency perturbed output voltage simulation waveforms: first order response. \((\omega_c/\omega_r): 1.654 \rightarrow 1.601, \phi: 1.414\) rad.

Fig. 18. Frequency perturbed output voltage simulation waveforms: first order response. \((\omega_c/\omega_r): 2.482 \rightarrow 2.409, \phi: 0.000\) rad.

A. Time Constant Versus Switching Frequency

The time constant of the first order model can be deduced from (25b) as

\[
\tau = \frac{1}{\omega_c} = \frac{C_1 R_L}{1 + \left(\frac{R_L}{X_c}\right)^2}.
\]  

(28)

Now this is compared with the measured value for various switching frequencies as shown in Figs. 26–29. The time constant is less affected by the switching frequency...
where the load resistance is small as can be predicted by (28). A little discrepancy between the predicted and the measured is seemed to be due to the harmonics and parasitic resistances which have not been counted in the model.
D. Boundary Switching Frequency Versus Output Capacitor

The boundary switching frequency is experimentally measured by finding the critical switching frequency when the overshoot eventually appears in the system response.

Applying the approximation used in (19) to (28) and setting the system time constant of (20) be the same as the first order time constant of (28) yield the following:

\[ \omega_d = \left[ 1 + \frac{1}{4Q^2} \left( \frac{\omega_s}{\omega_d} \right)^2 \right] \frac{1}{C_o R_L} \]  \quad (29)

Unfortunately (29) is not an explicit form, hence this can not but be solved by numerical computation. It is verified that (29) is enough for the rough prediction of the boundary switching frequency as shown in Figs. 33–34. And it is also identified that the practical SRC whose \( C_o \) is normally larger than 100 \( \mu F \) is a first order system almost everywhere except the narrow switching frequency band within 2 kHz from the resonant frequency, which corresponds to at most about 5\% of the resonant frequency.
VI. Conclusion

Throughout this paper the followings are newly proposed and verified.

1) The phasor transformation is newly proposed and utilized for the dc and ac analyses of the three control methods. So explicit and very simple analytical results are deduced from the equivalent time-invariant circuits obtained by the phasor transformation.

2) The analytical results are verified by both the simulations and the experiments with good agreement with the theories. A little discrepancy between the theories and the experiments may arises no problem since this occurs at the region not used in practice. The time constant and the gain expressions have been widely explored.

3) The system is modeled as the second order when \( \omega_c \) is close to \( \omega_s \), whereas it is modeled as the first order when \( \omega_c \) deviates from \( \omega_s \). Furthermore the system order depends on load capacitor and resistor.

4) The boundary switching frequency where the system order changes is estimated and verified by experiments that the system is practically first order almost everywhere except the very narrow frequency band near \( \omega_s \).

It can be concluded that the phasor transformation is one of the basic and powerful analysis techniques for single phase ac systems, especially for the SRC.

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References


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