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Contents
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ISBN 89-952189-1-6 98060
N411 Nonlinear Vibration Analysis of Running Viscoelastic Belts  
Yeon-Sun Choi (Korea)

N429 The effect of the number of nodal diameters on non-linear interactions in two asymmetric vibration modes of a circular plate  
Won Kyoung Lee, Myeong Hwan Yeo, Sergey Borisovich Samoilenko (Korea)

N858 A study on vibration characteristics caused by backlash of gearbox in escalator with chain-sprocket drive mechanism  
Yi-Sug Kwon, Seon-Ryong Park, Jong-Ho Suh, Seong-Wook Hong, No-Gill Park (Korea)

N938 Nonlinear Dynamic Model Establishment of Deployable Missile Control Fin  
Dae-Kwan Kim, Jae-Sung Bae, In Lee, Jae-Hung Han (Korea)

N1017 Coupled Lateral and Torsional Vibration Analyses of Speed Increasing Geared Rotor-Bearing System  
An Sung Lee, Jin Woong Ha, Dong-Hoon Choi (Korea)

N308 The solution of nonlinear vibration problem containing fractional derivative  
Dong Pyo Hong, Jae Jung Lee(Korea), Jizeng Wang (China)

Session: AN-8 (2)

Machinery Health Monitoring

N371 Condition Monitoring System for the Reciprocating Compressor  
Sang-Kwon Lee, Kyung-Rae Rho, Hee-Chul Kim, Boung-Ha An (Korea)

N408 Development of a diagnostic system for a fuelling machine  
Taehwan Kim, Jangbom Chai, Soonsung Hong, Unsik Seo, Wankyu Park (Korea)

N978 Development of remote diagnostic monitoring system for motor-operated valves  
Chanwoo Lim, Shinchul Kang, Seongki Kang, Jangbom Chai (Korea)

N1109 A use of the trispectrum in the monitoring of rotating machinery  
P.R. White (UK), S.A. McLenny (USA)

N328 Damage size estimation by the continuous wavelet transform of bending wave signals  
Ik Kyu Kim, Yoon Young Kim (Korea)

N401 Preliminary test results for rotordynamic coefficients and leakage performance of floating ring seals in the high pressure turbopump  
Sung-Kwang Shin, Yong-Bok Lee, Chang-Ho Kim, Hyun-Duck Kwak, Gunhee Jang (Korea)
[N938] Nonlinear Dynamic Model Establishment of Deployable Missile Control Fin

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ABSTRACT

A deployable missile control fin has some structural nonlinearities because of the worn or loose hinge and manufacturing tolerance. The structural nonlinearity has significant effects on the static and dynamic characteristics of the control fin. Thus, it is important to establish the accurate nonlinear dynamic model of the missile control fin. In the present study, the nonlinear dynamic model of a deployable missile control fin is established. The deployable missile control fin can be subdivided into two substructures and a nonlinear hinge with structural nonlinearities. From dynamic tests, the nonlinear hinge parameters are identified and the nonlinear hinge model is established by using Force-State Mapping Technique. The substructure models and the nonlinear hinge model are coupled to establish the nonlinear dynamic model of the fin by using Component Mode Synthesis (CMS). The nonlinear dynamic model of the deployable missile control fin is verified by modal and dynamic tests. It is shown that the natural frequencies obtained by CMS and measured experimentally are in good agreements.

KEYWORDS: system identification, force-state map, nonlinear stiffness, substructure synthesis
INTRODUCTION

Deployable missile control fins have been used primarily for the efficient use of space for past several decades. Most of deployable missile control fins have a complex hinge consisting of a torsional spring, a compression spring and several stoppers. Because of a worn or loose hinge and manufacturing tolerance, the hinge of this control fin has some structural nonlinearities such as preload, free-play, asymmetric bilinear stiffness, hysteresis, and coulomb damping, and these nonlinearities exert significant effects on the static and dynamic characteristics. Therefore, it is necessary to obtain an accurate dynamic model of the deployable missile control fin.

In general, most of practical engineering structures are huge and complicated, and may have some nonlinearities. Each structure can be divided into linear or nonlinear substructures to simplify analysis. Substructure synthesis is a modeling method permitting the representation of a relatively complex structure by a reduced number of degrees of freedom. Hunn [1] introduced the first partial modal coupling method. Hury [2] assumed that the motion of each substructure could be expressed by a linear combination of rigid-body modes, constraint modes, and normal modes. Craig and Bampton [3] treated the displacements of substructures as being composed of constraint modes and normal modes. Karpel, Newman and Raveh [4-6] suggested fictitious masses loaded at the interface coordinates of a central substructure of which the connection to additional substructures is statically determinate.

The purpose of this study is to establish the dynamic model of a deployable missile control fin, shown in Fig.1, divided into the lower wing (Sub-A), the upper wing (Sub-B) and the nonlinear hinge. After investigating the existence and types of the nonlinear characteristics of the control fin from modal tests, the nonlinear hinge model of the control fin is obtained by using a system identification method, Force State Mapping Technique [7,8]. Using the substructure synthesis method, the dynamic model of the deployable missile control fin, which consists of the nonlinear hinge and two substructures, is established, and verified through modal and dynamic tests.

IDENTIFICATION OF NONLINEAR HINGE

Force-State Mapping Technique

If the structural linkage such as a joint and hinge can be completely represented by their displacement and velocity, a single degree-of-freedom (DOF) system can be expressed by the nonlinear second-order ordinary differential equation of motion as the following:

\[ f(x, \dot{x}) = C(x, \dot{x})\ddot{x} + K(x, \dot{x})x = F(t) - M\ddot{x} \]  

(1)
where $M$, $C(x,\dot{x})$, $K(x,\ddot{x})$, $F(t)$, $x$, $\dot{x}$ and $\ddot{x}$ are the mass, damping, stiffness, applied force, displacement, velocity and acceleration, respectively. The restoring force $f(x,\dot{x})$ is a function of the displacement $x$ and velocity $\dot{x}$.

The force-state mapping technique [7,8] is to plot the three-dimensional surface “force-state map” of the restoring force for the displacement and velocity. Generally, each of the nonlinear properties produces the different type of the force-state map and the restoring force can be described by the linear combination of a several number of the linear and nonlinear force components such as a preload, linear or nonlinear springs and dampers, coulomb friction, material hysteresis damping, and so on.

Fig. 1 Configuration of deployable missile control fin.

Fig. 2 Experimental setup for dynamic test.

Experiments and Results

Fig. 2 shows the experimental setup for the dynamic test of a missile control fin. A force sensor, a laser displacement sensor, and an accelerometer are located at the tip of the fin to measure the experimental data of the applied force, displacement, and acceleration. A shaker is used to apply the sinusoidal excitation force to the fin and a function generator controls the frequency and amplitude of the input signal. The missile control fin is excited at four excitation frequencies and eight amplitudes for each frequency.

Fig. 3 shows the force-state map of the nonlinear hinge of the control fin for an excitation frequency of 30 Hz. From the curved surface of the force-state map, the nonlinearities such as a preload, nonlinear stiffness with free-play and coulomb friction can be observed and the nonlinear restoring force of the hinge can be represented by
\[ f = P + C_i \dot{x} + F_c \text{sign}(\dot{x}) + F_{\text{nonlinear}} \]  

(2)

with

\[
F_{\text{nonlinear}} = \begin{cases} 
K_1(x - s_1) & x < s_1 \\
K_2(x - s_1) & s_1 < x < s_2 \\
K_3(x - s_2) + K_2(s_2 - s_1) & s_2 \leq x 
\end{cases}
\]

where \( P \), \( C_i \), \( F_c \), \( K_{1,2,3} \) and \((s_2 - s_1)\) are a preload, viscous damping, coulomb damping, stiffness with free-play and free-play, respectively.

Table 1 Estimated values of the parameters of a nonlinear hinge for four excitation frequencies.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( K_1 )</th>
<th>( K_2 )</th>
<th>( K_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( C_i )</th>
<th>( F_c )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Hz</td>
<td>365.72</td>
<td>65.62</td>
<td>436.63</td>
<td>-0.0050</td>
<td>0.0194</td>
<td>0.211</td>
<td>0.192</td>
<td>-0.185</td>
</tr>
<tr>
<td>15 Hz</td>
<td>364.42</td>
<td>56.17</td>
<td>421.84</td>
<td>-0.0050</td>
<td>0.0196</td>
<td>0.131</td>
<td>0.194</td>
<td>-0.105</td>
</tr>
<tr>
<td>30 Hz</td>
<td>366.23</td>
<td>60.31</td>
<td>410.63</td>
<td>-0.0047</td>
<td>0.0197</td>
<td>0.102</td>
<td>0.212</td>
<td>-0.112</td>
</tr>
<tr>
<td>50 Hz</td>
<td>398.33</td>
<td>62.78</td>
<td>427.26</td>
<td>-0.0042</td>
<td>0.0187</td>
<td>0.086</td>
<td>0.203</td>
<td>-0.030</td>
</tr>
<tr>
<td>Average</td>
<td>373.67</td>
<td>61.22</td>
<td>424.09</td>
<td>-0.0048</td>
<td>0.0194</td>
<td>0.132</td>
<td>0.200</td>
<td>-0.108</td>
</tr>
</tbody>
</table>

Table 1 shows the estimated values of the parameters of Eq. (2) for four excitation frequencies. The parameters, except for the viscous damping, can be assumed to have constant values over the excitation frequencies. The viscous damping decreases as the excitation frequency increases. The nonlinear hinge has the free-play of 0.024 radians (1.38 degrees) and the stiffness properties out of the
free-play are different. Fig. 4 shows that the established model of the hinge is in good agreement with the dynamic test data.

**EXPANSION OF SUBSTRUCTURE SYNTHESIS METHOD**

To couple the components of the deployable missile control fin with the hinge, the substructure synthesis [3,6] should be expanded to consider the hinge that consists of torsional springs. Generally, the moment applied to rotational displacements by a torsional spring can be expressed as follows

\[ M_\theta = K_\theta (u_i - u_j) \quad (3) \]

where \( u_{i,j} \), \( K_\theta \) and \( M_\theta \) are two independent rotational displacements, a torsional spring coefficient, and moment, respectively. If two substructures (Sub-A and Sub-B) are coupled by torsional springs located at some of the interface coordinates, the interface coordinates of each substructure can be divided into the coordinates \( \{I_p\} \) with torsional springs and the other coordinates \( \{I_a\} \) without them. Then, the generalized equations of motion of Sub-A and B can be written in the form

Sub-A:

\[
\begin{bmatrix}
m_A & m_A^{(A)} & m_A^{(A)} \\
m_A^{(A)T} & m_A^{(A)} & m_A^{(A)} \\
m_A^{(A)T} & m_A^{(A)} & m_A^{(A)} \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\xi}_A \\
\ddot{\xi}_A^{(A)} \\
\ddot{\xi}_A^{(A)} \\
\end{bmatrix}
+ \begin{bmatrix}
\omega_A^2 & 0 & 0 \\
0 & k_A^{(A)} & k_A^{(A)} \\
0 & k_A^{(A)T} & k_A^{(A)} \\
\end{bmatrix}
\begin{bmatrix}
\xi_A \\
\xi_A^{(A)} \\
\xi_A^{(A)} \\
\end{bmatrix}
= \begin{bmatrix}
F_A^{(A)} \\
F_A^{(A)} \\
F_A^{(A)} \\
\end{bmatrix}
\]

(4)

Sub-B:

\[
\begin{bmatrix}
m_B & m_B^{(B)} & m_B^{(B)} \\
m_B^{(B)T} & m_B^{(B)} & m_B^{(B)} \\
m_B^{(B)T} & m_B^{(B)} & m_B^{(B)} \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\xi}_B \\
\ddot{\xi}_B^{(B)} \\
\ddot{\xi}_B^{(B)} \\
\end{bmatrix}
+ \begin{bmatrix}
\omega_B^2 & 0 & 0 \\
0 & k_B^{(B)} & k_B^{(B)} \\
0 & k_B^{(B)T} & k_B^{(B)} \\
\end{bmatrix}
\begin{bmatrix}
\xi_B \\
\xi_B^{(B)} \\
\xi_B^{(B)} \\
\end{bmatrix}
= \begin{bmatrix}
F_B^{(B)} \\
F_B^{(B)} \\
F_B^{(B)} \\
\end{bmatrix}
\]

(5)

where the generalized mass matrix \([m]\), stiffness matrix \([k]\), displacement vector \(\{\xi\}\) and the internal force vector \(\{F\}\) are partitioned according to the interior coordinates \(\{A,B\}\) and two interface coordinates \(\{I_p,I_a\}\).

Substitution of the compatibility equations relating to the interface coordinates \(\{I_p,I_a\}\) and coupling of these equations give
where \( [K_\theta] \) is a diagonal matrix of the torsional spring coefficients according to each \( I_\nu \). The natural frequencies and eigenvectors of the combined structure with torsional springs located at interface coordinates can be easily obtained from Eq. (6).

**COUPLING AND VERIFICATION OF DEPLOYABLE MISSILE CONTROL FIN**

**Coupling of Components**

The finite element models of Sub-A and Sub-B shown in Fig. 5 are obtained from MSC/NASTRAN®. Sub-A has Tria-3 and Quad-4 elements and Sub-B has Quad-4 elements. The finite element models of Sub-A and B and the nonlinear hinge model established from the system identification are coupled by using the expanded CB method outlined in the preceding section. Fig. 5 shows a scheme of the coupling process. The lowest mode of Sub-A and the lowest six modes of Sub-B are used to represent each substructure.

![Fig. 5 Schematic diagram of Substructure A, B and Nonlinear hinge model.](image)

![Fig. 6 The first natural frequency variation in fixed-boundary condition.](image)
Verification and Results

The coupled structural model is verified by two measurements. In these verifications, the damping terms in the nonlinear hinge model, Eq. (2), are ignored for the application of Eq. (6).

The first verification is the comparison of natural frequencies calculated from Eqs. (6) for the coupled structure model with modal test results of the deployable missile control fin in the free-boundary condition. In this modal test, impulse force is applied at the lower wing. Table 2 shows the natural frequencies of the coupled structural model calculated with three estimated linear hinge stiffness properties partitioned with the free-play and corresponding measured natural frequencies. It shows the reasonable agreements in the mode frequencies except for the first mode. The first mode cannot be measured and has the largest variation with the hinge stiffness properties.

The second verification is the comparison of the first natural frequency of the coupled structure model with the modal test results for the control fin in the fixed-boundary condition. In this modal test, the lower wing of the control fin is clamped at a fixture and a series of random excitations is applied at the tip of the upper wing. Table 3 shows the first natural frequencies of the coupled structural model and corresponding measured first natural frequencies for several input levels. The measured natural frequencies are varied with the input levels as shown in Fig. 6 and the variation might be caused by the asymmetrical bilinear hinge stiffness. It is clear that the variation of measured frequencies is within the frequency range of the coupled structural model.

Table 2 Comparison of natural frequencies between synthesis and experiment.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Synthesis</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$</td>
<td>$K_2$</td>
</tr>
<tr>
<td>1</td>
<td>329.5</td>
<td>106.2</td>
</tr>
<tr>
<td>2</td>
<td>1283.0</td>
<td>1255.7</td>
</tr>
<tr>
<td>3</td>
<td>1457.3</td>
<td>1435.1</td>
</tr>
<tr>
<td>4</td>
<td>2788.3</td>
<td>2785.6</td>
</tr>
<tr>
<td>5</td>
<td>3415.7</td>
<td>3414.2</td>
</tr>
<tr>
<td>6</td>
<td>3735.5</td>
<td>3721.9</td>
</tr>
</tbody>
</table>

Table 3 First natural frequencies of the coupled structure model in fixed-boundary.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Synthesis</th>
<th>Input Level (rad./sec^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$</td>
<td>$K_2$</td>
</tr>
<tr>
<td>1</td>
<td>154.8</td>
<td>61.4</td>
</tr>
<tr>
<td>2</td>
<td>1143.6</td>
<td>1139.2</td>
</tr>
</tbody>
</table>
SUMMARY AND CONCLUSION

The reduced coupled finite element model of the deployable missile control fin with a nonlinear hinge has been established. The process of establishing the finite element model is as follows. 1) The mathematical nonlinear hinge model is constructed from dynamic tests and Force State Mapping Technique. 2) In order to consider hinge stiffness existed at the interface coordinates between the substructures, Craig-Bampton's component mode synthesis method is expanded. 3) The finite element models of the substructures and the established nonlinear hinge model are coupled by using the expanded substructure synthesis method. The coupled finite element model verified with modal tests results shows the good agreements. The coupled structural model of the deployable missile control fin is efficiently applicable to the aeroelastic analysis.

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